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Undervaluation through Foreign Reserve Accumulation

Static Losses, Dynamic Gains

Anton Korinek
Luis Serven

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Abstract

This paper shows that real exchange rate undervaluation through the accumulation of foreign reserves may improve welfare in economies with learning-by-investing externalities that arise disproportionately from the tradable sector. In the presence of targeting problems or when policy choices are restricted by multilateral agreements, first-best policies such as subsidies to capital accumulation, or subsidies to tradable production are not feasible. A neo-mercantilist policy of foreign reserve accumulation "outsources" the targeting problem or overcomes the multilateral restrictions by providing loans to foreigners that can only be used to buy up domestic tradable goods. This raises the relative price of tradable versus non-tradable goods (i.e. undervalues the real exchange rate) at the static cost of temporarily reducing tradable absorption in the domestic economy. However, since the tradable sector generates greater learning-by-investing externalities, it leads to dynamic gains in the form of higher growth. The net welfare effects of reserve accumulation depend on the balance between the static losses from lower tradable absorption versus the dynamic gains from higher growth.

This paper—a product of the Macroeconomics and Growth Team, Development Research Group—is part of a larger effort in the department to understand the determinants of economic growth. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at lserven@worldbank.org.

Undervaluation through Foreign Reserve Accumulation:
Static Losses, Dynamic Gains*

Anton Korinek
University of Maryland

Luis Servén
World Bank

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1 Introduction

Over the past decades, a number of emerging economies, notably in Asia, have experienced fierce economic growth, while also accumulating large amounts of foreign reserves.[1] These observations contrast with standard neoclassical open economy growth models in which economies with rapid productivity growth are predicted to run current account deficits so as to import capital and accelerate the buildup of the domestic capital stock (see e.g. Gourinchas and Jeanne, 2007, for a critical analysis).

The literature has proposed two main categories of explanations for these facts: First, reserve accumulation might be a form of precautionary savings to insure against future country-specific adverse shocks.[2] However, it has been difficult to reconcile the massive amounts of reserves observed in the data with realistic magnitudes of shocks that a country might want to insure against.[3]

According to a second category of explanations, much of the recent reserve accumulation in Asia results from a form of "neo-mercantilist" policy to increase net exports so as to enhance economic growth, as argued for instance by Dooley et al. (2003) or Rodrik (2008).[4] These papers argue that developing countries might enjoy learning-by-doing externalities in the spirit of Arrow (1962) and Romer (1986). A policy of fostering exports by undervaluing the real exchange rate through foreign reserve accumulation would increase domestic production and lead to dynamic welfare gains due to these externalities.

The subject of our paper is to develop a formal dynamic model of the welfare effects of real exchange rate undervaluation and to assess the desirability of reserve accumulation and real exchange rate undervaluation as a second-best instrument to internalize such learning-by-doing externalities.[5]

Our setup is based on the notion that reserve accumulation and undervaluation solve a targeting problem: In a first-best world with a full set of instruments, government would like to subsidize investment to induce agents to internalize their learning-by-doing externalities. However, if policymakers face difficulties in targeting productive investment opportunities, an alternative mechanism is needed. By lending to for-

[1]China, for example, was sitting atop of USD 2.4 trillion of official foreign reserves by early 2010 - a whopping 49% of its GDP, and experienced growth of more than 9% on average over the past decade (data from the People's Bank of China). Its performance on both measures was followed closely by other Asian tiger economies such as Taiwan and South Korea.

[2]See e.g. Aizenman and Marion (2003), Durdu et al. (2009), Mendoza et al. (2009) or Carroll and Jeanne (2009) for proponents of this view.

[3]This is discussed e.g. in Jeanne and Rancière (2009). However, see Carroll and Jeanne (2009) for a more positive assessment.

[4]Mercantilism was a widespread view among economic thinkers in Europe during the period of $1500-1750$ and is still a frequent argument in the public discourse among non-economists.

[5]For the purposes of this paper, we equate the term "reserve accumulation" to "real exchange rate undervaluation," since reserve accumulation in our model of closed capital accounts requires that more tradable goods are exported, which makes tradable goods in the domestic economy scarcer and depreciates the real exchange rate.

eigners who spend only on tradable goods, government indirectly targets the tradable sector, which generates large learning-by-investing externalities and boosts aggregate saving and investment. In a way, the government "outsources" the targeting problem to foreigners.

The difficulty of targeting policy measures at specific sectors has long been emphasized by the economic literature. First, selective subsidies pose potentially severe agency problem, as they offer ample opportunities for rent extraction.[6] Secondly, sector-specific targeting imposes vast knowledge requirements on government, which are unlikely to be met in practice. Pack and Saggi (2006) survey the literature on this topic and present a detailed list of such requirements.[7] Furthermore, WTO rules have severely curtailed the ability of developing countries to deploy sector-specific taxes and subsidies, as any such actions – if they lead directly or indirectly to expanding exports – would fall by design under restrictions on "trade-distorting interventions."[8]

Our formal model describes a small economy with two intermediate goods sectors, a tradable and a non-tradable sector. The two intermediate goods can be combined to yield a composite final good that can be used for consumption and investment. Both intermediate sectors employ two factors, labor and capital, where our measure of capital includes all factors that can be accumulated, i.e. physical as well as intangible forms, such human capital in the form of schooling or training, organizational capital, institutional capital etc. This is a common interpretation of capital in the endogenous growth literature, since the accumulation of all these factors has the potential of spillover effects. As is common in many developing economies, we assume that capital accounts are closed for private agents, and only government can trade financial assets with the outside world.

We make two crucial assumptions in our analysis: First, we assume that the economy exhibits learning-by-investing externalities, i.e. that the level of technology in the economy is proportional to the amount of capital accumulated. This implies that the economy is of the AK-type as in Romer (1989), i.e. that growth is endogenous to the economic system and can be affected by policy. For evidence on such spillover effects in developing countries see e.g. Xu and Sheng (2010). Syverson (2010) provides a more general survey.

Secondly, we assume that tradable goods are more intensive in our measure of capital than non-tradable goods, which implies that the production of tradable goods

[6]Even in economies with highly developed institutions, these concerns are of major importance, as illustrated e.g. by the large number of fraudulent schemes seeking to profit from the European Union's Common Agricultural Policy (see e.g. New York Times, Oct. 27, 2009, "Fraud Plagues Sugar Subsidy System in Europe" or New York Times, Dec. 28, 2009, "Olive Growers' Claims Prompt Investigation").

[7]Klimenko (2004) describes conditions under which even a perfectly benevolent government that attempts to target specific industries may end up inefficiently steering a country away from its long-run comparative advantage, if information is imperfect.

[8]This point is noted also by Charlton and Stiglitz (2006) and Rodrik (2009). UNCTAD (2006) describes in detail the restrictions on national policies imposed by multilateral trade agreements.

generates greater learning-by-investing externalities than non-tradable goods. Note that we expect such externalities to be of particular importance for non-physical forms of capital, such as human capital. An undervalued real exchange rate raises the price of tradable goods and – in accordance with the Stolper-Samuelson theorem – the private returns on all forms of capital that are employed relatively more intensely in the tradable sector, i.e. it moves the private returns closer to the social returns of such capital that include the learning-by-investing effects. In response to this price signal private agents increase their saving and accumulation of such capital, leading to dynamic welfare gains.

To conduct our welfare analysis, we derive a simple analytical formula for welfare that directly captures the trade-off between the static distortions that our policy measures introduce into the economy and the dynamic gains that are reaped from higher growth. This trade-off can be elegantly captured in a diagram of static allocative efficiency versus dynamic growth.

In our framework, reserve accumulation permanently removes tradable goods from the economy in order to increase the relative price of tradables. This policy creates a first-order static welfare loss every period, as real resources that could otherwise have been consumed leave the economy. On the other hand, the undervalued exchange rate entails a first-order dynamic growth benefit.

We show that the net welfare effect of reserve accumulation may be positive under certain conditions, specifically in economies in which the tradable sector is significantly more intensive in our measure of capital and which exhibit a high willingness to substitute consumption intertemporally. We term economies that fulfill these conditions *trade-dependent* economies.

We analyze our model economy in a steady state in which reserve accumulation occurs every period and reserves are never repatriated. (Decumulation of reserves would lead to the opposite effects of accumulation, i.e. it would reduce growth.) This is equivalent to throwing tradable goods into the ocean and is – of course – an extreme assumption. In practice foreign reserves yield important insurance benefits (see e.g. Jeanne and Rancière, 2009), or could be used for imports at a later stage of the country's development. In such instances, a policy of reserve accumulation would be welfare-enhancing under considerably milder conditions.

We also consider the case of an economy that obtains an exogenous supply of tradable goods in addition to the regular output from the tradable production sector. Examples include natural resource discoveries, foreign aid, or speculative capital inflows. We find that such economies exhibit a form of "Dutch disease:" a higher supply of tradable goods in the economy reduces the domestic returns to capital and decreases the economy's growth rate. For trade-dependent economies, the resulting decrease in the growth rate is so large that they are worse off as a result of the inflow of additional tradable goods. Put differently, in countries for which reserve hoarding is welfare-improving, it is also the case that (untied) foreign aid, or resource discoveries, are welfare-reducing.

After establishing our main result, we relax the severity of the targeting problem and investigate optimal government policies: We continue to assume that the government is unable to subsidize capital formation, but we suppose that it can differentially intervene in the economy's sectors to distort the economy's real exchange rate, e.g. by imposing subsidies and taxes on tradable vs. non-tradable goods, or by reallocating government spending from non-tradable to tradable goods. In each of the two cases, the static efficiency losses from the price distortion in a given period are second-order, whereas the dynamic welfare gains of the higher growth that results from real exchange rate undervaluation are first-order. If government can correctly target policy measures at individual sectors, it is therefore always optimal to implement them.

Our analysis focuses on the real rather than the nominal exchange rate. In models with sticky nominal prices or wages, such as e.g. New Keynesian models, an exchange rate devaluation temporarily reduces the real cost of consumption or production, and the resulting lower real prices raise the demand for goods or labor, resulting in higher output. However, as all nominal variables adjust to their equilibrium values, the effect fades out. Our paper, by contrast, focuses on real exchange rate undervaluation and offers a structural explanation for how such a policy can provide a persistent boost to growth.

The effects of an undervalued currency resemble in many ways those of restrictive trade policy (for a detailed discussion see e.g. Mussa, 1985). An undervalued currency simultaneously encourages the domestic production of tradables, similar to a production or export subsidy, and discourages the domestic consumption of tradables, similar to a consumption tax or import tariff. Both effects increase the current account. However, unlike restrictive trade policy, an undervalued currency does not discriminate between locally and foreign-produced tradable goods.

Literature

Our work is related to the literature on export-led growth, which has typically focused on learning and improvements in human capital, higher competition, technological spill-overs, and increasing returns to scale (see e.g. Keesing, 1967). Among the general equilibrium models that have been developed to illustrate these effects are Romer (1989), who shows that free trade can enhance growth by increasing the number of intermediate goods and Grossman and Helpman (1991) and Edwards (1992), who demonstrate that an increase in technological spill-overs through trade can raise the long-run growth rate of an economy. The mechanism through which these spill-overs take place is more or less assumed exogenously. The model we propose here, by contrast, focuses on the capital accumulation process: higher savings rates in an endogenous growth environment in the style of Romer (1986) translate into higher growth.

Our framework is closely related to the models of inter-industry spillovers familiar from the infant industry literature (Succar, 1987; Young, 1991). Such models feature industries experiencing learning externalities, at rates that may vary across industries. Our paper embeds such a framework into an otherwise standard open economy endogenous growth model. The conditions characterizing our *trade-dependent* economies can be seen as the analogous of the Mill-Bastable test that determines whether government intervention in support of infant industries is welfare-improving.[9]

More recently, Rodrik (2008) has presented empirical evidence and has developed a model of growth through exchange rate undervaluation similar to ours. Our theoretical analysis differs in two main aspects: First, Rodrik does not investigate how undervaluation can help to internalize the learning-by-investing spillovers. Instead, he assumes that the returns to capital in developing countries are artificially depressed because of difficulties in appropriability in the tradable sector, and he proposes that real exchange rate undervaluation can reduce this distortion. In other words, Rodrik assumes the tradable sector is special because it suffers more from distortions; we assume the tradable sector is special because of its learning-by-investing spillover effects. Secondly, our paper contributes a welfare analysis of the static losses versus dynamic gains that arise from exchange rate undervaluation in economies with endogenous growth. We express both in a tractable analytical formula and in an intuitive graphical diagram.

Aizenman and Lee (2008) investigate the policy implications of learning-by-doing externalities in two/three-period models. The focus of their paper is on how different forms of learning-by-doing externalities call for different first-best policy interventions. We focus instead on the benchmark Romer (1989) learning-by-investing externality and focus on second-best policy interventions in the presence of a targeting problem. We also derive quantitative welfare and policy implications.

In the empirical literature, the question whether higher exports can lead to higher growth has not been conclusively settled. Though several more recent empirical studies are available, the perhaps most telling summary of this literature is given in a survey by Giles and Williams (2000), which concludes that "it is difficult to decide for or against [the export-led growth hypothesis], as the results are conflicting." Evidence on learning-by-doing externalities associated with exporting is likewise inconclusive, owing in large part to the difficulty in disentangling productivity-based selection into exporting from true learning-by-exporting effects (Harrison and Rodriguez-Clare, 2009).[10] Given the inconclusive results in the empirical literature, our paper aims to theoretically clarify the channels through which undervaluation can increase growth and welfare so as to better guide future empirical research.

[9] The Mill-Bastable test essentially states that the discounted stream of productivity gains generated through learning should exceed the discounted cost of the government intervention required to achive the learning; see e.g. Melitz (2005) for some specific applications.

[10] Rodrik (2009), however, finds that a large (tradable) manufacturing sector leads to positive growth externalities.

2 Model Structure

Our benchmark model describes an economy with a continuum of infinitely-lived representative consumer-workers of mass 1. There are two factors, labor and capital, that are used to produce two intermediate goods, tradable and non-tradable goods T and N. The two intermediate goods in turn can be combined to yield a final consumption/investment good, which also serves as a numeraire good and which we assume cannot be traded across borders.

2.1 Representative consumer-workers

Each representative consumer-worker maximizes the present discounted value of his utility, which consists of a CRRA period utility function with intertemporal elasticity of substitution $1/\theta$, discounted at factor β. Consumer-workers inelastically supply an amount $\bar{L} = 1$ of labor at the given market wage w and rent out their capital stock K at the given gross rental rate R and experience a depreciation rate δ. They choose how much of their factor income to consume C and how much of it to invest I to maintain and augment the capital stock. Note that our measure of capital K accounts not only for physical capital but also for human capital such as education or on-the-job training, for organizational capital and for all other factors that the agent can accumulate and that yield learning-by-investing externalities.

A representative agent's optimization problem, subject to his period budget constraint, his law of motion of capital, and a transversality condition that rules out Ponzi schemes is[11]

$$\begin{aligned} \max U = \max \sum_t \beta^t \frac{C^{1-\theta}}{1-\theta} \qquad (1) \\ \text{s.t. } C + I = w + RK \\ K_{t+1} = (1-\delta) K + I \\ \lim_{t\to\infty} (1 + R - \delta)^{-t} K_t = 0 \end{aligned}$$

The Euler equation determines the consumption growth rate γ^{DE} that private agents in the economy choose,

$$\frac{C_t}{C_{t-1}} = [\beta(1 + R_t - \delta)]^{\frac{1}{\theta}} =: 1 + \gamma^{DE} \qquad (2)$$

Consumption growth is an increasing function of the return to capital. Furthermore, the effect of the interest rate is stronger the higher the agent's willingness to engage in intertemporal substitution, as expressed by the elasticity $1/\theta$.

[11]Since capital accounts in the economy are closed, private agents cannot borrow or save abroad.

2.2 Intermediate goods sectors

The tradable and non-tradable intermediate goods trade at prices p_T and p_N respectively in the domestic economy. We define the real exchange rate q as the relative price of the two

$$q = p_T/p_N \tag{3}$$

Note that an appreciation of the real exchange rate is reflected as a decrease in q.

The tradable goods sector hires capital K_T and labor L_T using a Cobb-Douglas production function F_T with capital share of α and labor-augmenting technology A_T and solves the profit maximization problem

$$\max_{K_T, L_T} p_T K_T^{\alpha} \left(A_T L_T\right)^{1-\alpha} - R K_T - w L_T$$

Similarly, the non-tradable sector rents capital K_N and hires labor L_N to produce the non-tradable good N using a Cobb-Douglas production function with capital share η and labor-augmenting technology A_N. Non-tradable firms optimize profits according to the expression

$$\max_{K_N, L_N} p_N K_N^{\eta} \left(A_N L_N\right)^{1-\eta} - R K_N - w L_N$$

We make the following assumption:

Assumption 1 *The capital share in the tradable sector is greater than in the non-tradable sector, i.e. $\alpha > \eta$.*

If the tradable sector employs relatively more capital, then it will also draw more investment and will generate greater learning-by-investing externalities, as we discuss in more detail below.

Note that our assumption on relative capital intensities is especially likely to hold given that we interpret capital more broadly than what is captured by the notion of physical capital.

By dividing the first-order conditions of both sectors with respect to capital, we obtain the following necessary condition for the capital market to be in equilibrium, i.e. for capital to earn the same returns in both sectors (see appendix for details),

$$q = \frac{p_T}{p_N} = \frac{\eta K_N^{\eta-1} \left(A_N L_N\right)^{1-\eta}}{\alpha K_T^{\alpha-1} \left(A_T L_T\right)^{1-\alpha}} \tag{4}$$

In other words, the real exchange rate has to be more depreciated (i.e. q has to be higher) the more productive capital is in the non-tradable sector compared to the tradable sector. (If the marginal productivity of capital suddenly increased in the non-tradable sector, a decline in the relative price of non-tradables would re-equilibrate capital markets.) Similarly, by combining the first-order optimality conditions on labor we obtain an equilibrium condition for labor to earn the same returns in both sectors,

$$q = \frac{p_T}{p_N} = \frac{(1-\eta)\, K_N^{\eta} A_N^{1-\eta} L_N^{-\eta}}{(1-\alpha)\, K_T^{\alpha} A_T^{1-\alpha} L_T^{-\alpha}} \tag{5}$$

According to this expression, the real exchange rate q has to be higher (i.e. more depreciated) the more productive labor is in the non-tradable sector compared to the tradable sector.

2.3 Technology

In order to endogenize the economy's growth rate, we follow Arrow (1962) and Romer (1986) in assuming that the economy exhibits aggregate learning-by-investing spillover effects. Specifically, suppose that the aggregate level of productivity in the intermediate goods sectors rises in proportion to the change in the aggregate capital stock K so that $\Delta A_T \simeq \Delta A_N \simeq \Delta K$. Appropriately normalizing the units of T and N, we write[12]

$$A_T = A_N = K \tag{6}$$

2.4 Final goods sector

The final goods sector buys tradable goods T and non-tradable goods N at prevailing market prices and assembles them into final goods Z using a Cobb-Douglas production function with a share ϕ of tradable goods and $1-\phi$ of nontradable goods, using technology A_Z,

$$Z = F_Z(T, N) = A_Z T^{\phi} N^{1-\phi} \tag{7}$$

Since good Z is the numeraire, its price is $p_Z \equiv 1$. The strategy of firms in the final goods sector is to maximize profits

$$\max_{T,N} A_Z T^{\phi} N^{1-\phi} - p_T T - p_N N \tag{8}$$

Given the Cobb-Douglas technology, firms use inputs in proportion to their relative price,

$$q = \frac{p_T}{p_N} = \frac{\phi}{1-\phi} \cdot \frac{N}{T} \tag{9}$$

This optimality condition captures that the real exchange rate reflects the relative scarcity of tradable and non-tradable goods, i.e. it depreciates (q rises or tradable goods become more expensive) the scarcer tradable goods are relative to non-tradable goods.

[12] The assumption that technology in both sectors is equally affected by the learning-by-doing externality is necessary to obtain balanced growth, i.e. to ensure that the relative size of the two intermediate goods sectors remains constant over time and does not diverge.

3 Equilibrium

In this section, we use combine the optimality conditions of firms and consumers of the previous section to solve first for the economy's decentralized equilibrium, then for the optimum that would be chosen by a social planner. As a first step, we impose the economy's market clearing conditions.

3.1 Market clearing

Market clearing in the factor markets implies

$$\begin{aligned} K_T + K_N &= K & (10) \\ L_T + L_N &= \bar{L} = 1 & (11) \end{aligned}$$

Similarly, in the non-tradable intermediate sector we require

$$N = F_N \left(K_N, L_N \right) \tag{12}$$

In the tradable sector, the market clearing condition depends on the country's current account balance. In our benchmark solution, we assume that the economy's capital account is closed and therefore the current account is in balance. Later we will generalize this result. Under a balanced current account, market clearing requires that the entire supply $F_T(\cdot)$ of tradable goods is employed in the production of final goods,[13]

$$T = F_T \left(K_T, L_T \right) \tag{13}$$

Substituting the production functions from the two market clearing conditions (12) and (13) into the optimality condition (9) for the final goods sector, we obtain

$$q = \frac{\phi}{1-\phi} \cdot \frac{K_N^{\eta} \left(A_N L_N \right)^{1-\eta}}{K_T^{\alpha} \left(A_T L_T \right)^{1-\alpha}} \tag{14}$$

3.2 Equilibrium factor allocations

Throughout most of our analysis, it will prove convenient to capture the equilibrium allocations in the economy for given technology and factor endowments by two variables, the capital ratio $\kappa = K_N/K_T$ and the labor ratio $\lambda = L_N/\not{L}_T$ that describe how factors are allocated across the two intermediate goods sectors. For any factor

[13] The given model contains only one tradable good; therefore the only motive for trade is to transfer resources intertemporally. This implies that we abstract from all trade for reasons of static comparative advantage or of varieties, and the assumption of closed capital accounts implies that no international trade takes place between the economy and the rest of the world.

ratios κ and λ, it is straightforward to use the market-clearing conditions (10) and (11) to find the sectoral factor allocations

$$K_T = \frac{1}{1+\kappa}K \qquad K_N = \frac{\kappa}{1+\kappa}K \tag{15}$$

$$L_T = \frac{1}{1+\lambda}\bar{L} \qquad L_T = \frac{\lambda}{1+\lambda}\bar{L} \tag{16}$$

To obtain the optimal values for the factor ratios κ and λ, we combine the optimality conditions for the capital market (4) and the labor market (5) each with the goods market optimality condition (14) to eliminate q. We obtain optimal capital and labor ratios of

$$\kappa^* = \frac{1-\phi}{\phi} \cdot \frac{\eta}{\alpha} \tag{17}$$

$$\lambda^* = \frac{1-\phi}{\phi} \cdot \frac{1-\eta}{1-\alpha} \tag{18}$$

As is typical for Cobb-Douglas production technologies, the optimal ratio of factor allocations to the two sectors is determined by the relative shares of the two factors in final goods production. Following assumption 1, it is easy to see that $\kappa^* < \lambda^*$, i.e. the tradable sector is relatively more capital-intensive.

3.3 Consolidated production technology

For any pair (κ, λ) we can substitute the optimal factor allocations from (15) and (16) as well as the levels of technology A_T and A_N in the tradable and non-tradable production functions $F_T(K_T, L_T)$ and $F_N(K_N, L_N)$ and assemble the two intermediate goods using the final goods production function. This yields the economy's consolidated production technology for final goods[14]

$$F_Z(T, N) = A(\kappa, \lambda) K \tag{19}$$

where the social marginal return on capital $A = A(\kappa, \lambda)$ is a function solely of the sectoral capital and labor ratios κ and λ,

$$A(\kappa, \lambda) = A_Z \left(\frac{1}{1+\kappa}\right)^{\alpha\phi} \left(\frac{1}{1+\lambda}\right)^{(1-\alpha)\phi} \left(\frac{\kappa}{1+\kappa}\right)^{\eta(1-\phi)} \left(\frac{\lambda}{1+\lambda}\right)^{(1-\eta)(1-\phi)} L^{1-\tilde{\alpha}}, \tag{20}$$

and where we denote $\tilde{\alpha} = \alpha\phi + \eta(1-\phi)$ the weighted average capital share in the economy. This reflects that the economy's aggregate production technology for final goods is of the AK-form.

[14] For details see appendix A.2.

3.4 Decentralized equilibrium

We denote the social return on capital in the decentralized equilibrium as $A^* = A(\kappa^*, \lambda^*)$. However, from the perspective of individual agents, the aggregate capital stock K, and therefore the level of technology, is exogenous. In the decentralized equilibrium, the private return on capital R equals the marginal product of capital in both intermediate goods sectors so that $R = \alpha p_T K_T^{\alpha-1}(A_T L_T)^{1-\alpha}$ in accordance with the first-order conditions on capital for tradables. We substitute the optimality condition on tradable inputs $p_T = \phi\left(\frac{N}{T}\right)^{1-\phi}$ to solve for the private return on capital

$$R = \alpha\phi A_Z T^{\phi} N^{1-\phi}/K_T = \tilde{\alpha} A^* \tag{21}$$

where the last step follows from $\alpha\phi/K_T = \alpha\phi(1+\kappa^*)/K = \tilde{\alpha}/K$.

The private marginal return on capital R captures a fraction $\tilde{\alpha}$ of the social return A^* that is precisely the weighted average capital share in the production of final goods. By extension, the learning-by-investing externality is the remainder, $(1-\tilde{\alpha})A$. It is of equal magnitude to the weighted labor share $1-\tilde{\alpha}$ in final production, since we assumed technology to be labor-augmenting. Note that both the private and the social return to capital are independent of the level of the capital stock.

Given the private return on capital R, decentralized agents pick a level of investment that implements the optimal growth rate γ^{DE} from their Euler equation (2).

3.5 Social planner

A social planner in the described economy maximizes the same objective as the representative agent in section 2.1, but internalizes the learning-by-investing externalities. This implies that he recognizes that

$$RK + wL = A^* K$$

when determining the optimal amount of capital accumulation. The social marginal return on capital consists not only of the private return $R = \tilde{\alpha} A^*$ but also of higher wage income $d(wL)/dK = (1-\tilde{\alpha})A$ that is achieved from the resultant higher level of technology. This yields the social planner's Euler equation

$$\frac{C_t}{C_{t-1}} = \left[\beta\left(1 + A^* - \delta\right)\right]^{\frac{1}{\theta}} =: 1 + \gamma^{SP} \tag{22}$$

Since decentralized agents internalize only a fraction $R = \tilde{\alpha} A^* < A^*$ of the social return to capital, it is clear that the planner's growth rate γ^{SP} is greater than the decentralized growth rate γ^{DE} of equation (2). In other words, decentralized agents invest too little and consume too much. This leads to suboptimally slow growth in the economy and creates a natural case for policy intervention, which we will discuss further in section 4.

3.6 Steady state

Economies with an AK production technology exhibit a steady state in which the interest rate is constant and the capital stock, output and consumption grow at a constant rate γ (Romer, 1986). In the decentralized equilibrium this growth rate is γ^{DE} as determined by equation (2); in the social planner's equilibrium it is γ^{SP} as given by (22). Furthermore, in both equilibria the social return on capital is $A = A^*$. In this section we describe the steady state in such an economy for a given growth rate γ and social return A.

In order to implement a growth rate of γ, investment must make up for depreciation and augment the capital stock at that rate so that

$$I = (\gamma + \delta)\, K \tag{23}$$

The remaining output will be consumed every period. Aggregate output is the product of the capital stock K times the given social return on capital A; therefore we denote consumption as

$$C = AK - I = (A - \gamma - \delta)\, K \tag{24}$$

Given an initial level of capital K_0, we express the capital stock and consumption in the economy as

$$K_t = (1+\gamma)^t\, K_0 \qquad \text{and} \qquad C_t = (A - \gamma - \delta)\,(1+\gamma)^t\, K_0$$

The evolution of the economy is therefore fully determined by the pair (A, γ). We can express welfare in the economy as a function of these two variables:[15]

$$\begin{aligned} U(\gamma, A) = & \sum \beta^t \frac{C^{1-\theta}}{1-\theta} = \sum \beta^t \frac{\left[(A-\gamma-\delta)(1+\gamma)^t K_0\right]^{1-\theta}}{1-\theta} = \\ = & \frac{1}{1-\theta} \cdot \frac{\left[(A-\gamma-\delta)K_0\right]^{1-\theta}}{1-\beta(1+\gamma)^{1-\theta}} \end{aligned} \tag{25}$$

It is clear that this expression is an increasing function of A, i.e. that welfare is higher the greater the social return on capital every period. On the other hand, the dependence of welfare on the steady state growth rate γ is non-monotonic: for a given social return A, welfare is maximized when the growth rate γ is the one chosen by the

[15] For the case of $\theta = 1$ the period utility function becomes Cobb-Douglas, and the expression for welfare is

$$\begin{aligned} U(\gamma, A) = & \sum \beta^t \log C = \sum \beta^t \left\{\log\left[(A-\gamma-\delta)K_0\right] + t\log(1+\gamma)\right\} = \\ = & \frac{\log(A-\gamma-\delta) + \log K_0}{1-\beta} + \frac{\log(1+\gamma)}{(1-\beta)^2} \end{aligned}$$

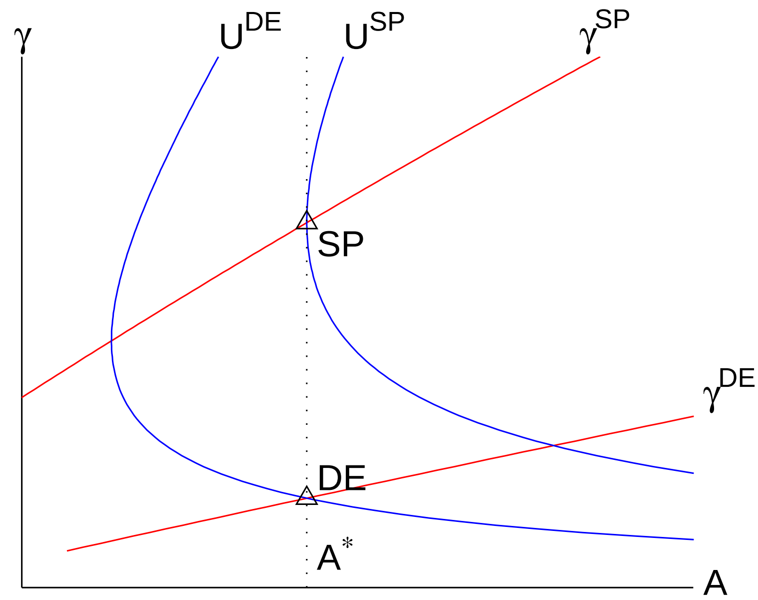

Figure 1: Iso-utility curves in (A, γ)-space

social planner (22). For lower growth rates, e.g. for the one chosen by decentralized agents according to their Euler equation (2), welfare is an increasing function of the growth rate; once the socially optimal level has been surpassed, welfare is a declining function of the growth rate.

This non-monotonic relationship stems from a trade-off between current consumption and future growth: A higher growth rate raises future consumption, which raises future welfare; this effect is captured by the denominator in (25) and is dominant for low growth rates $\gamma < \gamma^{SP}$. On the other hand, implementing a higher growth rate requires higher investment, and therefore lower levels of initial consumption, which reduces welfare; this is reflected in the numerator of expression (25) and dominates for high growth rates $\gamma > \gamma^{SP}$. The social planner chooses the optimal tradeoff between short-term consumption and long-run growth.

In figure 1 we present a diagram with the resulting iso-utility curves in the (A, γ)-space. The two upward sloping lines $\gamma^{DE}(A)$ and $\gamma^{SP}(A)$ depict the growth rates that decentralized agents and the social planner would pick for different levels of productivity A in the economy, as determined by their Euler equations (2) and (22). If we indicate the social return on capital A^* in the economy by the dotted vertical line, the decentralized equilibrium DE and the social planner's optimum SP lie at the intersections of this line with the $\gamma^{DE}(A)$ and $\gamma^{SP}(A)$ schedules.

We have also drawn iso-utility curves through these two equilibria. The level of utility in the decentralized equilibrium is below that in the social optimum, as rightward movements in the graph correspond to higher levels of utility. Note that the iso-utility curves are c-shaped, and an iso-utility curve requires the lowest social product of capital precisely at the point where the curve intersects with the social planner's $\gamma^{SP}(A)$-line. This is because the growth rate chosen by the social planner is optimal for a given level of A.

4 First-best Benchmark

The decentralized equilibrium exhibits an inefficiently low rate of investment since decentralized agents do not internalize the social returns to capital that stem from learning-by-investing externalities. In the absence of targeting problems, first-best policy responses would aim to induce decentralized agents to internalize these externalities by eliminating the wedge between the private and social returns to investment. In our model, this could be achieved through subsidies on capital holdings or on the returns to capital, an investment tax credit, or subsidies to production.

Suppose government imposes a subsidy s_K to capital holdings that is financed by a lump-sum tax T.[16] This raises the private returns to capital and therefore induces agents to save more. The agent's optimization problem can be modified accordingly by expressing his budget constraint as

$$C_t = [1 + R + s_K - \delta]\, K_t + w - K_{t+1} - T$$

This implies the Euler equation

$$1 + \gamma(s_K) = \frac{C_t}{C_{t-1}} = [\beta(1 + R + s_K - \delta)]^{\frac{1}{\theta}} \tag{26}$$

The subsidy unambiguously raises growth, since the higher returns on capital induce a substitution effect that increases capital investment, but no income effect because of the lump-sum tax. Using (23), the steady-state level of consumption can be derived as

$$C_t = (R + s_K)K_t + w - T - I_t = RK_t + w - I_t = [A^* - \gamma(s_K) - \delta]\, K_t$$

A subsidy on capital in the amount of $s_K^* = (1-\tilde{\alpha})A^*$ raises the returns on capital to the social level $R + s_K^* = \tilde{\alpha}A^* + (1-\tilde{\alpha})\, A^* = A^*$ and therefore implements the socially optimal growth rate, given by equation (22).

In our model, the following policies are equivalent to subsidies on capital accumulation itself: A subsidy on the returns to capital in the amount of $s_R = (1-\tilde{\alpha})/\tilde{\alpha}$ per dollar of interest income would raise the returns on capital to $R(1 + s_R) = \tilde{\alpha}A^*(1 + \frac{1-\tilde{\alpha}}{\tilde{\alpha}}) = A^*$. An investment tax credit $c_I = 1 - \tilde{\alpha}$ per dollar invested would lower the private cost of investment from I to $(1-c_I)I = \tilde{\alpha}I$ and would eliminate the difference between the private and social returns to capital (Saint-Paul, 1992). Similarly, a production subsidy at rate $s_Z = (1-\tilde{\alpha})/\tilde{\alpha}$ would raise the private returns on

[16] Since labor supply is inelastic in our framework, lump-sum taxes can equivalently be viewed as taxes on either wage income or consumption, both of which would be non-distortionary. To complement our analysis here, we will analyze distortionary taxation in the following section.

capital (and also labor) and would restore the socially optimal savings incentives for decentralized agents. All of these measures would push the decentralized investment rate toward the social optimum.

Proposition 1 *A subsidy s_K on holding capital, an investment tax credit c_I, or a subsidy s_Z on production increase the private return on capital and raise growth in the decentralized equilibrium. The social planner's equilibrium can be implemented by setting $s_K = (1-\tilde{\alpha})A^*$ or $s_R = (1-\tilde{\alpha})/\tilde{\alpha}$ or $c_I = 1-\tilde{\alpha}$ or $s_Z = (1-\tilde{\alpha})/\tilde{\alpha}$.*

By the same token, taxing capital, interest income, investment, or production has the opposite effects from what we just described: for example, a capital tax τ_K corresponds to a negative subsidy $s_K = -\tau_K$ in the calculation above, a tax τ_R on the returns to capital corresponds to a negative subsidy $s_K = -\tau_R R$ per unit of capital, a tax on investment is equivalent to a negative subsidy of $s_K = -A^*\tau_I$, or a production tax τ_Z is equivalent to a negative subsidy on capital of $s_K = -\tilde{\alpha}A^*\tau_Z$ plus a lump-sum tax in the amount of $-\tau_Z(1-\tilde{\alpha})A^*K$. Each of these policy measures reduces the private return on capital and the economy's growth rate:

Corollary 2 *A tax τ_K on holding capital, a tax τ_R on the returns to capital, a tax τ_I on investment, or a tax τ_Z on final goods production reduce the private return on capital and lower growth in the decentralized equilibrium of the economy.*

In figure 1 first-best policy measures can be described as a vertical movement along the A^*-line from the decentralized equilibrium DE to the social optimum SP: government revenue is raised in a non-distortionary manner so that the social productivity of capital remains constant at A^*, whereas the growth rate in the economy increases from γ^{DE} to γ^{SP}. Welfare is clearly increased.

Targeting Problem The discussed first-best policy measures assume that the government possesses very precise information, and that its institutional capacity to overcome agency problems, and prevent corruption and abuse, is similarly very high. For example, in an environment where some agents have socially wasteful investment opportunities that do not generate productive output, a general investment subsidy may be welfare-reducing because it provides incentives for such wasteful projects to be implemented. By the same token, targeting a specific sector may be difficult because it is hard for government to verify whether a given expenditure is indeed intended to create capital for the sector in question. These problem are especially severe in our framework given our broad notion of capital, which includes human capital and various other forms of intangible capital.

The targeting problem can be overcome if the private sector has superior information and plays a role in the allocation of subsidies. One such measure is to raise the domestic price of tradable goods through foreign reserve accumulation: foreigners purchase only tradable goods; therefore the policy measure targets precisely that sector. Furthermore, foreigners only spend their money on useful goods; therefore they filter out wasteful investment expenditures that do not yield any output. In the following section 5, we describe conditions under which real exchange rate undervaluation through reserve accumulation is indeed welfare-improving.

Public Capital Accumulation Let us discuss one further policy option that is sometimes proposed as a first-best measure for internalizing learning-by-investing externalities: that government makes up for the inefficiently low private level of investment through public investment in the capital stock. Assume that government invests I^G financed by lump-sum taxation, that it rents out the accumulated capital stock K^G to the intermediate goods producers at the prevailing market interest rate R, and that it transfers the resulting returns to the representative agent in lump sum fashion. For a given level of the private capital stock K, this would increase the aggregate capital stock to $K + K^G$ and would seemingly raise the economy's growth rate to the socially optimal rate γ^{SP}.

However, if we solve the decentralized agent's optimization problem augmented by this policy measure (see appendix A.3), it can be seen that the decentralized agent's Euler equation is unchanged from the one representing the no-intervention decentralized equilibrium (2). In other words, given that he internalizes only a return to capital of $R = \tilde{\alpha} A^*$, the decentralized agent does not want to see his consumption grow at a rate faster than γ^{DE}. Whenever government increases its investment by ΔI, the private agent would reduce his investment in an equal amount in order to return to his private optimum. In our framework, government accumulation of capital therefore fully crowds out private investment.[17]

These results hold for public investment in capital that aims to act as a substitute for private investment. On the other hand, if government invests in forms of capital that are complementary to private capital accumulation, such as upgrading a country's infrastructure or improving the institutional environment, then it increases the incentives for private agents to invest and mitigates the distortions in the economy that stem from the learning-by-investing spillovers.

5 Foreign Reserve Accumulation

If government cannot target subsidies directly to capital accumulation or to specific capital-intensive sectors of the economy, then foreign reserve accumulation may be a viable second-best alternative to increase returns in the tradable sector and stimulate private investment. More formally, we assume the following two restrictions on the government's set of instruments:

Restriction 1 *Government cannot distinguish profitable private investments from socially wasteful subsidy-seeking investments.*

[17] If government purchases of capital were financed by distortionary taxation, then the aggregate capital stock would actually decline, as both the income effect of future transfers from governmental capital income and the tax distortion would induce decentralized agents to invest less.

This restriction makes it impossible to subsidize capital accumulation.

Restriction 2 *Government cannot distinguish proper tradable goods from subsidy-seeking scams.*

Restriction 2 prevents the government from directly targeting subsidies to the tradable sector, which is more capital-intensive.

By accumulating foreign reserves government can "outsource" these targeting problems to foreigners. In an environment where capital accounts are closed, i.e. where private agents are not allowed to borrow or lend abroad, accumulating foreign reserves is tantamount to granting credit to foreigners to finance exports of domestic tradable goods.This reduces the quantity of tradable goods in the economy and therefore increases their price, i.e. it depreciates the real exchange rate. Since the tradable sector is more capital-intensive, a depreciated real exchange rate raises the returns to capital and provides incentives for domestic agents to raise investment closer to the socially optimal level.

Suppose that government accumulates foreign reserves by providing loans to foreigners to purchase a quantity V of tradable goods. Assume furthermore that the government revenue necessary to finance these loans is raised via lump-sum taxation.[18] Denoting the quantity of intervention as a fraction v of domestic tradable production so that $V = v \cdot F_T(\cdot)$, the market clearing condition for tradable goods (13) is modified to $T = (1-v) F_T(K_T, L_T)$. As a result, the equilibrium condition (9) for the final goods sector is divided by the factor $(1-v)$:

$$q(v) = \frac{\phi}{1-\phi} \cdot \frac{F_N(K_N, L_N)}{F_T(K_T, L_T)(1-v)} = \frac{q}{1-v}$$

Running a current account surplus appreciates the real exchange rate by making tradable goods in the economy scarcer. Using this modified final goods equilibrium condition we can express the optimal ratios of capital and labor employed in the intermediate goods sectors as

$$\kappa(v) = (1-v)\,\kappa \qquad \text{and} \qquad \lambda(v) = (1-v)\,\lambda \tag{27}$$

The two ratios decline in v as factors flow into the tradable sector so as to make up for the domestic shortage of tradable goods that results from the government's exports. In other words, the more the government raises v, the higher the demand for tradable intermediate goods and therefore the lower the fraction of both capital

[18] In practice, the accumulation of foreign reserves often occurs through "unsterilized intervention," i.e. the government finances the additional exports with newly issued domestic currency. This implies that the source of finance for V is effectively seigniorage, leading to higher inflation and distorting the level of money holdings in the economy. However, our model does not include nominal variables and we do not specify in the detail the precise source of government revenue.

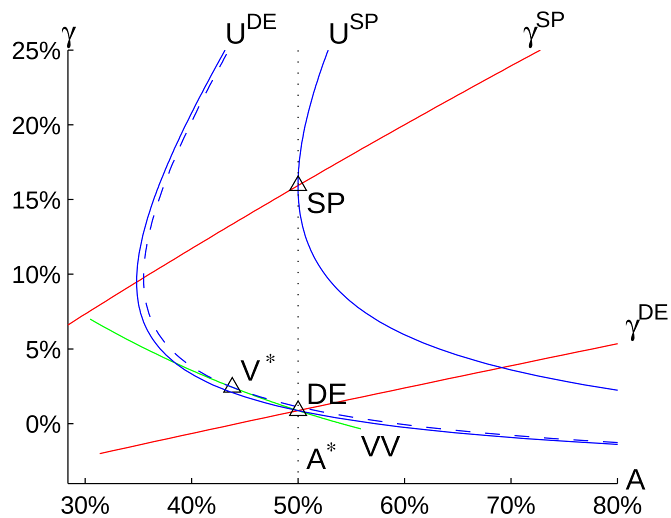

Figure 2: Effects of current account intervention

and labor allocated to the non-tradable sector. We denote the domestically available social product of capital under current account intervention as

$$A_V(v) = (1-v)^{\phi} A\left(\kappa(v), \lambda(v)\right)$$

Exporting of fraction v of tradable intermediate inputs leaves a fraction $(1-v)$ for domestic production, which entails a first-order decline in final goods production. However, the private interest rate that results from this policy is

$$R\left(v\right) = \frac{\alpha\phi + (1-v)\eta(1-\phi)}{\left(1-v\right)^{1-\phi}} \cdot A\left(\kappa(v), \lambda(v)\right)$$

The factor pre-multiplying $A(\cdot)$ in this expression captures the share of intermediate goods output that accrues to capital. As long as assumption 1 ($\alpha > \eta$) is satisfied, this factor experiences a first-order increase in accordance with the Stolper-Samuleson theorem (see appendix A.4 for a detailed derivation). The factor $A(\kappa(v), \lambda(v))$ experiences a second order decline as the intervention distorts the optimal allocation of factors into the tradable/non-tradable goods sectors. For small v, this entails a first-order increase in the interest rate and by extension in the economy's growth rate.

The total welfare effects of current account intervention depend on the relative magnitude of the dynamic welfare gain from mitigating the learning-by-doing externality and raising the growth rate compared to the static welfare loss from giving up a fraction of tradable goods that could otherwise be consumed/invested. Figure 2 depicts the effects of current account intervention graphically: increasing v corresponds to a movement upwards and to the left along the VV curve, starting from the decentralized equilibrium DE. As long as the VV curve is steeper than the agent's iso-utility curves, the static welfare loss (i.e. the movement to the left) is more than offset by the dynamic gain from higher growth (i.e. the upwards movement). The slopes of the iso-utility curve and of the VV curve in the decentralized equilibrium is reported in table 1 for different parameter values, together with a comparison of which of the two curves is steeper. The optimum amount of current account intervention can be determined as the point where the VV curve is a tangent to the representative agent's iso-utility curves, as indicated by the point V^* in the figure.

Analytically, we determine the slope of representative agent's indifference curves in the decentralized equilibrium by implicitly differentiating the agent's welfare function (25) for constant welfare $\bar{U}$,

$$\left.\frac{d\gamma}{dA}\right|_{\bar{U}} = -\frac{1-\beta\,(1+\gamma)^{1-\theta}}{\beta\,(1+A-\delta)\,(1+\gamma)^{-\theta}-1}$$

Similarly, the output/growth trade-off of current account intervention is captured by the slope of the VV locus (see appendix for details),

$$\left.\frac{d\gamma}{dA_V(v)}\right|_{v=0} = \left.\frac{d\gamma/dv}{dA_V(v)/dv}\right|_{v=0} = -\frac{\beta^{\frac{1}{\theta}}(1-\phi)(\tilde{\alpha}-\eta)}{\phi\theta(1+R-\delta)^{\frac{\theta-1}{\theta}}}$$

Definition 3 (Trade-dependent economy) *We call an economy* trade-dependent *if the dynamic growth benefit from removing tradable goods from the economy is larger than the static welfare loss in the decentralized equilibrium.*

This is the case whenever the representative agent's indifference curve in the decentralized equilibrium is more negatively sloped than the output/growth trade-off locus of current account intervention, or analytically

$$\left.\frac{d\gamma_V}{dA}\right|_{\bar{U}} > \left.\frac{d\gamma}{dA}\right|_{VV}$$

Proposition 4 *A small current account surplus of amount V depreciates the real exchange rate q, raises the private returns on capital R and increases the growth rate γ in the economy. If the economy is trade-dependent (definition 3), this raises welfare.*

Table 1 illustrates the desirability of current account intervention for a wide range of parameters. The first six columns report the parameter values used for the economy, where we target A^* by adjusting A_Z appropriately. The next three columns report the resulting aggregate capital share $\tilde{\alpha}$ as well as the growth rates in the economy's decentralized equilibrium and the social optimum. The last three columns report the marginal dynamic utility gain from increasing the growth rate by raising the amount of intervention $MU_{\gamma(v)}$, the marginal static utility loss from the resource loss that occurs when v is increased $MU_{A(v)}$, and the optimal v^*, if available. (If the static loss from undervaluation already exceeds the dynamic gain in the decentralized equilibrium, then no welfare-improving amount of reserve accumulation $v > 0$ is available.)

ϕ	α	η	A^*	β	θ	$\tilde{\alpha}$	γ^{DE}	$MU_{\gamma(v)}$	$MU_{A(v)}$	v^*
Benchmark economy										
0.4	0.8	0.3	0.4	0.96	1	0.50	7.5%	22.08	-16.34	0.28
0.4	0.8	0.6	0.4	0.96	1	0.68	14.4%	7.44	-22.77	.
0.4	0.4	0.3	0.4	0.96	1	0.34	1.4%	4.93	-13.06	.
0.4	0.9	0.5	0.4	0.96	1	0.66	13.7%	15.24	-21.82	.
0.4	0.5	0.2	0.4	0.96	1	0.32	0.6%	14.97	-12.74	0.22
Relatively closed economy										
0.2	0.8	0.3	0.4	0.96	1	0.40	3.7%	15.81	-7.06	0.68
0.2	0.8	0.6	0.4	0.96	1	0.64	12.9%	5.19	-10.47	.
0.2	0.4	0.3	0.4	0.96	1	0.32	0.6%	3.33	-6.37	.
0.2	0.9	0.5	0.4	0.96	1	0.58	10.6%	11.02	-9.34	0.27
0.2	0.5	0.2	0.4	0.96	1	0.26	-1.7%	10.35	-5.94	0.68
Patient economy										
0.4	0.8	0.3	0.4	0.975	1	0.50	9.2%	60.36	-28.07	0.44
0.4	0.8	0.6	0.4	0.975	1	0.68	16.2%	21.03	-40.56	.
0.4	0.4	0.3	0.4	0.975	1	0.34	3.0%	13.26	-22.04	.
0.4	0.9	0.5	0.4	0.975	1	0.66	15.4%	42.85	-38.65	0.08
0.4	0.5	0.2	0.4	0.975	1	0.32	2.2%	40.20	-21.46	0.51
Lower elasticity of substitution economy										
0.4	0.8	0.3	0.4	0.96	2	0.50	3.7%	9.00	-26.92	.
0.4	0.8	0.6	0.4	0.96	2	0.68	7.0%	1.40	-24.90	.
0.4	0.4	0.3	0.4	0.96	2	0.34	0.7%	5.39	-35.06	.
0.4	0.9	0.5	0.4	0.96	2	0.66	6.6%	3.11	-24.94	.
0.4	0.5	0.2	0.4	0.96	2	0.32	0.3%	19.34	-37.12	.

Table 1: Optimal reserve accumulation for selected parameter values

The table is split into four blocks, each of which consists of five lines. The five lines within each block reflect alternative values of the capital intensities α and η in the two intermediate goods sectors. The first line assumes an economy with a non-tradable sector of standard capital intensity of 0.3, whereas the tradable sector is considerably more capital-intensive, .8, reflecting e.g. a higher importance of human capital in that sector. In lines 2 and 3, we narrow the gap between the tradable and non-tradable capital intensities by raising the non-tradable capital intensity/reducing the tradable capital intensity. Lines 4 and 5 increase/lower the capital intensities in both sectors in tandem. The resulting aggregate capital share in the economy is reflected by $\tilde{\alpha}$; it has an important impact on the growth rate γ^{DE} in the decentralized equilibrium as agents internalize a smaller or larger share of the social product of capital A^*.

Block 1 contains what we employ as our benchmark economy, reflecting a size of the tradable sector of .4, which is a standard value in the literature (see e.g Mendoza, 2005). We chose a value of $A^* = 0.4$ for the social product of capital to obtain growth rates in the decentralized equilibrium that vary within a reasonable range of values observed in developing countries, from 0% to 14%. In our benchmark configuration, the table shows that the dynamic gain $MU_{\gamma(v)}$ exceeds the static loss $MU_{A(v)}$ of reserve accumulation only in lines 1 and 5, i.e. for economies with capital shares that differ significantly between tradable and non-tradable sectors but are relatively low $\tilde{\alpha} < .5$ in aggregate. The relatively low aggregate capital share implies that there are significant learning-by-investing externalities; the large difference in capital intensities implies that policy measures that affect the real exchange rate have a large Stolper-Samuelson effect on the returns to capital.

In block 2 we represent a relatively closed economy with a value share of tradables of $\phi = .2$. This significantly lowers the cost of intervention, as a given amount of tradables exported can achieve a larger movement of the real exchange rate in the domestic economy. As a result intervention is also desirable for the economy with parameters in line 4, in which the aggregate capital share is relatively high.

Block 3 repeats our calculations for an economy that is more patient than the benchmark, with $\beta = .975$. Since agents discount the future at a lower rate, the dynamic utility gains from higher growth $MU_{\gamma(v)}$ increase significantly. (Since $MU_{A(v)}$ captures the present discounted value of all future static losses, it also rises in absolute value, but by less than the dynamic utility gain.) Foreign reserve accumulation is desirable under the same circumstances as in the relatively closed economy.

Finally, block 4 captures an economy with a lower intertemporal elasticity of substitution $\theta = 2$. This reduces the dynamic gains from future growth significantly, as agents are less willing to substitute current for future consumption. Current account intervention is undesirable in all five lines of this block. One interpretation of this result is that exchange rate undervaluation is most desirable in countries where policymakers place an important weight on future growth and are highly willing to give up current consumption for this goal.

Our quantitative results are derived under the assumption that tradable goods are permanently removed from the economy and provide no future benefit to domestic agents. This is of course an extreme assumption. In particular, if the country also derives insurance benefits from holding reserves, or if the learning-by-investing externalities cease at some point, say when the economy has reached the world technology frontier (Acemoglu et al., 2006), then reserves can be repatriated without jeopardizing future growth and yield significant utility benefits that we have not captured in our specification.[19] Under such circumstances, undervaluation through reserve accumulation is *a fortiori* desirable under much weaker conditions than what we found above in table 1.

Relationship to Trade Policy Measures

The role of current account intervention in our setup is to induce foreigners to remove tradable goods from the domestic economy so as to push up their relative price and increase the return on capital, thereby solving a targeting problem. Foreigners ensure that only firms that indeed produce useful tradable goods and that are productive enough to export (see e.g. Melitz, 2003) will benefit from the sectoral subsidy created by current account intervention. This avoids the selection problems that arise for most government policies targeted at a specific sector.

However, the growth channel through which current account intervention affects welfare in our setup is fundamentally distinct from standard channels of restrictive trade policy. Import tariffs, for example, take advantage of a country's monopsony power; they aim to push down the relative world market price of a country's imports in terms of its exports so as to improve the country's terms of trade. In doing so they discriminate between domestically and foreign produced tradable goods. By contrast, in our setup there is no role for terms of trade effects, since there is a single homogenous tradable good.[20]

If the economy under consideration was large compared to the rest of the world, the only international price that reserve accumulation in our model would affect is the international interest rate, since the domestic government increases the supply of credit to the world economy. However, we assumed the economy is small compared to the rest of the world and takes the world interest rate as given.

[19] If the learning-by-doing externality is still active when reserves are decumulated and tradable goods are repatriated, this would trigger the same effects that we described in reverse: importing large amounts of tradable goods would appreciate the real exchange rate, depress the domestic interest rate and reduce capital accumulation and growth.

[20] More generally, if there were multiple tradable goods in the world economy and the country under consideration had market power over its exports, export subsidies would lower the world market price of the country's exports and would *deteriorate* the domestic country's terms of trade. While this would raise aggregate welfare in importing countries, it typically draws heavy criticism from those sectors in importing countries that are hurt by the measure. These open economy considerations are not the focus of the present paper.

5.1 Capital account openness and capital flows

So far we have assumed that the capital account of the economy under consideration is closed for all agents except the government. We abstracted from all forms of private intertemporal trade.

Let us now investigate whether and under what circumstances capital account restrictions are indeed desirable in our model. Assume that the economy described so far is a small open economy that faces an exogenous world interest rate of r^w, and let us analyze the role of capital flows in this context.

In our model economy with learning-by-doing externalities, we discussed that the aggregate prodution technology is A^*K, i.e. linear in the economy's capital stock. This implies that the normal equilibrating mechanism that results from decreasing returns to scale (i.e. that the rate of return is a decreasing function of the capital stock) is not present. However, there is a different equilibrating mechanism at work: when the world interest rate is higher (lower) than the interest rate faced by decentralized agents in the small open economy, they have incentive to lend (borrow) abroad by exporting (importing) tradable intermediate goods. As the quantity of tradable goods in the economy in a given period shrinks (rises), the same forces that we discussed above will raise (lower) the returns to capital.

More specifically, assume that the interest rate in the domestic economy is above the world interest rate; this implies that tradable goods will flow into the economy; the relative price of tradable goods will fall; since tradable goods are more capital-intensive, a Stolper-Samuelson-like effect will entail a decline in the interest rate. This process will take place until domestic and foreign interest rates are equalized. Given constant parameter values, note that capital market equilibrium in such an economy would require a persistent trade deficit to equalize rates of return. At some point it is likely that borrowing constraints on the domestic economy become binding and/or lead to an increase in domestic interest rates and the flow of resources stops.

Depending on the level of the world interest rate r^w, we distinguish three cases:

Case 1: $r^w \geq A^* - \delta$ If the world rate of interest is greater than the economy's social return on capital net of depreciation, then it would be optimal for the economy to lend abroad, i.e. to export tradable goods. The reduction in the domestic availability of tradable goods will raise the domestic return on capital until the return differential vanishes. Note that this case is rather unlikely, since A^* captures the social returns on capital, i.e. both wages and the private return on capital, whereas the private interest rate r^w captures only private returns on capital.

Case 2: $A^* - \delta > r^w \geq \tilde{\alpha}A^* - \delta$ If capital flows were deregulated, decentralized agents would export capital so long as the private net return on capital $R - \delta = \tilde{\alpha}A^* - \delta$ is less than the world interest rate r^w. Given assumption 1, the resulting outflow of tradable goods would increase the private return on capital until equilibrium is obtained when $R - \delta = r^w$. However, since decentralized agents internalize only the private returns to capital and not the increased wage earnings obtained from a higher capital stock, the resulting capital outflow is socially inefficient and reduces the economy's welfare. A social planner would recognize that the social return on domestic capital is in fact higher than the world interest rate. This would create a strong rationale for keeping the economy's capital accounts closed for private agents so as to restrict capital flight.

Case 3: $\tilde{\alpha} A^* - \delta > r^w$ Finally, if the world interest rate is less than the economy's private net return on capital, decentralized agents would want to borrow abroad and increase their capital stock. Any capital inflow would entail not only higher earnings on capital, but via the learning-by-investing externality it would also raise wages.

5.2 Resource curse

Our model is also well suited to study the effects of exogenous changes in the domestic supply of tradable goods, such as what is captured by the so-called resource curse, aid curse, or by a surge in private capital flows into the country. Analytically, an exogenous increase in the supply of tradables represents the direct opposite of a government policy of reserve accumulation, as analyzed earlier in section 5. We can interpret a sudden discovery of resources or a surge in aid inflows as an additional exogenous supply of tradable goods $\bar{T}$, which implies that the market clearing condition for tradable intermediate goods becomes $T = \bar{T} + F_T(K_T, L_T)$.

The situation can be analyzed in terms of the model laid out earlier in this section if we set $v = -\bar{T}/F_T$. In particular, an exogenous inflow of tradable resources will entail a static welfare gain for the economy as the total amount of resources available rises, but the economy will suffer a dynamic loss as the increased supply of tradable intermediate goods reduces the domestic return on capital and pushes factors into the production of non-tradables. (Note that what matters for this conclusion is not the capital intensity of natural resources, but the capital intensity of those tradable goods that were previously domestically produced and are imported after the discovery of natural resources.)

Corollary 5 *In an trade-dependent economy, a small exogenous inflow $\bar{T}$ of tradable resources unambiguously reduces welfare.*

A direct implication of this finding is that if reserve accumulation improves welfare in a given economy, then untied foreign aid unambiguously reduces welfare.

6 Sector-Specific Interventions

As the institutional capacities of a government develop, its ability to implement well-targeted taxes and subsidies may improve. In particular, it is common practice among industrialized countries that specific government policies are targeted at specific sectors. This section investigates the scope for second-best government intervention if we continue to assume government faces restriction 1 (on targeting investment), but drop restriction 2 (on targeting specific sectors).

This opens the possibility for government to engage in second-best policies that share the following feature: they raise the private returns to capital R and induce decentralized agents to invest more, which increases the economy's growth rate γ above γ^{DE} and leads to a first order welfare gain. They do so at the cost of introducing a distortion into the economy's factor allocation that reduces the social product of capital A below the optimum level A^*. In figure 3 this corresponds to a movement upward and to the left of the decentralized equilibrium DE along the TT curve.

Such policies raise welfare as long as the dynamic welfare gain from higher growth justifies the static loss in productivity, i.e. as long as $d\gamma/dA|_{TT}$ along the second-best frontier is steeper than the slope of the indifference curve $d\gamma/dA|_U$ at a given point (γ, A). The optimum level of government intervention is is reached at the point of the TT curve where the two slopes coincide, i.e. where the respective iso-utility curve forms a tangent to the TT locus. In the following we apply this principle to a range of second-best government policies.

6.1 Differential taxation of intermediate goods

We start by assuming that government levies differential taxes/subsidies (τ_T, τ_N) on the purchase of tradable and non-tradable intermediate goods for final production, where subsidies are represented by negative tax rates. The optimization problem of final goods producers can be expressed as

$$\max_{T,N} A_Z T^{\phi} N^{1-\phi} - (1+\tau_T)\, p_T T - (1+\tau_N)\, p_N N$$

which yields the two first-order conditions

$$\text{FOC}(T): \phi A_Z \left(\frac{N}{T}\right)^{1-\phi} = (1+\tau_T) p_T \tag{28}$$

$$\text{FOC}(N): (1-\phi) A_Z \left(\frac{T}{N}\right)^{\phi} = (1+\tau_N) p_N$$

Dividing the two first-order condition, we obtain the equilibrium condition (FF) for the final goods sector, which – in the case of differential taxation of intermediate goods – reads as

$$q = \frac{p_T}{p_N} = \frac{\phi}{1-\phi} \cdot \frac{1+\tau_N}{1+\tau_T} \cdot \frac{N}{T} \tag{29}$$

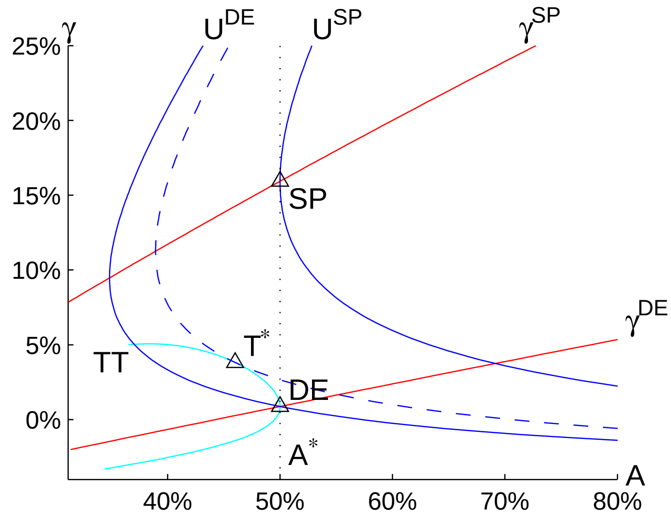

Figure 3: Iso-utility curves in (A, γ)-space

The optimization problem of intermediate goods producers is unaffected; hence the equilibrium conditions (RR) and (ww) for factor markets remain unchanged. We can combine these with the modified final goods market condition (29) to find that the effect of intermediate goods taxation on the capital and labor ratios of the two sectors is

$$\kappa(\tau_T, \tau_N) = \frac{1+\tau_T}{1+\tau_N} \cdot \kappa^* \qquad \text{and} \qquad \lambda(\tau_T, \tau_N) = \frac{1+\tau_T}{1+\tau_N} \cdot \lambda^*$$

Naturally, the greater the tax $\tau_T > 0$ on tradable goods relative to the tax on nontradables goods τ_N, the higher the fraction of factors allocated to the non-tradable sector. By the same token, the greater (in absolute value) the subsidy $\tau_T < 0$ to the tradable sector, the higher the fraction of factors allocated to that sector. Since the sectoral factor allocations κ^* and λ^* in the decentralized equilibrium were socially optimal, reallocating them through tax policy introduces a second-order distortion into the economy that lowers the aggregate social return on capital to[21]

$$A_\tau = A(\kappa(\tau_T, \tau_N), \lambda(\tau_T, \tau_N)) \leq A^*$$

with strict inequality whenever $\tau_T \neq \tau_N$. The resulting interest rate is

$$R(\tau_T, \tau_N) = \frac{\alpha\phi\left[1+\kappa(\tau_T, \tau_N)\right]}{1+\tau_T} \cdot A_\tau = \left[\frac{\alpha\phi}{1+\tau_T} + \frac{\eta(1-\phi)}{1+\tau_N}\right] \cdot A_\tau \qquad (30)$$

A tax (subsidy) on either intermediate goods sector lowers (raises) the returns to capital. The return to capital R in this expression consists of the sum of the returns

[21] This follows directly from the envelope theorem: since κ^* and λ^* were chosen to maximize $A(\kappa, \lambda)$, small changes to the two parameters entail only second-order deviations from A^*.

to capital in the tradable and in the non-tradable sector, where the relative weights $\alpha\phi$ and $\eta(1-\phi)$ reflect the shares of tradable capital and non-tradable capital in final goods production (α is the capital share in tradable goods and ϕ is the share of tradables in final goods, and similarly for non-tradable goods).

If the two tax rates (subsidies) are identical $\tau_T = \tau_N$, then the measure is equivalent to a general tax τ_Z (or subsidy s_Z) on production and the condition collapses to $R = \tilde{\alpha}A^*/(1+\tau_Z)$, as discussed in section 4 on first-best policy measures. In general, such policy measures require that government can rebate the tax revenue (or raise the revenue required for the subsidy) in a lump-sum fashion. In the following we analyze the potential to manipulate the relative price of intermediate goods through a revenue-neutral pair of taxes/subsidies on intermediate goods.

Revenue-neutral taxes/subsidies on intermediate goods

Definition 6 *A pair of sectoral taxes/subsidies* (τ_T, τ_N) *on intermediate goods is revenue-neutral if*

$$\tau_T p_T T + \tau_N p_N N = 0 \qquad \mathit{or} \qquad \frac{\tau_N}{\tau_T} = -\frac{p_T T}{p_N N} \tag{31}$$

Each revenue-neutral pair (τ_T, τ_N) defines a unique wedge between the prices of tradable and non-tradable goods in expression (29). Furthermore, we find:

Lemma 7 *Any pair* $(\hat{\tau}_T, \hat{\tau}_N)$ *that does not satisfy restriction (31) can equivalently be represented as a revenue-neutral pair* (τ_T, τ_N) *together with a uniform tax/subsidy on final goods production* τ_Z*.*

The economic effects of τ_Z have already been analyzed in corollary 2 in the section on first-best policy measures.

Since the value shares of the two intermediate goods entering final goods production is constant, a revenue-neutral pair (τ_T, τ_N) satisfies (see appendix A.5)

$$\tau_N = -\frac{\tau_T \phi}{1-\phi+\tau_T} \tag{32}$$

In other words, picking a positive subsidy $-\tau_T$ defines a unique tax τ_N such that the measure is revenue-neutral and vice versa.[22]

The effects of such a measure on the private interest rate R and by extension on growth are described by the following proposition (see appendix A.5 for a proof):

Proposition 8 *A small revenue-neutral pair* (τ_T, τ_N) *of subsidies on tradable goods* $\tau_T < 0$ *and taxes on non-tradable goods* $\tau_N > 0$ *raises the private interest rate and stimulates growth if and only if assumption 1 is satisified, i.e. if* $\alpha > \eta$*.*

[22]This holds as long as the subsidies satisfy $\tau_T > -(1-\phi)$ or $\tau_N > -\phi$ respectively. Subsidies that violate these conditions are too expensive to be financed by taxes levied exclusively on the other sector.

ϕ	α	η	A^*	γ^{DE}	τ_T^*	τ_N^*	$\Delta A/A$	γ_τ^*	ΔU
0.4	0.8	0.3	0.4	3.69%	-0.20	0.20	-1.50%	4.46%	3.92%
0.4	0.8	0.6	0.4	6.97%	-0.04	0.03	-0.05%	7.03%	0.13%
0.4	0.4	0.3	0.4	0.69%	-0.08	0.06	-0.24%	0.74%	0.54%
0.4	0.9	0.5	0.4	6.61%	-0.10	0.08	-0.33%	6.89%	0.70%
0.4	0.5	0.2	0.4	0.3%	-0.21	0.22	-2.03%	0.78%	5.99%

Table 2: Sectoral tax/subsidy measures for different parameter values

The different capital intensities among the two sectors imply that the subsidy to tradable goods falls relatively more on capital, whereas the tax on non-tradables falls relatively more on labor. In other words, the policy represents a redistribution from labor to capital.The proposition is a version of the Stolper and Samuelson (1941) theorem: manipulating the relative price of the capital-intensive versus the labor-intensive good moves the relative return to capital compared to labor in the same direction. In accordance with the Euler equation of decentralized agents (2), a higher private interest rate R raises the private savings rate and therefore the growth rate of the economy. Since the decentralized savings and growth rates were suboptimally low, increasing them entails a first-order dynamic welfare gain.

Table 2 reports the optimal pair of second-best taxes/subsidies on intermediate goods for the same parameter values as in a block of table 1. For each of the cases, we report the optimal pair of subsidies and taxes (τ_T^*, τ_N^*), where negative values represent subsidies, the percentage decline in the social product of capital $\Delta A/A$, as well as the growth rate under the specified optimal policy. The last column contains the increase in welfare ΔU in terms of the equivalent permanent increase in consumption that results from the optimal policy.

Row 1 represents an economy in which the capital intensity of the two sectors reflects our benchmark case. Given the relatively large difference in capital intensities, a set of subsidies and taxes in the amount of 20% each is called for, and this raises the growth rate in the economy by a third of a percentage point while reducing the social product of capital by 1.5%. The equivalent increase in welfare is close to 4%.

From the remaining examples in the table we can see that a greater difference in capital intensity among the two sectors makes the optimum size of policy intervention larger, whereas a larger aggregate capital share in the economy reduces optimal policy measures, as the economy is already closer to the first best.

We illustrate our findings graphically in figure 3: a subsidy on tradable relative to non-tradable goods moves the decentralized equilibrium along the second-best frontier TT up and to the left. The dynamic growth effect (i.e. the upward movement) has first-order positive welfare effects, since the decentralized equilibrium exhibits a socially inefficient growth rate. The distortion to the sectoral factor allocation that reduces the social product of capital (i.e. the movement to the left) has a second-order welfare cost, since the decentralized equilibrium was characterized by the socially optimal factor allocation between the two sectors. By implication, the policy is unambiguously welfare-improving for small tax rates. The point marked by T^* indicates the optimal level of subsidies/taxes in the given example, which can be found as the tangency point of the TT locus with the representative agent's indifference curves. We have drawn the indifference curve going through this point as a dotted line.

Figure 3 also illustrates the findings of Rodrik (2008), who argues that developing countries suffer from distortions in the appropriability of returns, which are particularly pronounced in the tradable sector. He models these distortions as a tax that discriminates against the tradable sector and suboptimally shifts the economy's factor allocation towards non-tradables. In the figure this would be reflected as a move along the lower arm of the second-best frontier TT moving down from the decentralized equilibrium DE. Undoing this distortion by raising the relative price of tradables (i.e. depreciating the real exchange rate) can restore the decentralized equilibrium DE and increase welfare because it both improves the sectoral factor allocation and raises the growth rate by increasing the private return on capital.

While the analysis of Rodrik (2008) addresses the appropriability problem in the tradable sector, he remains silent on how policy action can induce agents to internalize the learning-by-investing externality that is present in both his and our framework. Addressing this externality is the only way to move the economy closer to the first-best equilibrium SP that would be chosen by a social planner.

A subsidy $-\hat{\tau}_T$ on tradable goods that is financed by a distortionary tax on general output τ_Z is equivalent to a revenue-neutral pair of taxes (τ_T, τ_N) where $1 + \tau_T = (1 + \hat{\tau}_T) / (1 + \tau_Z)$ and $\tau_N = \tau_Z$. Following the argument of proposition 8, we find the following:

Corollary 9 *A subsidy on tradable goods that is financed by a general tax τ_Z will raise the returns to capital and increase growth if and only if $\alpha > \eta$.*

6.2 Composition of government spending

Another way of influencing the real exchange rate and thereby affecting the relative return to capital is through the composition of government spending. Assume that the government purchases the amounts G_T and G_N of tradable and non-tradable goods at prevailing market prices every period and employs them to produce a public good G using a production function

$$G = F_G (G_T, G_N)$$

In order to keep our focus strictly on the effects of reallocations in government spending, we assume that government needs to provide a fixed amount of public spending $G = \bar{G}$ to keep the economy running, but any spending beyond this threshold has no effects on welfare. Furthermore, we assume that government revenue is raised via lump-sum taxation. (We have already discussed the effects of distortionary output taxation in corollary 2.)

Let us define a frontier GG of factor inputs (G_T, G_N) that satisfies the required level of government spending so that $F_G(G_T, G_N) = \bar{G}$. By reallocating governmental demand for intermediate inputs from non-tradable towards tradable goods, the government can influcence the real exchange rate, the private return to capital, and growth. However, such reallocations are costly as they involve deviations from the bundle of inputs that minimizes the cost of public goods provision.

Analytically, we define the fractions of tradable and non-tradable production absorbed by the government as $g_T = G_T/F_T(\cdot)$ and $g_N = G_N/F_N(\cdot)$. Market clearing in the two intermediate goods sectors implies that only the fractions $(1-g_T)F_T(K_T, L_T)$ and $(1-g_N)F_N(K_N, L_N)$ are available for production of the private final good Z. The resulting equlibrium condition in the final goods sector is

$$q = \frac{\phi}{1-\phi} \cdot \frac{1-g_N}{1-g_T} \cdot \frac{F_N(K_N, L_N)}{F_T(K_T, L_T)} \tag{FF$_G$}$$

The ratios of capital and labor inputs into the two sectors are

$$\kappa_G(g_T, g_N) = \frac{1-g_T}{1-g_N} \cdot \kappa^* \qquad \text{and} \qquad \lambda_G(g_T, g_N) = \frac{1-g_T}{1-g_N} \cdot \lambda^* \tag{33}$$

The more government shifts its absorption of intermediate goods towards one sector, the more production factors flow into that sector. The resulting level of private final goods production is

$$A_G(g_T, g_N)K = (1-g_T)^{\phi}(1-g_N)^{1-\phi} A\left(\kappa_G(g_T, g_N), \lambda_G(g_T, g_N)\right) K$$

Assume that from a static point of view, the optimal allocation of intermediate goods between government absorption and final goods production is captured by the pair

$$(g_T^*, g_N^*) = \arg\max A_G(g_T, g_N) \qquad \text{s.t.} \qquad F_G(g_T F_T(\cdot), g_N F_N(\cdot)) = \bar{G}$$

In other words, in the absence of the dynamic externality, the amounts $g_T^* F_T(\cdot)$ and $g_N^* F_N(\cdot)$ of intermediate goods would be the cheapest way for government to produce the required level of spending $\bar{G}$. If the government increases its absorption of tradable goods by moving along its factor input frontier GG, more capital and labor is allocated to the tradable sector, i.e. κ and λ rise. Substituting expressions (33) in the tradable sector's first-order condition on capital (35), the private return to capital is

$$R = \left[\alpha\phi\left(\frac{1-g_N}{1-g_T}\right)^{\phi} + \eta(1-\phi)\left(\frac{1-g_T}{1-g_N}\right)^{1-\phi}\right] \cdot A\left(\kappa_G, \lambda_G\right)$$

The term $\frac{1-g_N}{1-g_T}$ captures the Stolper-Samuelson effect, i.e. that higher demand for the capital-intensive good causes a first-order rise in the rate of return on capital,

ϕ	α	η	$\bar{G}$	γ^{DE}	g_T	g_N	ΔA_G	γ_G	ΔU
0.4	0.66	0.33	33%	5%	0.36	0.31	-0.28%	5.12%	0.52%
0.4	0.8	0.2	33%	5%	0.4	0.3	-1.04%	5.48%	2.14%
0.2	0.8	0.2	33%	5%	0.49	0.3	-2.56%	5.93%	4.25%
0.4	0.66	0.33	20%	5%	0.22	0.19	-0.1%	5.07%	0.35%

Table 3: Second-best policy measures for different parameter values

which increases savings and growth. The term $A_G(g_T, g_N) < A_G(g_T^*, g_N^*)$ captures the second-order distortion in the sectoral allocation of capital and labor.

We conclude that a reallocation of government spending towards the tradable sector achieves a first-order dynamic growth effect at a second-order static efficiency cost. Therefore a small reallocation unambiguously raises welfare.

Graphically, the locus of factor inputs (G_T, G_N) that produces the required amount of government spending looks similar to the TT-locus in figure 3. Table 3 illustrates the optimal reallocation in government spending for economies with different parameters values for ϕ, α and η and for A_Z that is calibrated to yield the indicated growth rate γ^{DE} in the economy. In addition we vary the fraction of government spending in total output as indicated in column 4 by the variable $\bar{G}$. In order to simplify the interpretability of our results, we assume that government spending employs the same production function (7) as final goods. This implies that the most cost-effective way of producing $\bar{G}$ is to employ intermediate goods in the same proportions as final goods producers do, so that $g_T = g_N = \bar{G}/Z$. The columns marked by g_T and g_N indicate the optimal shares of intermediate goods that a government following a second-best policy chooses. This results in a static distortion ΔA_G to the economy's social product of capital, but a dynamic increase in the growth rate to γ_G. The overall welfare gain expressed as the equivalent permanent increase in consumption is reported in the last column.

In the example presented in the first row, the optimal second-best expenditure policy uses 36% of the economy's output of tradables and only 31% of non-tradables. This causes output to rise by .12% and leads to a relatively small increase in welfare of .52%. If we increase the difference in capital intensities among the two sectors (row 2), the government's optimal expenditure-switching policy as well as the resulting welfare effects are markedly stronger. This holds even more if the tradable sector is small, as represented by $\phi = .2$ (row 3). Lastly, it is natural that the smaller the size of government expenditure, the less significant the effects of sectoral reallocations (row 4).[23]

[23]More generally, the social cost of sectoral reallocations in government spending and therefore the optimal level of reallocations also depend on the substitutability of tradable and non-tradable goods in the government's production function F_G.

6.3 Sector-specific factor taxation

Another way for government to affect relative prices and the return to capital in the economy would be by imposing sector-specific taxes or subsidies on the returns to the production factors. While this technically violates our restriction 1, our analysis is highly relevant for economies in which the non-tradable sector is predominantly informal so that all formal policy measures are likely to disproportionately affect the tradable sector.

We denote the bundle of tax rates on the returns on capital and labor in the tradable and non-tradable sectors as $(\tau_{TK}, \tau_{TL}, \tau_{NK}, \tau_{NL})$, where a negative tax rate represents a subsidy. We continue to assume that any revenues or costs are rebated in lump-sum fashion. By repeating the steps outlined in subsection 2.2, we find that the equilibrium conditions (4) and (5) for the two factor markets are modified to

$$q = \frac{1+\tau_{TK}}{1+\tau_{NK}} \cdot \frac{\eta K_N^{\eta-1} \left(A_N L_N\right)^{1-\eta}}{\alpha K_T^{\alpha-1} \left(A_T L_T\right)^{1-\alpha}}$$

$$q = \frac{1+\tau_{TL}}{1+\tau_{NL}} \cdot \frac{(1-\eta) K_N^{\eta} A_N^{1-\eta} L_N^{-\eta}}{(1-\alpha) K_T^{\alpha} A_T^{1-\alpha} L_T^{-\alpha}}$$

Combining these two equations with the equilibrium condition (14) for the final goods market, which remains unchanged, results in capital and labor ratios of

$$\kappa\left(\tau_{TK}, \tau_{NK}\right) = \frac{1+\tau_{TK}}{1+\tau_{NK}} \cdot \kappa^* \qquad \text{and} \qquad \lambda\left(\tau_{TL}, \tau_{NL}\right) = \frac{1+\tau_{TL}}{1+\tau_{NL}} \cdot \lambda^*$$

as well as an equilibrium interest rate (see appendix A.6 for details) of

$$R\left(\{\tau_{ij}\}\right) = \left[\frac{\alpha\phi}{1+\tau_{TK}} + \frac{\eta(1-\phi)}{1+\tau_{NK}}\right] \cdot A(\kappa, \lambda) \tag{34}$$

Taxes on labor enter this expression only indirectly through the social return on capital $A(\kappa, \lambda)$. As can be seen from the expression for $\lambda(\tau_{TL}, \tau_{NL})$, the inelastic labor supply entails that wage taxation is irrelevant for the social return on capital A, the interest rate R and therefore welfare as long as both sectors are taxed at the same rate – the tax rates in the expression for λ cancel out and the tax acts as a lump-sum tax. On the other hand, if the tax rates on labor differ across the two sectors, welfare is unambiguously reduced: labor will be allocated inefficiently between the two sectors, which introduces a second-order distortion to the social return on capital $A(\cdot)$ without any direct effects on the private interest rate $R(\cdot)$. This lowers both the return to capital and growth in the economy.

By contrast, taxing (subsidizing) the returns to capital in any sector reduces (increases) the economy-wide interest rate and by implication savings, with the strength of the effect depending on the capital share of the relevant sector, as specified by equation (34). If the tax rates on capital in the two sectors differ, a second-order static distortion is introduced into the sectoral capital allocation, as captured by the expression for $\kappa(\tau_{TK}, \tau_{NK})$. Furthermore, note that taxing non-tradable capital and subsidizing tradable capital (or vice versa) in a revenue-neutral fashion does not have a first-order effect on the interest rate, since capital is unspecific in our model: the aggregate return to capital cannot be increased by taking from capital owners and giving back to them; such a policy only introduces a second-order distortion into the economy.

More generally, any bundle of sector-specific factor taxes can equivalently be represented as a pair of taxes on capital and labor (τ_K, τ_L) together with a revenue-neutral pair of taxes on intermediate goods (τ_T, τ_N). The effects of these two sets of policy measures are discussed in sections 4 and 6.1 respectively.

Our analysis of sector-specific factor taxation suggests that in countries in which the non-tradable sector is predominantly informal (e.g. small shops and street vendors) and therefore difficult to subject to taxes or subsidies, subsidizing (formal) tradable capital and raising the revenue by taxing (formal) tradable labor would constitute another second-best policy option: the policy would achieve a first-order increase in the private return to capital (in both sectors, since capital is unspecific) at the cost of second-order distortions to the capital ratio κ (as excess capital flows into the formal tradable sector to take advantage of the subsidy) and the labor ratio λ (as labor flees into the informal non-tradable sector to avoid taxation).

7 Conclusions

This paper has established conditions under which policy measures to undervalue a country's real exchange rate generate dynamic gains in terms of increased economic growth by internalizing learning-by-investing externalities. Our findings depend on two critical properties, (i) that technological progress in the economy is subject to learning-by-investing externalities and (ii) that the tradable sector in the economy is more capital-intensive and therefore generates a disproportionate amount of these externalities.

If first-best policy measures are are not feasible because of targeting problems, or because multilateral agreements constrain tax-cum-subsidy measures, reserve accumulation may be desirable if certain deep parameter restrictions are met regarding the magnitude of growth externalities, the relative capital-intensity and size of the tradable sector, the willingness of agents to intertemporally smooth and their rate of time preference. The conditions under which it is desirable for a country to accumulate reserves are identical to those under which countries suffer from a foreign aid curse or resource curse when they experience an exogenous inflow of tradable resources.

Our paper analyzes these issues from the perspective of a small open economy that does not affect equilibrium in world capital markets. An interesting next step on our research agenda is to evaluate the welfare effects of real exchange rate undervaluation in a given country on other countries. In a two-country setting, we conjecture that reserve accumulation in a country that is subject to learning-by-investing externalities may constitue a Pareto improvement if the second country is free of such growth externalities, as the second country benefits from a lower world interest rate and the first country from internalizing the externality. In turn, in a multi-country setting, all countries that exhibit learning-by-investing effects impose negative externalities on each other when they engage in reserve accumulation, whereas countries free of growth externalities benefit from other countries' reserve accumulation.

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A Mathematical Appendix

A.1 Equilibrium Conditions for Firms

We express the first order conditions of tradable firms as functions of the product rent and the product wage,

$$\alpha K_T^{\alpha-1}\left(A_T L_T\right)^{1-\alpha} = \frac{R}{p_T} \tag{35}$$

$$\left(1-\alpha\right) K_T^{\alpha} A_T^{1-\alpha} L_T^{-\alpha} = \frac{w}{p_T} \tag{36}$$

and similarly the first order conditions for non-tradable firms,

$$\eta K_N^{\eta-1}\left(A_N L_N\right)^{1-\eta} = \frac{R}{p_N} \tag{37}$$

$$\left(1-\eta\right) K_N^{\eta} A_N^{1-\eta} L_N^{-\eta} = \frac{w}{p_N} \tag{38}$$

Dividing the first order conditions on capital yields equilibrium condition (4) for the capital market; dividing the remaining two conditions yields the equilibrium condition (5) for the labor market.

The final goods sector's first order conditions imply that the marginal product of each intermediate input has to equal its price,

$$\phi A_Z \left(\frac{N}{T}\right)^{1-\phi} = p_T \tag{39}$$

$$\left(1-\phi\right) A_Z \left(\frac{T}{N}\right)^{\phi} = p_N \tag{40}$$

Combining the two conditions we obtain equilibrium condition (14) for the final goods sector.

A.2 Aggregate Production Technology

If we substitute for the endogenous levels of technology (6) and the private factor allocations (15) and (16) in the tradable and non-tradable production functions and assemble the two intermediate goods into final goods using production function (7), it can be seen that the economy's combined production technology is

$$A(\kappa,\lambda) = F_Z(T,N)/K = A_Z\left[K_T^{\alpha}(KL_T)^{1-\alpha}\right]^{\phi}\left[K_N^{\eta}(KL_N)^{1-\eta}\right]^{1-\phi}/K =$$
$$= A_Z\left[\left(\frac{1}{1+\kappa}\right)^{\alpha}\left(\frac{1}{1+\lambda}\right)^{1-\alpha}\right]^{\phi}\left[\left(\frac{\kappa}{1+\kappa}\right)^{\eta}\left(\frac{\lambda}{1+\lambda}\right)^{1-\eta}\right]^{1-\phi} L^{1-\tilde{\alpha}}$$

where we substituted the definition of the aggregate weighted capital share $\tilde{\alpha} = \alpha\phi+\eta(1-\phi)$ to obtain the expression in equation (19). If the capital and labor ratios are at the socially optimal levels κ^* and λ^*, this expression can be further reduced to

$$A_Z\frac{(\phi\alpha)^{\phi\alpha}\left[(1-\phi)\eta\right]^{(1-\phi)\eta}}{\tilde{\alpha}^{\tilde{\alpha}}}\cdot\frac{\left[\phi(1-\alpha)\right]^{\phi(1-\alpha)}\left[(1-\phi)(1-\eta)\right]^{(1-\phi)(1-\eta)}}{(1-\tilde{\alpha})^{1-\tilde{\alpha}}}L^{1-\tilde{\alpha}}$$

A.3 Public accumulation of capital

We extend the model of investment and capital accumulation from section 2.1 by labeling private investment and capital I^P and K^P and by introducing governmental investment I^G and capital K^G that follow a law of motion similar to that of private capital, i.e.

$$K_{t+1}^G = (1-\delta)K_t^G + I_t^G$$

Assuming that government investment I^G is financed by lump-sum taxes and the returns on governmental capital are distributed to agents in lump-sum fashion, the representative agent's budget constraint can be expressed as

$$C_t = \left[K_t^P + K_t^G\right](1+R_t-\delta) + w - \left[K_{t+1}^P + K_{t+1}^G\right]$$

Substituting this into the agent's maximization problem (1) and taking the first-order condition with respect to private capital K_t we find the Euler equation

$$\frac{C_t}{C_{t-1}} = \left[\beta(1+R_t-\delta)\right]^{\frac{1}{\theta}}$$

This optimality condition is identical to the decentralized agent's Euler equation (2) in the absence of government intervention. Furthermore, if the series of values for the capital stock $\{K_t\}_{t=0}^{\infty}$ is the solution to the decentralized maximization problem, then any series $\{K_t^P + K_t^G\}_{t=0}^{\infty}$ where $K_t^P + K_t^G = K_t$ solves the modified problem with governmental accumulation of capital. In other words, for every increase in the governmental capital stock ΔK^G, private agents reduce their capital stock K^P by an identical amount so as to solve their optimization problem – public investment fully crowds out private investment.

A.4 Effects of Current Account Intervention

We derive the expression for the price of tradable goods from (39) using the modified equilibrium condition (27) for the final goods sector:

$$p_T(v) = \phi A_Z \left(\frac{F_N(\cdot)}{(1-v)F_T(\cdot)} \right)^{1-\phi}$$

The equilibrium interest rate can then be derived from (35) as

$$\begin{aligned} R(v) = \alpha p_T \frac{F_T(\cdot)}{K_T} &= \frac{\alpha\phi A_Z}{(1-v)^{1-\phi}} \cdot \frac{1+\kappa(v)}{K_T} \cdot F_T(\cdot)^{\phi} F_N(\cdot)^{1-\phi} = \\ &= \frac{\alpha\phi + (1-v)\eta(1-\phi)}{(1-v)^{1-\phi}} \cdot A\left(\kappa(v), \lambda(v)\right) \end{aligned}$$

Small interventions v introduce a distortion into the capital/labor allocation, which reduces $A(\kappa(v), \lambda(v))$. However, since $\kappa(v)$ and $\lambda(v)$ are chosen optimally given the intervention, the envelope theorem implies that this effect is second order and $dA(\cdot)/dv|_{v=0} = 0$. On the other hand, the factor pre-multiplying $A(\cdot)$ experiences a first-order increase so that the interest rate under current account intervention is larger than the free market interest rate,

$$\begin{aligned} \left.\frac{dR(v)}{dv}\right|_{v=0} &= (1-\phi) \left.\frac{-\eta(1-v) + [\alpha\phi + \eta(1-\phi)(1-v)]}{(1-v)^{2-\phi}}\right|_{v=0} \cdot A^* = \\ &= (\tilde{\alpha} - \eta)(1-\phi)A^* > 0 \end{aligned}$$

which is positive as long as the capital share in the tradable sector is larger than in the non-tradable sector, as we assumed in assumption 1.

The slope of the VV locus at $v = 0$ can be expressed by the implicit function theorem as $\frac{dA_V}{d\gamma} = \frac{dA_V/dv}{d\gamma/dv}$ where

$$\begin{aligned} \left.\frac{dA_V}{dv}\right|_{v=0} &= -\phi A^* \\ \left.\frac{d\gamma}{dv}\right|_{v=0} &= \frac{1}{\theta}\beta^{\frac{1}{\theta}}(1+R-\delta)^{\frac{1-\theta}{\theta}} \cdot \frac{dR(v)}{dv} \end{aligned}$$

A.5 Differential taxation of intermediate goods

We first derive the expression linking a subsidy on tradables to a tax on non-tradables such that the pair (τ_T, τ_N) is revenue-neutral. For this, combine the revenue-neutrality condition (31) with the equilibrium condition in the intermediate goods market (29) to find

$$\begin{aligned}
-\frac{\tau_N}{\tau_T} &= \frac{p_T T}{p_N N} = \frac{\phi}{1-\phi} \cdot \frac{1+\tau_N}{1+\tau_T} \\
(1-\phi)(1+\tau_T)\,\tau_N &= -\phi\,(1+\tau_N)\,\tau_T \\
\tau_N &= -\frac{\phi \tau_T}{1-\phi+\tau_T} \\
\text{or} \qquad 1+\tau_N &= \frac{(1-\phi)(1+\tau_T)}{1-\phi+\tau_T}
\end{aligned}$$

We substitute this into equation (30) and find:

$$R(\tau_T, \tau_N) = \frac{\alpha\phi + \eta\,(1-\phi+\tau_T)}{1+\tau_T} \cdot A_\tau$$

Note that the social product of capital A_τ is only affected to a second-order degree by taxing/subsidizing intermediate goods and equals A^* for $\tau_T = \tau_N = 0$. If we differentiate the interest rate with respect to τ_T, we therefore find that for small revenue-neutral sectoral taxes:

$$\begin{aligned}
\frac{dR(\tau_T, \tau_N)}{d\tau_T} &= \frac{\eta\,(1+\tau_T) - \alpha\phi - \eta\,(1-\phi+\tau_T)}{(1+\tau_T)^2} A_\tau + O^2(\tau_T) = \\
&= \phi \frac{\eta - \alpha}{(1+\tau_T)^2} A_\tau + O^2(\tau_T)
\end{aligned}$$

As long as $\alpha > \eta$ (assumption 1) is satisfied, a small revenue-neutral tax (subsidy) on tradable production lowers (raises) the private return to capital.

A.6 Sector-specific factor taxation

Since there is no distortion in the production of final goods, we obtain the price of tradables p_T from equation (39) and substitute this into the expression for the interest rate that follows from the first-order condition on tradable capital (35) with a tax rate τ_{TK}:

$$\begin{aligned}
R(\{\tau_{ij}\}) &= \frac{\alpha}{1+\tau_{TK}} \cdot \frac{T}{K_T} = \frac{\alpha\phi}{1+\tau_{TK}} A_Z \left(\frac{N}{T}\right)^{1-\phi} \frac{(1+\kappa)\,T}{K} = \\
&= \frac{\alpha\phi\,(1+\kappa)}{1-\tau_{TK}} A(\kappa, \lambda) = \\
&= \left[\frac{\alpha\phi}{1+\tau_{TK}} + \frac{\eta\,(1-\phi)}{1+\tau_{NK}}\right] \cdot A(\kappa, \lambda)
\end{aligned}$$