WPS8574

Policy Research Working Paper 8574

Tax-Transfers Schemes, Informality, and Search Frictions in a Small Open Economy

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World Bank Group
Poverty and Equity Global Practice
September 2018

Policy Research Working Paper 8574

Abstract

This paper evaluates the impact of market-oriented structural reforms, in particular labor market policies, social assistance programs, and trade liberalization on long run unemployment, wage inequality, and the distribution of employment across sectors in a small open economy with search frictions and idiosyncratic productivity shocks. The paper builds a search and matching model of a labor market with a large informal sector and estimates the model using Colombian household-level data. Changes in labor taxes may have sizable aggregate, compositional, and distributional effects if workers associate high payroll taxes with more valuable and efficient social security services. The higher is the valuation of the services, the higher is the reduction in the log-wage gap. An expansion of public health insurance to informal sector workers has minor aggregate and distributional effects. Changes in relative prices that negatively affect the relative profitability of the formal sector have quite sizable aggregate effects, producing more long-run unemployment and informality, and increasing unemployment duration.

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Tax-Transfers Schemes, Informality, and Search Frictions in a Small Open Economy*

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Keywords: Developing Economy, Unemployment, Informality, Payroll Taxes, Subsidized Health, Wage Inequality

JEL Classification Numbers: E24, E26, E27, J31, J32, J46, J64, F16

*Thanks to James Albrecht and Susan Vroman for their valuable contributions. I also wish to thank participants at Georgetown GSO and Macroeconomic Seminars, MEA Annual Meetings, IJAS Conference at Harvard University, LACEA-LAMES Annual Meetings, GCER Georgetown Conference, and seminar audiences at the Colombian Central Bank, Pontificia Universidad Javeriana, U.S. Department of Labor and the World Bank.

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1 Introduction

This paper seeks to understand the impact of market-oriented structural reforms on steady state unemployment, wage dispersion and the distribution of employment across sectors in a small open economy with a persistently large informal sector. To this end, I formulate and estimate an equilibrium search and matching model of a labor market with search frictions and idiosyncratic productivity shocks.

The central idea behind modeling the labor market with a search model (instead of a neoclassical one) is that, given the existence of heterogeneity, frictions or imperfect information, search is a costly activity for both firms and workers who must spend resources before job creation and job destruction occurs. In these types of models equilibrium unemployment in the steady state emerges naturally as a result of the transitions in and out of unemployment, since some existing jobs break up before new matches are made. None of these properties characterize Walrasian labor markets.

The model is particularly relevant for developing economies, in particular Latin American (LA) economies, where wage inequality, high unemployment, job instability and large informal sectors are longstanding concerns, particularly in the last two decades (Inter-American Development Bank, 2004)[1].

After the 1990s, many LA countries followed a sequence of market-oriented structural reforms, including changes in labor market legislation, social security programs, and changes in the degree of trade and financial openness. The extent to which these countries' labor market, trade and social security reforms contribute to deteriorating labor market conditions is still a debatable question in the literature. Some other factors including aggregate and idiosyncratic market shocks, demographic changes affecting the size of the labor force, or skill biased-technological change may also play an important role[2].

The model is simulated for the Colombian economy, a country that in the 1990s and 2000s implemented substantial market-oriented reforms whose main goal was to deregulate labor and financial markets.

There are some previous empirical studies focusing on the effects of Colombian structural reforms on labor markets: Cardenas and Gutierrez (1996); Cardenas, Kugler and Bernal (1998); Kugler (1999); Eslava, Haltiwanger, Kugler and Kugler (2010); Camacho, Conover and Hoyos (2013). Most of these studies employ reduced form estimation and none of them build a search model as an analytical framework to understand incentives behind flows between the formal and informal sectors. I provide a new lens to study the impact of the reforms by building and estimating

[1] The informal sector is particularly large in Latin American economies (from 30 to 70 percent of GDP) according to some studies (Maloney, 2004). Other studies (Schneider, 2005) suggest also that the size of the informal sector in other economies is not negligible. They estimate that, for the period 1999-2000, the average size of the informal economy (as percent of GDP) is 41.2 percent in African countries, 26.3 percent in Asian countries, 37.9 percent in transitional economies and 16.7 percent in OECD countries.

[2] For a survey of the impact of labor market regulations in Latin America see Heckman and Pages (2004), on the impact of trade liberalization and job turnover, see Haltiwanger et al (2004). For a survey of the impact of trade on wage inequality and informality in Latin America and other developing economies see Attanazio, Goldberg and Pavcnik (2004).

a structural model.

Some recent studies have looked at informality in Latin American economies using a flow approach to unemployment: Bosch and Maloney (2007, 2008, 2010); Bosch, Goni and Maloney(2007); Fiess, Fugazza and Maloney (2008).

This model contributes to a growing recent literature that combines informality with labor market search frictions in emerging economies: Albrecht, Navarro and Vroman (2009); Bosch and Esteban-Pretel (2012); Meghir, Narita and Robin (2015); Cosar, Guner and Tybout (2016).

I develop a substantially modified version of the search and matching model with endogenous job destruction by Mortensen and Pissarides (1994)(MP1994 hereafter). The main differences between this model and MP1994 are the following: a) An informal sector is added; b) There is a continuum of worker types; c) The idiosyncratic productivity shock process is modeled differently. While in MP1994 matches start at the "state of the art" or, at the fixed maximum productivity level, in this model they start with a draw from a distribution (stochastic job matching).

The model is similar in spirit to the one developed by Albrecht, Navarro, Vroman (2009) (ANV2009 hereafter) with an informal sector, but the modeling of the informal sector is different, capturing the idea that the informal sector is a 'disadvantaged' sector of a dualistic or segmented labor market,[3] instead of an unregulated self-employment sector. Even though the informal sector has close linkages to the formal sector, it is still a 'disadvantaged' sector characterized by low entry barriers in terms of skills, and therefore, populated by workers with low productivity levels who are excluded or segregated from the formal economy.

The main differences with ANV2009 are the following: 1) In this model there are direct flows from formal to informal, while in their model there are no flows between sectors;[4] 2) In this model there is ex-post match-specific heterogeneity, so a worker's type (individual specific characteristics relevant to the labor market such as human capital level) is not the same as match-specific productivity; workers of a given type are not certain of whether they will be 'good' or 'bad' at a specific job, so I assume initial productivity in a match is a draw from a distribution and not a fixed value (stochastic job matching); 3) Workers in both sectors have productivity shocks that may be 'good' or 'bad' with respect to the match productivity, while in their model, only workers in the formal sector are subject to shocks, and a worker's current productivity can never exceed their type; 4) To understand the impact of social assistance programs and adapt the model to the Colombian case, workers in this model contribute to social security in the formal sector and have access to subsidized health in the informal sector, features that are not present in ANV2009.

Counterfactual experiments show that changes in labor taxes may have sizable aggregate, compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient social security services. The higher the valuation of the SS services, the more progressive

[3] There is some supporting empirical evidence of a segmented labor market for the Colombian case. Most of the arguments are supported by the fact that the informal sector is countercyclical, absorbing labor during downturns. See Fiess, Fugazza, Maloney (2010) and Mondragon, Pena, Wills (2010).

[4] I estimate that, among the informal-sector workers who switch jobs in a 12-month-period (June 2002 to June 2003), 59.1 percent flow from the formal-sector.

these labor market policies become. An expansion of subsidized health to informal sector workers has minor aggregate and distributional effects. Changes in relative prices that affect the profitability of the formal sector negatively have quite sizable aggregate effects, producing more long run unemployment and informality, and increasing unemployment duration.

The paper proceeds as follows. Section 2 introduces some stylized facts about the Colombian labor markets and a brief summary of the policy reforms. Sections 3 details the theoretical model. Section 4 describes the data and details the estimation procedure and results. Section 5 concludes.

2 Stylized Facts

Some stylized facts about the Colombian urban labor markets in the last two decades include: increasing long-run unemployment and unemployment duration, a decreasing but persistently high informality rate, a modest rise in relative earnings favoring informal sector workers, and a decrease in overall wage inequality[5] driven by a reduction in within-sector inequality.

Figure 1 presents the evolution of the unemployment rate from the first quarter of 1984 to the third quarter of 2013. At the beginning of 1984, approximately 13.7 percent of the labor force was unemployed. A period of steadily declining unemployment started. In the fourth quarter of 1993 the unemployment rate had dropped to 7.8 percent. A long period of rising unemployment followed, reaching a peak of 20.5 percent in the third quarter of 2000. The decline of unemployment in the following quarters was slow, reaching a rate of 10.4 percent in the second quarter of 2013.

Notably, short-term unemployment did not fall near its average levels: 11.8 percent over the period 1984-1999 versus 13.9 percent over the period 2000-2013. Mean unemployment duration also increased from 31 weeks to 35 weeks, well above pre-recession norms (see Figure 2).

Figure 3 shows the evolution of the employment rate. The long run employment rate followed an upward trend, increasing from 52 percent in the period 1984-1999 to 55.8 percent in the period 2000-2013.

Informality has been a persistent phenomenon over the last two decades: regardless of the definition of informality used,[6] approximately one of every two workers are considered informal. Figure

[5]Labor market statistics are constructed using data from Colombian household surveys. These surveys are repeated cross-sections carried out by the Colombian Statistics Department (DANE) on employed and unemployed individuals, and include: Encuesta Nacional de Hogares (ENH) for the period 1984:QI to 2000:Q2, Encuesta Continua de Hogares (ECH) for the period 2000:M1 to 2006:M12, and Gran Encuesta Integrada de Hogares (GEIH) for the period 2007:M1-2013:M6. Informality statistics are constructed using the Informality module in the surveys, available every two years before 1996, and yearly afterwards. The statistics presented in this section are for all urban workers.

[6] The following definitions will be used in the paper:

"Social Protection" (SP) Informality Definition: A worker is considered informal if either of the following two conditions hold:

- *Health Affiliation*: Is not affiliated to a health plan, or if affiliated does not make any contributions to the system (either because is part of subsidized regime or beneficiary of the contributory regime)
- *Pension Affiliation*: Not affiliated to a pension fund

This variable is primarily a proxy for non-compliance to labor regulations in Colombia.

"Firm Size and Occupation" (FSO) Informality Definition: A worker is considered informal if the following two criteria hold:

4 shows the evolution of the informality rate based on alternative criteria. The average informality rate based on the health criterion diminished slightly from 49.7 percent in 1984-1999 to 48.7 percent in 2000-2013; informality based on pension decreased from 60.2 percent to 57.5 percent in 2000-2013; informality based on health and pension decreased from 60.9 percent to 58.7 percent.

When looking at the sectoral log-wage distributions, two facts are worth emphasizing.

First, there are substantial differences in mean relative earnings, favoring formal-sector workers. This is evidenced by the size of the log wage premium, which varies between 0.71 and 1.43 over the whole period 1984-2013 (see Figure 5). The formal-informal sector mean log wage gap, although diminishing, is still persistently high: the gap decreased only by 0.04 log points in the period under consideration.[7]

Second, overall wage inequality improved even though there were no significant improvements in between-sector inequality. Figure 6 shows the variance of the earning distribution in the overall economy. This change was mostly driven by lower wage dispersion within each sector. While between-sector variance decreased only by 9.4 percent, within-sector variance decreased by 32.3 percent, leading to an overall fall in log-wage variance of 29.3 percent. Figure 7 shows this variance broken down into the between and within-sector components.

Finally, mean real wages rose in both sectors, suggesting an improvement in living standards. Mean hourly wages increased by 0.15 and 0.13 log-wage points in the informal and formal sector respectively (See Figure 8).

In the period under consideration, a sequence of market-oriented structural reforms was implemented: the labor reforms of 1990, 2002 and 2012, the social security reforms of 1993, 2003 and 2007, and the trade liberalization reform that started in 1991.

Prior to the labor reforms, employment protection in Colombia was promoted by labor regulations that imposed high severance payments, early retirement and restrictions on temporary work, affecting labor markets' flexibility, with potential adverse effects on employment.

In this context, the recent labor and social security reforms were intended to contribute to the flexibility and efficiency of the labor markets, while still maintaining some level of worker protection.

The labor reform of 1990 was primarily aimed at stimulating job creation by instituting a more flexible system of hiring and layoffs. This was achieved by allowing short-term contracts, promoting a more flexible wage regime, and more importantly, decreasing severance payments. The reform required that formal sector firms make an annual contribution to a private severance fund (includ-

- Firm Size: Works in firms with five or fewer employees
- *Occupation*: Works as domestic employee, self-employed, employer or unpaid family worker.

This definition is the one used by the Colombian Statistics Department, DANE, and is consistent with the one used by the ILO, but it does not include any criteria related to non-compliance with regulations.

[7] Here I use the social security definition of informality, constructed using only health contributions since there is no information for pensions before 1996. The measure of earnings in the survey not only include monetary wages for workers (including tips, commissions) but also remuneration for self-employed, so there is some measurement error in wages.

ing interest payments) instead of paying severance at the time employment terminated.[8]

The social security reform of 1993, implemented in 1994, introduced major changes to the health and pension systems, monopolized by the government until then. The reform increased pension and health contributions and thereby non-wage labor (hereafter NWL) costs with the main goal of expanding social security coverage. Aiming to reach universal health coverage, the reform also created two coexisting health regimes: a contributive regime (CR) and a subsidized regime (SR).

In the CR, employers must provide health insurance by law, regardless of occupation,[9] and the cost is shared between employers and employees. In the SR, 'poor' individuals who meet certain poverty criteria have access to subsidized health, where 'poor' is determined by a poverty index score based on the Census of the Poor (SISBEN).[10] This system is financed with transfers from the contributive regime.

The labor reform of 2002 created a system of social protection, aimed to protect unemployed workers and to promote employment in recessionary periods but had limited impact on job creation, since most of the measures applied to a very limited group of workers and NWL costs were unaffected. The pension reform of 2003 increased the age requirements for retirement, the length of service required for pension, and the pension contribution rate (gradually until 2008), thus increasing employer and employee NWL costs. The health reform of 2007 increased employer health contributions, raising employer NWL costs.

After three decades of rising NWL costs,[11] the more recent labor reform of 2012 reduced employer NWL costs substantially by 8.96 percent through the elimination[12] of para-statal contributions,[13] with the main goal of stimulating employment.

Changes in social security contributions caused sizable changes in employer and employee NWL costs, potentially affecting job creation and job destruction in the formal and the informal sectors.

Figures 9 and 10 show the evolution of employer and employee NWL costs in the period 1984-2013.[14] In the period 1985-1990, employer NWL costs were 47.08 percent of the wage. The reduction in severance payments introduced by the reform of 1990 caused a reduction in the average NWL cost paid by the employer from 1991 to 1993, distributing this payment linearly over time. The implementation of this change has been gradual, as discussed before. As a result, NWL costs were reduced to 45.98 percent in 1993. In the period 1994-1996, these costs began to rise to reach a level of 53.41 percent in 1996, since the increasing pension and health contributions implemented

[8]However, the law applies only to workers hired after 1991, which means that the reduction in severance payments depends on the turnover rate, and the number of workers who voluntarily switch to the new regime. Hence the reduction in severance payments was gradual rather than immediate.

[9]According to the law, self-employees must also contribute to the system.

[10] Individuals may qualify for fully subsidized health if they meet the following criteria: being part of a SISBEN level 1 or 2 household, not being affiliated to the CR, not having an employment relationship, not being a retiree and not being a beneficiary of the CR.

[11]The reduction in severance payments introduced by the Reform of 1990 is the exception.

[12]This reduction only applies to workers earning less than 10 times the minimum wage.

[13] In addition to social security, paid vacation and mandatory bonuses, employers must pay taxes to finance social programs. These para-statal contributions include training programs, family allowances and in-kind transfers for low income households.

[14]For a detailed disaggregation of these costs see Tables 12 and 13.

by the social security reform of 1993 more than compensated for the gradual reduction of severance payments. In the period 1996-2003, they remained constant. Finally, in the period 2004-2012, these costs continued to climb primarily because of increasing social security contributions, until they reached a peak of 55.78 percent in 2012. The reform of 2012 counteracted the trend by reducing them significantly to 50.78 percent.[15]

The employee NWL costs were constant at 4.5 percent during the period 1990-1993, since they were not affected by the labor reform of 1990. From 1994 to 1996 they started to increase, driven primarily by increasing pension contributions introduced by the social security reform. During the period 1996-2003 they were stable, and after 2004 they begun to climb until reaching a level of 9 percent in 2013.

Colombia also started a dramatic process of trade liberalization and financial openness in 1991.[16] This may have caused important changes in the real exchange rate, defined as the price of tradables relative to non-tradables. Since the informal sector is mainly comprised of non-tradables, [17] the real exchange rate is a key relative price in this small open economy, and can be quite relevant when it comes to explaining movement of labor across sectors. A sharp increase is observed before 2003, followed by a real appreciation afterwards (see Figure 11).

In recent years, investment in social assistance programs (cash transfers, in-kind transfers and subsidized health) has been growing in Colombia. In particular, since 1993, there has been an expansion of subsidized health (Subsidized Regime) to workers who are not part of the contributive regime (informal under the SP definition). The government is expanding access to health with the main goal of achieving universal health care in the near future. Figure 12 shows the growing percentage of informal-sector workers who are affiliated to the subsidized regime. While in the second quarter of 2001 approximately 19.42 percent of informal-sector workers were affiliated to the SR, in the second quarter of 2013 this was 47.65 percent.[18]

Higher payroll taxes, changes in relative prices affecting the relative profitability of the two sectors, and expanding social assistance programs to informal workers[19] may explain some of the stylized facts observed in the labor markets in recent years.

[15] These payroll taxes are high when compared to the US, where contributions range between 15 and 20 percent of the gross wage, and similar to the levels observed in some EU countries (i.e. Sweden, Belgium, France), where contributions are close to 50 percent.

[16] Interest rate ceilings, exchange rate controls and restrictions on foreign direct investment were eliminated, as well as requirements to invest in government securities. International trade was largely liberalized, due to significant reductions in import tariffs.

[17] According to a recent Informality survey by Batini, Kim, Levine and Lotti (2010), 87 percent of self-employed and informal workers in Colombia are concentrated in non-tradables (i.e commerce and services).

[18] Author's calculations based on Colombian Household Survey. No data are available before 2001.

[19] An inflexible wage structure in the formal sector may also be a factor since previous literature (Maloney, Nunez, 2001) suggests that minimum wages in Colombia are high and binding. Here I don't consider any source of wage rigidity.

3 Model

3.1 Workers

In this economy agents live forever, discount the future at a constant rate r, and live in a stationary environment where there are no dynamic changes to the structural parameters. The labor force, L , is assumed to be constant and normalized to unity.

In equilibrium, each agent can be in one of three states: unemployed, employed in the formal sector, or employed in the informal sector. I want to allow flows from and to each possible state (except from informal to formal-employment), so I have a total of five transitions in the model.

There is worker heterogeneity ex-ante and ex-post. Ex-ante, workers differ in individual-specific characteristics relevant to the labor market, such as their human capital level, which are called worker types. Ex-post, workers differ in their labor market status and their initial productivity at the job (ex-post match-specific heterogeneity), as well as their future productivity since they are also subject to idiosyncratic productivity shocks.

Let y be the worker's type, where y is an iid draw from a distribution $F(y)$ with support on the range $[0 \leq y \leq \infty]$. Let y' be the initial productivity of the job (match-specific), where y' is a draw from a distribution $H_i(y' \mid y)$, for $i = F, I$.[20]

Let u be the measure of unemployed workers, v_i be the measure of vacancies in sector i, and n_i be the measure of workers in sector i.[21] Let θ and ϑ be the parameters that measure labor market tightness in the formal and informal sectors respectively, where[22]

$$\theta = \frac{v_F}{u}; \vartheta = \frac{v_I}{u + n_F}$$

Let $U(y)$ and $N_i(y', y)$ be the present-discounted value of the expected income stream of an unemployed worker of type y and of an employed worker of type y with match-specific productivity y' in sector i, respectively.

While unemployed, workers enjoy returns b[23] and receive offers from both sectors, regardless of their type. Let α be an exogenous Poisson rate at which informal sector offers arrive to the

[20] First-order stochastic dominance is assumed, i.e. if $y_1 > y_2$, then $H_i(y' \mid y_1) < H_i(y' \mid y_2)$. In the calibration, these distributions are assumed to be log-normal with a scale parameter varying linearly with $log(y)$, so the first-order stochastic dominance assumption is satisfied.

[21] Notice that v_i is not the total measure of vacancies in the economy, but only the vacancies in sector i. Also, since the labor force is normalized to 1, u is also equal to the aggregate unemployment rate.

[22] The tightness measures the number of vacancies relative to potential job seekers in each sector. While job seekers in the formal sector are only unemployed workers, job seekers in the informal are unemployed workers and formal sector workers affected by a 'bad' shock that causes them to look into informal sector opportunities.

[23] This is also usually interpreted as an unemployment insurance benefit but there was no such insurance in Colombia before 2003, so b is interpreted as the opportunity cost of leisure. Unemployed workers in Colombia can also receive subsidized health, but there is no data available to measure the size of the subsidy, or the fraction of unemployed who receive it.

unemployed,[24] and $m(\theta)$ be the endogenous rate at which formal sector offers arrive.[25] Once contact is made between a worker of type y and a potential employer in the formal sector, a productivity for the prospective match, y', is drawn.

The details of the job creation process are as follows. Because of the existence of a productivity distribution for new matches, not all meetings create a match. Let $R_{UF}(y)$ and $R_{UI}(y)$ be the minimum productivities in the formal and informal sectors below which neither the firm nor the type-y worker want to start a match (endogenous reservation productivities). If the realization of the productivity draw for a worker of type y is sufficiently 'high',[26] the worker and the firm in sector i decide to match, and the worker gets a capital gain of $N_i(y', y) - U(y)$; otherwise, the worker returns to the pool of unemployed, and the job remains vacant.[27]

The flow value of unemployment for a worker of type y is:

$$rU(y) = b + \alpha Emax[N_I(y', y) - U(y), 0] + m(\theta)Emax[N_F(y', y) - U(y), 0].$$

Given the assumptions on the match-specific productivity this gives:

$$rU(y) = b + \alpha \int_{R_{UI}(y)}^{\infty} [N_I(y', y) - U(y)]dH_I(y' \mid y) + m(\theta) \int_{R_{UF}(y)}^{\infty} [N_F(y', y) - U(y)]dH_F(y' \mid y). \quad (1)$$

While employed in sector i, a worker of type y and current productivity y' enjoys flow utility $u_i(y', y)$. Then, the match that started at productivity y' may continue or be destroyed. The job destruction rate is endogenized by introducing idiosyncratic shocks to job productivities:[28] productivity shocks arrive to jobs in sector i at Poisson rate λ, changing the productivity to a new level x. These new productivities are *iid* draws from the conditional distribution $H_i(x \mid y)$.[29]

The same mechanism that governs the job creation process applies to job destruction: a match ends when it is in the mutual interest of the worker and the firm to do so, i.e., when a sufficiently bad draw of x is realized.[30] The threshold productivities for match dissolution in the informal and the formal sector are $R_{IU}(y)$ and $R_{FU}(y)$, respectively.

[24] I am assuming no congestion effects in the informal sector (α is not a function of ϑ), so the measure of job seekers does not make it harder for an individual to find an informal sector opportunity. It may be the case that while job seekers are eager to find a formal sector job, they are not eager to find an informal sector job.

[25] The matching function has standard properties, so $m(\theta)$ is increasing and concave in θ. In the calibration I assume a Cobb Douglas matching function given by $m(\theta) = A\theta^{1-\alpha_m}$.

[26] Sufficiently 'high' means $y' \geq R_{UF}(y)$ for the formal sector and $y' \geq R_{UI}(y)$ for the informal sector.

[27] Another way of modeling this choice is by assuming that workers choose whether or not to accept jobs based on a reservation wage. This is analogous to the reservation productivity concept.

[28] There are two reasons why the productivity of a job may fall below the reservation value: idiosyncratic or aggregate shocks. Previous evidence for Colombia using plant-level data from the Annual Manufacturing survey estimates that the actual impact of reforms through factor adjustment on aggregate productivity was modest (Eslava, Haltiwanger, Kugler and Kugler, 2010), so introducing idiosyncratic shocks instead of aggregate shocks in the model seems more reasonable.

[29] Notice that I assume that productivity shocks affect both sectors symmetrically: workers receive shocks at the same rate, regardless of the sector in which they are in. Also, I assume that the idiosyncratic productivity that is drawn after the shock arrives is independent of the initial productivity y' and is irreversible (the firm must produce at the new productivity or shut down), where $x \in [0, \infty]$.

[30] This means $x < R_{IU}(y)$ for the informal sector, and $x < R_{FU}(y)$ for the formal sector.

I introduce a feature in the model that captures the view of the informal sector as a 'disadvantaged' sector of a segmented labor market that expands during downturns to absorb displaced workers from the formal sector. When affected by a 'bad' shock, the formal-sector worker has to give up his job before learning his productivity on a potential informal-sector opportunity already available to him. Then, when the productivity is realized, he may choose whether to take the prospective informal-sector job or to become unemployed. On the other hand, the informal-sector worker does not have the option to move directly to the formal sector; he must become unemployed. Also, while the formal-sector worker must make contributions to the social security system, the informal-sector worker receives some subsidized health without incurring any cost.

The flow value of taking a formal-sector job for a worker of type y and current productivity y' (new hire)[31] is:

$$rN_F(y',y) = u_F(y',y) + \lambda H_F(R_{FU}(y) \mid y) Emax[N_I(x,y) - N_F(y',y), U(y) - N_F(y',y)] \quad (2)$$

$$+\lambda \int_{R_{FU}(y)}^{\infty} [N_F(x,y) - N_F(y',y)] dH_F(x \mid y)$$

where

$$Emax[\cdot\,] = \int_0^{R_{FI}(y)} [U(y) - N_F(y',y)] dH_I(x \mid y) + \int_{R_{FI}(y)}^{\infty} [N_I(x,y) - N_F(y',y)] dH_I(x \mid y)$$

When affected by a 'good' shock, the formal-sector worker stays in his job and gets the capital gain $N_F(x,y) - N_F(y',y)$. When affected by a 'bad' shock, he may decide to transition to the informal-sector to get a capital gain of $N_I(x,y) - N_F(y',y)$, or to become unemployed and get $U(y) - N_F(y',y)$.

I assume that $u_F(y',y)$ depends on effective current labor income, net after paying SS contributions, and is adjusted by a subjective valuation of the total (employer and employee) contributions to the system.[32] It can be expressed as:

$$u_F(y',y) = w_F(y',y)[(1-\delta_2) + \tau(\delta_1 + \delta_2)]$$

where $w_F(y',y)$ is the hourly wage in the formal sector, δ_2 and δ_1 are the employee and employer NWL costs as a percentage of the wage,[33] τ is a parameter that measures workers valuation of total social security contributions (including employer and employee contributions), $0 \leq \tau \leq 1$.[34]

Let $\hat{\delta}_2$ be the non-wage labor costs as a percentage of the wage adjusted by the worker's valuation of the benefits that he/she receives as a result of the total contributions to the social

[31] This is the flow value for a worker not affected yet by a shock.

[32] I assume the utility function is linear in income and total SS benefits constitute a linear function of the total (employer and employee) contributions.

[33] These costs consist of social security contributions (health and pension) and non-SS contributions (para-statal contributions and others). See Tables 12 and 13 for details.

[34] The valuation of these contributions reflects the value of these contributions and the efficiency of the services provided.

security system,[35] so:

$$\hat{\delta}_2 = \delta_2(1-\tau) - \tau\delta_1$$

The instantaneous utility can be rewritten more compactly as:

$$u_F(y', y) = w_F(y', y)(1 - \hat{\delta}_2) \tag{3}$$

An incumbent worker in the formal sector has a different value function than a new hire since, as will be explained later, the wage (and therefore utility) determined in a Nash bargaining negotiation is different. When the firm negotiates with an old hire it must pay severance if they mutually decide to discontinue the match. Therefore, severance weakens the firm's bargaining position.

Let $w_F^s(x, y)$ and $u_F^s(x, y)$ be the wage and utility of an incumbent worker of type y and current productivity x, respectively.[36] The flow value of continuing in a formal-sector job for this worker is:

$$rN_F(x, y) = u_F^s(x, y) + \lambda H_F(R_{FU}(y) \mid y) Emax[N_I(x', y) - N_F(x, y), U(y) - N_F(x, y)] \tag{4}$$

$$+\lambda \int_{R_{FU}(y)}^{\infty} [N_F(x', y) - N_F(x, y)] dH_F(x' \mid y)$$

where x' is a another draw from the distribution $H_F(. \mid y)$ and

$$u_F^s(x, y) = w_F^s(x, y)(1 - \hat{\delta}_2) \tag{5}$$

The flow value of taking an informal-sector job for a worker of type y and match specific productivity y' is:

$$rN_I(y', y) = u_I(y', y) + \lambda H_I(R_{IU}(y) \mid y)[U(y) - N_I(y', y)] + \lambda \int_{R_{IU}(y)}^{\infty} [N_I(x, y) - N_I(y', y)] dH_I(x \mid y) \tag{6}$$

The flow utility for a worker of type y and current productivity y' in the informal sector can be expressed as:

$$u_I(y', y) = w_I(y', y)[1 + \hat{\delta}_3], \text{with} \tag{7}$$

$$\hat{\delta}_3 = \mu\delta_3$$

where $w_I(y', y)$ is the hourly wage[37] in the informal sector for a worker of type y and match-specific productivity y', δ_3 is the amount of social assistance (subsidized health) that workers receive from

[35] Notice that $\hat{\delta}_2 < 0$ if $\tau(\delta_1 + \delta_2) > \delta_2$. Therefore, if the value of the SS services received is higher than the cost of contributing to the system, $\hat{\delta}_2$ works as a transfer and not as a tax.

[36] The superscripts indicate that the firm would have to make a severance payment if its match with this worker ended.

[37] Since there are no severance payments in the informal sector, there is no distinction between wages for a new hire versus an incumbent worker.

the government as percentage of their wage in the informal sector,[38] and μ is a parameter that measures the workers valuation of the social assistance benefits received, where $0 \leq \mu \leq 1$.[39]

So, informal workers are a "vulnerable" population in the sense that, even if they may have access to partial insurance against health shocks due to a government subsidized health program, they do not have any insurance against unemployment shocks (i.e., severance payments) or aging (i.e., mandatory pension or retirement accounts).

3.2 Firms

The small economy produces two composite goods: tradables and non-tradables. There are two productive sectors in this economy: formal and informal. The formal and informal sectors are assumed to produce tradables and non-tradables, respectively.

Each sector has a continuum of small firms in the unit interval, which are identical in all respects within each sector. Each firm has one job and maximizes the present discounted value of profits and chooses whether to open a job vacancy and hire a worker or not, so the number of jobs/firms is endogenous. Since the profit maximization condition requires that the marginal value of a vacancy must be zero, this is exactly equivalent to a zero-profit condition for firm entry.

Firms can only adjust to meet demand through changes in the extensive margin (number of jobs offered/employed people), but not through the intensive margin.

The main difference between firms across sectors is that informal-sector firms are not affected by labor market regulations such as social security contributions and severance payments. Distributions of productivity and wages are also different.

Let V_i be the present-discounted value of expected profit from a vacant job in sector i, and $J_i(y', y)$ be the present-discounted value of expected profit from a filled job in sector i with a worker of type y and match-specific productivity y'.

Firms in sector i open vacancies and search among the pool of job seekers, which involves a hiring cost c, assumed to be constant.[40] They also face some uncertainty when meeting a job seeker, since they do not know with certainty the type of worker they will meet, and conditional on type, how productive that worker will be at the job.

[38] I assume the benefit from social assistance is proportional to the informal-sector wage. For the case of Colombia, δ_3 corresponds to allocated health expenditures in the Subsidized Regime program as percentage of nominal wage. However, not all workers receive the subsidy: only those workers whose families are categorized as "poor" as determined by a Poverty Index Score, using the Census of the Poor (SISBEN).

[39] I am assuming that employee valuations of these services may be lower, equal or higher than in the SS system ($\mu < \tau$, $\mu = \tau$, $\mu > \tau$), depending on the perception of the efficiency of the services provided by the public sector. In the case where $\mu = 0$, informal workers do not value the services offered by the government. In this case, any change in subsidized health expenditures, δ_3, will not change the flow income in the informal sector. If $\mu > 0$, an expansion in subsidized health expenditures will cause a higher flow income in the informal sector with a consequent behavioral impact.

[40] The hiring cost can also depend on productivity or wages over the business cycle, but it is reasonable to assume is constant in the steady state.

The flow value of having a vacancy in the formal sector is as follows:

$$rV_F = -c + \frac{m(\theta)}{\theta} Emax[J_F(y', y) - V_F, 0]$$

Formal sector vacancies meet searching workers (only unemployed workers) at the rate $\frac{m(\theta)}{\theta}$. If the job is filled, the firm gets the expected capital gains, $J_F(y', y) - V_F$.

Given the assumption on the match-specific productivity this is equivalent to:[41]

$$rV_F = -c + \frac{m(\theta)}{\theta} \int_0^{\infty} \int_{R_{UF}(y)}^{\infty} [J_F(y', y) - V_F] dH_F(y' \mid y) dF_U^*(y) \tag{8}$$

where $F_U^*(y)$ is the distribution of y among the unemployed. Using Bayes rule:

$$dF_U^*(y) = \frac{u(y)f(y)}{\int_0^{\infty} u(y)f(y)dy} dy,$$

where $u(y)$ is the unemployment rate conditional on y.

On the other hand, informal-sector firms meet not only unemployed but also formal-sector workers affected by a 'bad' shock. The rate at which informal-sector vacancies meet unemployed workers is $\frac{\alpha}{\vartheta}$, while the rate at which informal-sector vacancies meet formal-sector workers of type y affected by a 'bad' shock is $\frac{\lambda H_F(R_{FU}(y)|y)}{\vartheta}$.[42]

The flow value of having a vacancy in the informal sector is:

$$rV_I = -c + \frac{\alpha}{\vartheta} Emax[J_I(y', y) - V_I, 0] + \frac{\lambda H_F(R_{FU}(y) \mid y)}{\vartheta} Emax[J_I(x, y) - V_I, 0]$$

Given the assumption on the productivity shock, this is equivalent to:

$$rV_I = -c + \frac{\alpha}{\vartheta} \left[\frac{u}{u + n_F} \right] \int_0^{\infty} \int_{R_{UI}(y)}^{\infty} [J_I(y', y) - V_I] dH_I(y' \mid y) dF_U^*(y) + \tag{9}$$

$$+ \frac{\lambda}{\vartheta} \left[\frac{n_F}{u + n_F} \right] \int_0^{\infty} \int_{R_{FI}(y)}^{\infty} H_F(R_{FU}(y) \mid y)[J_I(x, y) - V_I] dH_I(x \mid y) dF_F^*(y)$$

where $F_F^*(y)$ is the distribution of y among the job seekers in the formal sector that can be expressed as:

$$dF_F^*(y) = \frac{n_F(y)f(y)}{\int_0^{\infty} n_F(y)f(y)dy} dy.$$

A formal-sector firm matched with a worker of type y and match-specific productivity y' receives some net return for a job, $\pi_F(y', y)$, given by the market value of output minus the net cost of labor

[41] Notice also that the value of the vacancy does not depend on y.

[42] Informal-sector vacancies meet formal-sector workers at rate $\frac{\lambda H_F(R_{FU}(y)|y)}{\vartheta}$ because every time a formal-sector worker experiences a productivity shock, he meets an informal-sector opportunity only after experiencing a 'bad' shock.

(after paying SS contributions). A positive or negative productivity shock arrives at rate λ, and two possible cases arise: if the shock is 'good', the match continues with the capital gain (or loss) $J_F(x,y) - J_F(y',y)$; if the shock is 'bad', the match ends, the firm pays firing costs, s, and posts a new vacancy, so the capital loss suffered is $V_F(y) - J_F(y',y) - s$.

The flow value of a filled job in the formal sector with a worker of type y and match-specific productivity y' (new hire) is:

$$rJ_F(y',y) = \pi_F(y',y) + \lambda H_F(R_{FU}(y) \mid y)[V_F - J_F(y',y) - s] + \lambda \int_{R_{FU}(y)}^{\infty} [J_F(x,y) - J_F(y',y)] dH_F(x \mid y) \tag{10}$$

where $\pi_F(y',y)$ is the nominal value of a job's output in the formal sector, which can be defined as:

$$\pi_F(y',y) = p_F y' - w_F(y',y)(1+\delta_1) \tag{11}$$

and p_F is the price of a formal-sector good (price of tradable good).

The flow value of a filled job in the formal sector with a worker of type y and productivity x (incumbent) is:

$$rJ_F(x,y) = \pi_F^s(x,y) + \lambda H_F(R_{FU}(y) \mid y)[V_F - J_F(x,y) - s] + \lambda \int_{R_{FU}(y)}^{\infty} [J_F(x',y) - J_F(x,y)] dH_F(x' \mid y) \tag{12}$$

where

$$\pi_F^s(x,y) = p_F x - w_F^s(x,y)(1+\delta_1) \tag{13}$$

Notice that $rJ_F(x,y)$ differs from $rJ_F(y',y)$ because the wage associated with each function is different[43].

The flow value of a filled job in the informal sector with a worker of type y and current productivity y' is as follows:

$$rJ_I(y',y) = \pi_I(y',y) + \lambda H_I(R_{IU}(y) \mid y)[V_I - J_I(y',y)] + \lambda \int_{R_{IU}(y)}^{\infty} [J_I(x,y) - J_I(y',y)] dH_I(x \mid y) \tag{14}$$

where the nominal value of a job's output in the informal sector, $\pi_I(y',y)$ is defined as:

$$\pi_I(y',y) = p_I y' - w_I(y',y) \tag{15}$$

and p_I is the price of the informal sector good (non-tradable good).

The informal-sector job yields net return for the firm firm $\pi_I(y',y)$. The match may break (without any firing cost involved) if a 'bad' productivity shock arrives (x below some productivity threshold $R_{IU}(y)$), and the firm posts a new vacancy, so the capital loss suffered is $[V_I - J_I(y',y)]$. Otherwise, the match continues and the firm gets the corresponding capital gain or loss, $J_I(x,y) - J_I(y',y)$.

[43] I will later show that $w_F^s(x,y) \geq w_F(x,y)$.

3.3 Wage Determination

3.3.1 Formal Sector

Formal-sector wages in the steady state are determined by workers and firms using Nash bargaining, given the exogenous worker bargaining parameter, β.

An unemployed worker of type y with match-specific productivity y' and a formal sector firm decide to form a match if it is worth it for both, that is if the joint surplus is positive, or equivalently, if $R_{UF}(y) \leq y' \leq \infty$. If they match, they decide how to split the surplus and negotiate a wage contract using Nash bargaining, given the exogenous worker bargaining parameter β.

The initial wage[44] is given by:

$$\max_{w_F(y',y)} [N_F(y',y) - U(y)]^{\beta}[J_F(y',y) - V_F]^{1-\beta}$$

The standard sharing rule using the free entry condition ($V_F = 0$) implies

$$(1-\beta)(1+\delta_1)[N_F(y',y) - U(y)] = \beta(1-\hat{\delta}_2)J_F(y',y)$$

After doing some algebra, the corresponding formal-sector wage equation for a new hire of type y and current productivity y' is:

$$w_F(y',y) = \beta\left[\frac{p_F y'}{1+\delta_1} - \frac{\lambda s}{(1+\delta_1)}\right] + (1-\beta)\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y)\int_{R_{FI}(y)}^{\infty}[N_I(x,y) - U(y)]\, dH_I(x \mid y)}{(1-\hat{\delta}_2)}\right] \tag{16}$$

The wage negotiated in the formal sector is a weighted average between the productivity of the worker (adjusted by the expected severance cost) and the worker's continuation value (adjusted by a term that captures the flows from formal to informal).

The expected future severance cost reduces the benefits the firm gets if it accepts the bargain with the new hire, improving its bargaining power in the negotiation, and therefore reducing the negotiated wage.

The continuation value reflects not only the value for the worker if he does not accept the bargain (the flow value of unemployment), but also the benefits for the worker if he accepts the job, including the possibility of later moving to the informal sector. After being employed in the formal sector, the worker may be affected by a 'bad' productivity shock, and either flow to unemployment or to the informal sector. The expected gains of these two possible states worsen the worker's bargaining position. If the worker expects to get larger gains after being affected by a shock while working in the formal sector, the continuation value is reduced and the worker is willing to accept a lower formal-sector wage.

In general, if the worker has low productivity, a low continuation value, low bargaining power, and has to pay low non-wage labor costs as a formal employee, the worker is willing to accept a

[44] This is the wage for new hires in the formal sector that have not been affected by a shock.

lower wage in the negotiation.

Since it is assumed that the wage is renegotiated every time a productivity shock arrives, wages for employed workers of type y and current productivity x[45] are determined by workers and firms using Nash bargaining, given the exogenous worker bargaining parameter β, as follows:

$$\max_{w_F^s(x,y)} [N_F(x,y) - U(y)]^{\beta}[J_F(x,y) - (V_F - s)]^{1-\beta}$$

The corresponding standard sharing rule using the free entry condition ($V_F = 0$) is:

$$(1-\beta)(1+\delta_1)[N_F(x,y) - U(y)] = \beta(1-\hat{\delta}_2)[J_F(x,y) + s]$$

The formal-sector wage equation for an incumbent worker of type y and current productivity x is:

$$w_F^s(x,y) = \beta\left[\frac{p_F x}{1+\delta_1} + \frac{rs}{(1+\delta_1)}\right] + (1-\beta)\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y)\int_{R_{FI}(y)}^{\infty}[N_I(x',y) - U(y)]\, dH_I(x' \mid y)}{(1-\hat{\delta}_2)}\right] \tag{17}$$

Notice that $w_F^s(x,y) \geq w_F(x,y)$ since the severance tax now worsens the firm's bargaining position and reduces the benefits the firm gets if it does not accept the bargain with the old hire, thereby reducing its bargaining strength in the negotiation and increasing the negotiated wage.

3.3.2 Informal Sector

I assume there is bargaining over wages in the informal sector.[46]

The wage for a worker type y solves:

$$\max_{w_I(y',y)} [N_I(y',y) - U(y)]^{\beta}[J_I(y',y) - V_I]^{1-\beta}$$

The F.O.C (assuming interior solution), gives us the following standard sharing rule:

$$(1-\beta)[N_I(y',y) - U(y)] = \beta(1+\hat{\delta}_3)[J_I(y',y) - V_I]$$

The wage equation for the informal sector is:

$$w_I(y',y) = \beta\left[p_I y'\right] + (1-\beta)\left[\frac{rU(y)}{(1+\hat{\delta}_3)}\right] \tag{18}$$

In this case, neither the productivity nor the continuation value term needs to be adjusted since there is no severance tax or flows from informal to formal. However, high benefits in the form of

[45] This is the wage for incumbent workers who have been affected by a shock, so $R_{FU}(y) \leq x \leq \infty$.

[46] I conceive the informal sector as a 'disadvantaged' countercyclical sector in which low-skilled workers are negotiating wages with small firms, rather than as unregulated pro-cyclical self-employment. In Colombia, lack of compliance with social security is also a small firm phenomenon: approximately 87.2 percent of those informal under the SP definition are in small firms (≤ 5 employees).

better access to subsidized health (high $\hat{\delta}_3$) worsen the worker's bargaining position, which leads to a lower wage in the bargaining process.

3.4 Optimal Decision Rules and Reservation Productivities

Optimal decision rules are characterized by a reservation value property.
Reservation productivities are obtained by:

$$\begin{aligned} R_{FU}(y) : N_F(R_{FU}(y), y) = U(y) \quad &\Longleftrightarrow \quad J_F(R_{FU}(y), y) = V_F - s = -s \\ R_{UF}(y) : N_F(R_{UF}(y), y) = U(y) \quad &\Longleftrightarrow \quad J_F(R_{UF}(y), y) = V_F = 0 \\ R_{IU}(y) : N_I(R_{IU}(y), y) = U(y) \quad &\Longleftrightarrow \quad J_I(R_{IU}(y), y,) = V_I = 0 \\ R_{UI}(y) : N_I(R_{UI}(y), y) = U(y) \quad &\Longleftrightarrow \quad J_I(R_{UI}(y), y) = V_I = 0 \\ R_{FI}(y) : N_I(R_{FI}(y), y) = U(y) \quad &\Longleftrightarrow \quad J_I(R_{FI}(y), y) = V_I = 0 \end{aligned}$$

The Nash surplus sharing rule guarantees mutual agreement between the worker and the firm. Notice that $R_{IU}(y)$=$R_{UI}(y)$=$R_{FI}(y)$. On the other hand $R_{UF}(y) \neq R_{FU}(y)$ since when the match breaks, the formal sector firm has to pay a severance cost.

The corresponding reservation wages can be obtained through the Nash bargaining wage rule which maps productivities into wages, conditioning on type.

3.4.1 Reservation Productivity, $R_{FU}(y)$

A match in the formal sector is continued if and only if it is in the mutual interest of the worker and the firm to do so, so a necessary condition for match continuation is that the joint surplus must be non-negative. In equilibrium, the surplus is an increasing function of x,[47] and by definition $R_{FU}(y)$ is the threshold productivity above which the joint surplus is never negative. Therefore, $R_{FU}(y)$ is defined by the zero surplus condition:

$$N_F(R_{FU}(y), y) - U(y) + J_F(R_{FU}(y), y) + s = 0$$

Using the sharing rule combined with the free entry condition, I get:

$$J_F(R_{FU}(y), y) = -s$$

Substitution and integration by parts gives:

$$R_{FU}(y) = \frac{A_{FU}(y)}{(1-\hat{\delta}_2)P_F} - \frac{\left[1-\beta(1-\hat{\delta}_2)\right] rs}{(1-\beta)P_F} - \frac{\lambda}{r+\lambda} \int_{R_{FU}(y)}^{\infty} \left[1 - H_F(x' \mid y)\right] dx' \tag{19}$$

[47] See Appendix 6.1 for the surplus analytical expression.

The term $A_{FU}(y)$ captures the worker's outside option adjusted by a term that captures flows from formal to informal as follows:

$$A_{FU}(y) = (1+\delta_1)\left[rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} \big[N_I(x', y) - U(y)\big]\, dH_I(x' \mid y)\right] \tag{20}$$

This corresponds to a modified version of the standard upward-sloping job destruction curve, in which, for a given y, higher θ implies better worker's outside opportunities (higher $U(y)$) , and therefore more marginal jobs are destroyed (higher $R_{FU}(y)$). There is a secondary effect in play caused by the movement from formality to informality, since higher $R_{FU}(y)$ means higher probability of discontinuing the formal sector match, which negatively affects $A_{FU}(y)$. This equilibrium effect mitigates the impact of θ on $R_{FU}(y)$.

Given $U(y)$, the reservation productivity when transitioning from formal to unemployment is an increasing function of δ_1 and $\hat{\delta}_2$, and a decreasing function of $\hat{\delta}_3$, s and P_F, conditional on y.

The higher the firm's non-wage labor costs and the lower the price of the formal sector good or the severance cost, the higher the reservation productivity firms require to maintain the match after a productivity shock hits. Firms become pickier about the workers they are willing to 'retain' or 'continue employing'.

The lower the expected utility of the worker in the current match, either because of higher non-wage labor costs (adjusted by valuations), or lower $\hat{\delta}_3$ (receiving less benefits when moving to the IS), the higher the minimum productivity workers require to maintain a match (workers become pickier as well).

Also, $R_{FU}(y)$ is increasing in y if the adjusted outside option term $A_{FU}(y)$ dominates the integral term ('labor hoarding effect') in equation (19), since both terms are increasing in y.[48]

3.4.2 Reservation Productivity, $R_{UF}(y)$

Analogously, the reservation productivity $R_{UF}(y)$ is:

$$R_{UF}(y) = \frac{A_{UF}(y)}{(1-\hat{\delta}_2)P_F} + \frac{\Big[1-\beta(1-\hat{\delta}_2)\Big]\lambda s}{(1-\beta)P_F} - \frac{\lambda}{r+\lambda}\int_{R_{FU}(y)}^{\infty} \big[1 - H_F(x' \mid y)\big]\, dx' \tag{21}$$

where

$$A_{UF}(y) = (1+\delta_1)\left[rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} [N_I(x, y) - U(y)]\, dH_I(x \mid y)\right] \tag{22}$$

So:

$$R_{UF}(y) = R_{FU}(y) + \frac{\Big[1-\beta(1-\hat{\delta}_2)\Big](\lambda + r)s}{(1-\beta)P_F} \tag{23}$$

[48] This is the case for the particular set of parameter values chosen in the estimation.

Notice that $R_{UF}(y) \geq R_{FU}(y)$ if the second term is weakly positive. Formal sector firms are pickier when hiring a worker than when laying him or her off because in the first case they do not have to pay a firing cost.

3.4.3 Reservation Productivity $R_{IU}(y)$

Informal sector matches are destroyed when idiosyncratic productivity $x < R_{IU}(y)$, so the reservation productivity $R_{IU}(y)$ is defined by the condition:

$$J_I(R_{IU}(y), y) = 0$$

Substitution gives:

$$R_{IU}(y) = \frac{A_{IU}(y)}{(1+\hat{\delta}_3)P_I} - \frac{\lambda}{r+\lambda} \int_{R_{IU}(y)}^{\infty} [1 - H_I(x \mid y)]\, dx \tag{24}$$

where $A_{IU}(y) = rU(y)$.

Given $U(y)$, the reservation productivity when transitioning from the informal sector to unemployment is a decreasing function in $\hat{\delta}_3$ and P_I. The higher the worker's social assistance benefits (adjusted by valuations), the lower the minimum productivity workers require to maintain a match with an informal-sector firm. Firms are also less picky when profitability is high (high P_I).

3.5 Job Creation Conditions

In this section, I derive the job creation conditions in the formal and the informal sectors that allow us to pin down equilibrium labor market tightness, θ and ϑ respectively.

In equilibrium, formal sector firms open vacancies until rents are exhausted, so free entry implies $V_F = 0$. The free entry condition and equation (8) imply:

$$\frac{c\theta}{m(\theta)} = \int_0^{\infty} \int_{R_{UF}(y)}^{\infty} J_F(y', y) dH_F(y' \mid y) dF_U^*(y)$$

Equilibrium θ is such that the expected cost of hiring a worker in the formal sector is equal to the expected benefit of hiring an unemployed worker.

Expressing $J_F(y', y)$ as a function of reservation productivity $R_{UF}(y)$ in the latter equation gives:

$$c = \frac{m(\theta)}{\theta} \int_0^{\infty} \int_{R_{UF}(y)}^{\infty} \frac{(1-\beta)p_F[y' - R_{UF}(y)]}{r+\lambda} dH_F(y' \mid y) dF_U^*(y) \tag{25}$$

In the informal sector, free entry and equation (9) gives:

$$c = \frac{\alpha}{\vartheta} \left[\frac{u}{u+n_F}\right] \int_0^{\infty} \int_{R_{UI}(y)}^{\infty} J_I(y', y) dH_I(y' \mid y) dF_U^*(y)$$

$$+\frac{\lambda}{\vartheta}\left[\frac{n_F}{u+n_F}\right]\int_0^\infty\int_{R_{FI}(y)}^\infty H_F(R_{FU}(y)\mid y)J_I(x,y)dH_I(x\mid y)dF_F^*(y)$$

Expressing the firm's value function in terms of reservation productivities:

$$c=\frac{\alpha}{\vartheta}\left[\frac{u}{u+n_F}\right]\int_0^\infty\int_{R_{UI}(y)}^\infty\frac{(1-\beta)p_I\left(y'-R_{IU}(y)\right)}{r+\lambda}dH_I(y'\mid y)dF_U^*(y) \tag{26}$$

$$+\frac{\lambda}{\vartheta}\left[\frac{n_F}{u+n_F}\right]\int_0^\infty\int_{R_{FI}(y)}^\infty H_F(R_{FU}(y)\mid y)\left[\frac{(1-\beta)p_I\left(x-R_{IU}(y)\right)}{r+\lambda}\right]dH_I(x\mid y)dF_F^*(y)$$

3.6 Steady State Conditions

In this section I derive the equilibrium rate of unemployment, given the other equilibrium objects. Let $u(y)$ be the fraction of type-y workers in unemployment, $n_I(y)$ be the fraction in informal-sector employment and $n_F(y)$ be the fraction in formal-sector employment, so that $u(y)+n_I(y)+n_F(y)=1$. In the steady state, the mean rate of unemployment conditional on y is constant, so two steady state conditions apply.

First, the sum of the flows into unemployment must equal the sum of the flows out of unemployment:

$$\lambda H_I(R_{IU}(y)\mid y)n_I(y)+\lambda H_F(R_{FU}(y))H_I(R_{UI}(y)\mid y)n_F(y)=$$

$$=\alpha[1-H_I(R_{UI}(y)\mid y)]u(y)+m(\theta)[1-H_F(R_{UF}(y)\mid y)]u(y)$$

Second, the analogous condition for the formal sector:

$$\lambda H_F(R_{FU}(y))n_F(y)=m(\theta)[1-H_F(R_{UF}(y)\mid y)]u(y)$$

Solving the above equations gives:

$$u(y)=\frac{\lambda H_F(R_{FU}(y)\mid y)H_I(R_{IU}(y)\mid y)}{\alpha[1-H_I(R_{UI}(y)\mid y)]L(y)+m(\theta)[1-H_F(R_{UF}(y)\mid y)]K(y)+\lambda H_F(R_{FU}(y))H_I(R_{IU}(y)\mid y)} \tag{27}$$

$$n_F(y)=\frac{m(\theta)[1-H_F(R_{UF}(y)\mid y)]K(y)}{\alpha[1-H_I(R_{UI}(y)\mid y)]L(y)+m(\theta)[1-H_F(R_{UF}(y)\mid y)]K(y)+\lambda H_F(R_{FU}(y))H_I(R_{IU}(y)\mid y)} \tag{28}$$

$$n_I(y)=\frac{\alpha[1-H_I(R_{UI}(y)\mid y)]L(y)}{\alpha[1-H_I(R_{UI}(y)\mid y)]L(y)+m(\theta)[1-H_F(R_{UF}(y)\mid y)]K(y)+\lambda H_F(R_{FU}(y))H_I(R_{IU}(y)\mid y)} \tag{29}$$

where $L(y)$ and $K(y)$ are

$$L(y)\equiv H_F(R_{FU}(y)\mid y)$$

$$K(y) \equiv H_F(R_{FU}(y) \mid y) + H_I(R_{IU}(y) \mid y)\,[1 - H_F(R_{FU}(y) \mid y)]$$

Equation (27) corresponds to a modified version of the Beveridge curve, a negative relation between labor market tightness and unemployment, or alternatively, between vacancies and unemployment. When θ increases, $m(\theta)$ increases as well, encouraging job creation and reducing $u(y)$ ('job creation' effect). Stochastic job matching and endogenous job destruction give us an additional counteracting effect ('reservation productivity' effect): an increase in θ also increases $rU(y)$ (more outside opportunities), which increases $R_{UF}(y)$, $R_{UI}(y)$, and $R_{FU}(y)$. The first two discourage job creation and the latter encourages job destruction, both of which increase unemployment.[49]

Aggregate unemployment can be obtained by aggregating across types:

$$u = \int_0^\infty u(y)f(y)dy$$

Intuitively, in equilibrium there will be some imperfect sorting of workers among sectors based on their types, y. 'High' type workers are more likely to take formal-sector jobs, while 'low type' workers are more likely to take informal-sector jobs, and medium type workers will take both.[50]

In addition to compositional effects, changes in policy parameters affect the steady state distributions of productivities among sectors, affecting also the distributions of wages across formal and informal employment.

3.7 Determination of the Value of Unemployment

The only thing needed to solve the model is to determine $U(y)$ for all values of y, as a function of reservation productivities and labor market tightness parameters.

Equation (1), together with equations (26) and (32), imply:

$$rU(y) = b + \alpha\beta(1+\hat{\delta}_3)\int_{R_{UI}(y)}^{\infty} \frac{p_I\,(y' - R_{IU}(y))}{r+\lambda} dH_I(y' \mid y) + \tag{30}$$

$$+m(\theta)\beta(1-\hat{\delta}_2)\int_{R_{UF}(y)}^{\infty} \frac{p_F\,(y' - R_{UF}(y))}{(1+\delta_1)(r+\lambda)} dH_F(x \mid y)$$

[49]I assume that the first effect dominates the second effect (since empirical evidence supports a downward-sloping Beveridge curve). Stochastic job matching means that changes in labor taxes, subsidized health or prices may shift the relationship between vacancies and unemployment, and that the effect of a change in the relevant policy parameter on u via job offers is mitigated, since there are additional effects that offset this demand-side effect.

[50]This is more or less consistent with the empirical fact that the formal sector is skilled labor intensive while the informal sector is unskilled labor intensive in Colombia: average years of schooling for a FS worker is 12.14 , while for an IS worker it is 9.12 years (Source: Author's calculations based on ECH, second quarter of 2003).

To solve the model, some following functional form assumptions are made. Specifically:

$$
\begin{aligned}
F(y) &= \Phi\left(\frac{lny - \mu_y}{\sigma_y}\right) \text{ for } y > 0 \\
H_i(x \mid y) &= \Phi\left(\frac{lnx - \mu_i(y)}{\sigma_i}\right) \text{ for } x > 0, i = F, I \\
H_i(y' \mid y) &= \Phi\left(\frac{lny' - \mu_i(y)}{\sigma_i}\right) \text{ for } y' > 0, i = F, I \\
\mu_i(y) &= B_i log(y) \\
m(\theta) &= A\theta^{1-\alpha_m}
\end{aligned}
$$

Once the model is solved numerically,[51] the empirical strategy consists of estimating the model using micro data, and then simulating how labor market policies, subsidized health programs and relative prices affect the division of the labor force into unemployment, informal and formal sectors (aggregate effects), the mix of worker types in the two sectors (compositional effects) and the distribution of wages (distributional effects).[52]

4 Empirical Strategy

4.1 Descriptives

To estimate the model, I use data from the Encuesta Continua de Hogares for the second quarter of 2003, representative for 13 metropolitan areas.[53] I choose this period to approximate the pre-reform's steady state equilibrium because: 1) An informality module is available;[54] 2) Information on health access disaggregated by contributive or subsidized regime is also available;[55] 3) The first three quarters of 2003 have a roughly constant unemployment rate, and the model is derived under a steady state assumption.

In addition to standard demographic socio-economic variables (age, gender, marital status, educational attainment, etc), the sample is described by the following labor market variables:

$$(\{W_s\}_{s \in E_i}; \{t_{es}\}_{s \in E_i}; \{t_{nes}\}_{s \in E_i}; \{t_{us}\}_{s \in U}; \{h_s\}_{s \in E_i})_{i=F,I}$$

W_s: Accepted monthly wages for individual s, where $s \in E_i$, so each individual can be employed in sector i (formal or informal)

t_{es}: Employment duration[56] for individual s, where $s \in E_i$ (right -censored)

[51] For details on the computational algorithm to solve the model see Appendix, Section 6.5.

[52] I simulate the steady state distributions of equilibrium productivity and wages in both sectors, instead of deriving them them analytically. See Appendix, Sections 6.2 and 6.3 for the analytical derivations.

[53] These areas include the following cities and their metropolitan areas: Bogota, Cali, Medellin, Barranquilla, Bucaramanga, Manizales, Pasto, Pereira, Cucuta, Villavicencio, Ibague, Monteria and Cartagena.

[54] An informality module is available for the second quarter of every year after 2001, and every two years before 2001.

[55] Before 2001, there is no information on health access by contributive or subsidized regimes.

[56] This variable measures months in the current job.

t_{nes}: Non-Employment duration[57] of previous employment for individual s, where $s \in E_i$.

t_{us}: Unemployment duration for individual s, where $s \in U$ (incomplete spells for unemployed, right -censored)

h_s: Weekly hours of work for individual s, where $s \in E_i$.

Even though the data are not longitudinal, retrospective questions about previous unemployment and employment status for both employed and unemployed individuals are available, allowing for the construction of transition flows across the different states. There is no retrospective information on social security contributions, so to determine whether a worker was considered formal or informal before being unemployed or employed in the current job, the definition of informality based on firm size and occupation is used.[58]

I choose a subsample of full-time, male workers with completed primary education, and at most 20 years of schooling.[59] I also drop observations that correspond to the lowest 10% and the highest 2% of wages in each human capital level.[60] This trimming is performed to reduce measurement error and other sources of observed heterogeneity, since according to the model, agents are heterogeneous ex-ante only because of different human capital levels.[61]

The size of the subsample is 20,011 observations, representing 9.7 million people.

Table 1 shows the descriptive statistics for the employed population by sector.

When using the SP definition, the estimated informality rate for 2003 is 60.19 percent (s.e. of .0038), or the informal sector is 1.5x the size of the formal sector.

In comparison with the formal sector, the informal sector is characterized by:

- *Lower and Less Dispersed Wages*: Log hourly wages are on average lower and less dispersed than in the formal sector. The log-wage gap in the raw data without any controls is 0.63 (s.e. 0.044)[62]

- *More job stability*: Average job duration in the informal sector is 92.3 months (7.69 years) while in the formal sector it is 82.7 months (6.89 years)

- *Higher non-employment duration*: If unemployed/inactive in the previous year, informal sector workers face higher unemployment/inactivity duration on average than their formal sector counterparts (7.34 vs. 6.33 months)

- *Lower human capital levels*: Mean years of schooling among informal-sector workers is 9.13 vs. 12.15 among formal-sector workers.

[57] This variable measures months without employment between the current job and the previous job (retrospective question), so I cannot identify whether it refers to unemployment or inactivity duration. I also have unemployment duration for the unemployed population; however, this variable is right-censored.

[58] In this period, approximately 85.92 percent of informal sector-workers based on the SP definition are also informal based on the FSO definition.

[59] I drop workers from the bottom of the distribution because of measurement error in the wage data for these type of workers; I also drop workers from the top due to the small sample sizes.

[60] The reason for the asymmetry is that I want to avoid measurement error in high and low wages, but I also want to avoid wages on temporary or other jobs that are correctly measured, but fall outside the model scope.

[61] Female and part-time workers are also more likely to select themselves out of the labor force.

[62] This measure of wages includes tips and commissions, but excludes non-monetary payments.

Table 2 shows the variance of the earning distribution in the overall economy, broken down by two components: a) Variability within sectors; b) Variability between sectors. Despite the substantial differences in relative earnings, 97.5 percent of variance in log wages is explained by within-sector variance.[63]

Table 3 shows the descriptive statistics for the unemployed population. The estimated unemployment rate for the year 2003 is quite high: 18.5 percent (s.e. of .00274). Mean unemployment duration is quite high (51.5 weeks),[64] and higher than the mean non-employment duration for the employed population (6.9 months).

4.2 Labor Market Dynamics

Standard static analysis of labor markets that analyze stocks of workers in different states does not tell us anything about where those workers arrived from, how long they will stay, or where they will go next. I construct a set of statistics to analyze labor market dynamics and derive some stylized facts about movement across sectors in the Colombian labor markets, using the retrospective questions from the cross-section database.

4.2.1 Annual Discrete Transition Matrix

Individuals in the sample fall into four labor market states: unemployed (U), employed in the formal sector (F), employed in the informal sector (I), and other (O).[65]

I estimate an annual discrete transition matrix by estimating gross worker flows across states.[66]

Let m_{ij} be the number of individuals in the 2003 sample who were in state i in 2002 and state j in 2003. I observe some, but not all the transitions between states from 2012 to 2013.[67]

When considering transitions into unemployment, I observe all transitions m_{UU}, m_{FU}, m_{IU} and m_{OU}. When considering transitions into the formal sector, I observe m_{FF}, m_{IF} and $m_{UF} + m_{OF}$. When considering transitions into the informal sector, I observe m_{FI}, m_{II} and $m_{UI} + m_{OI}$. I also observed the following transitions into the 'other' state: m_{OO} and m_{UO}.

To recover m_{UI}, m_{OI}, m_{UF} and m_{OF} the following assumptions are made:

$$\frac{m_{OF}}{m_{UF} + m_{OF}} = \frac{m_{OI}}{m_{UI} + m_{OI}}$$

[63] It would be interesting if workers were assigned randomly to sectors, to see whether selection reduces or increases inequality, or what the aggregate wage variability would be if sectoral labor force quality were held constant (see Heckman & Sedlacek, 1985).

[64] This estimate can be upward biased because: 1) Unemployment duration is only observed for the currently unemployed; 2) It is more likely to have workers with higher lengths of unemployment in a sample of unemployed workers at a particular point in time (a 'stock' sample).

[65] Other includes the inactive population.

[66] Another approach may be to think about an underlying continuous-time Markov process that generates the discrete time mobility process, and therefore, estimate the transition probabilities accordingly. See Maloney and Bosch, 2007.

[67] In the survey, I have the following questions for the employed: 1) How long (months) have you been employed continuously in the current job?; 2) How long (months) were you without employment between your current job and the previous job? With these questions I can identify job-to-job transitions, and non-employment (unemployment or inactive) to job transitions.

$$m_{FU} + m_{IU} + m_{OU} = m_{UF} + m_{UI} + m_{UO}$$

The transitions from the 'other' employment state as a fraction of the total transitions from non-employment to employment are the same across sectors, and the sum of the flows into unemployment are equivalent to the sum of the flows out of unemployment (steady state assumption).

Table 4 contains the estimated transitions m_{ij}, for all i and j.

Once all transitions are recovered, the probability of transitioning from state i in 2012 to state j in 2013 can be estimated using:

$$\pi_{ij} = \frac{\hat{m_{ij}}}{\hat{n_i}},$$

where $\hat{n_i}$ is the estimated number of individuals in the 2003 sample who were in state i in 2002.[68]

Let N_{2003} be the number of observations in the 2003 sample and p_i the fraction of the 2002 sample in state i. Then:

$$\hat{n_i} = p_i N_{2003}$$

The estimated transition probabilities can be used to estimate the hazard (separation probabilities) out of sector i, conditional on being in state i up to period t, $h_{ii}(t)$, as follows:

$$\widehat{h_{ii}(t)} = 1 - \widehat{\pi_{ii}(t)}$$

Table 5 shows the estimated annual transition probabilities. The probability that an individual who was unemployed in the second quarter of 2002 remains unemployed in 2003 is 72 percent. Ignoring time aggregation bias,[69] this statistic suggests a hazard rate out of unemployment conditional on being unemployed of 0.28.

Mobility between sectors is dominated by mobility from the formal to the informal sector. When conditioning on working in the formal sector, 81 percent of formal sector workers stay formal over the course of a year. Even if persistence is very high, 6 percent transit to informal sector jobs and only 3 percent to unemployment. These figures imply a hazard out of the formal sector of 0.19.

Transitions from informality to formality are less important, quantitatively. Only approximately 4 percent of informal workers transit to formal sector jobs. Given that persistence is very high (93 percent of informal sector workers stay informal over the course of a year), the estimated hazard out of the informal sector is 0.07, lower than in the formal sector.

Table 6 shows the estimated hazards.

This empirical evidence supports the assumption in the model of direct transitions from formal to informal, without intervening unemployment. Transitions from informal to formal are less important.

It is worth noticing that these probabilities are not instantaneous transition probabilities, since

[68] In longitudinal data I observe n_i, but that is not the case in cross-section data since I have a different sample of individuals.

[69] Time aggregation bias may arise because the yearly measurement may combine multiple transitions into a single aggregate transition, so transitions that occur at a higher frequency are not captured here.

there are many workers who change status within a year whose transitions are not captured here.

4.2.2 Mean Employment and Unemployment Duration

Let t_i be the duration spell for state i. Using a partial model of expected unemployment duration where the hazard h_{ii} is constant, it can deduced that:

$$G(t_i) = 1 - exp\{-h_{ii}t\}$$

so mean duration in state i can be estimated using the estimated separation rate as follows:

$$\widehat{E(t_i)} = \frac{1}{\hat{h_{ii}}}$$

In the data there are three sources of bias in unemployment duration since: 1) Duration is only observed for the currently unemployed (incomplete spells); 2) The total length of the duration is not observed (right censoring); 3) Workers with longer unemployment spells are oversampled at at particular point in time. The first and third effects produce an upward bias in the estimate, while the second produces a downward bias in the estimate. Assuming exponential durations, the first two biases cancel out since under-sampling of short durations "cancels" with the underestimate of long durations.

Data on non-employment duration (complete spells for the employed population) and employment duration for workers in the formal and informal sectors (incomplete spells) are also available.

Table 7 shows the estimates for employment and unemployment duration, using duration data and estimated hazards. The estimated mean unemployment duration is 52 weeks using duration data, much lower than the estimates produced using hazards (183 weeks). This suggests that time aggregation bias can be quite significant.[70]

When using the FSO definition, a formal sector job lasts 69 months on average, an estimate close to the one using hazards rates. Mean length of informal employment (FSO definition) varies more when using the two methodologies: 100 vs. 167 months, suggesting that time aggregation in the informal-sector may also be quantitatively important.

4.3 Estimation

In the estimation, I am particularly interested in matching aggregate unemployment and employment rate figures, getting reasonable labor market tightness parameters for international standards,[71] and matching selected moments of the wage distribution in both sectors.

[70] There are many workers who make transitions from unemployment to other states that are not included in the hazard estimate, leading to a downward bias in the hazard rate of unemployment, and an upward bias in unemployment duration.

[71] There are no reliable estimates for the Colombian case since there is no data on job vacancies.

The parameter space is defined by the following 21 parameters:

$$\Omega = \{r, \lambda, b, \beta, A, \alpha_m, \mu_y, \sigma_y^2, B_F, B_I, \sigma_F, \sigma_I, P_F, P_I, \tau, \mu, \delta_1, \delta_2, \delta_3, c, \alpha\}$$

I partition the parameter space of the benchmark model in two groups: Ω_1 and Ω_2. In the first group, parameters are fixed based on previous results from micro studies or data. In the second group, parameters are estimated to match selected empirical moments, including the division of the labor force among unemployment, informal and formal-sector employment, employment and unemployment duration, and selected moments of the wage distribution.

Let Ω_1 be defined as:

$$\Omega_1 = \{r, P_F, P_I, \mu_y, \sigma_y^2, \lambda, b, \beta, A, \alpha_m, \delta_1, \delta_2, \delta_3, \tau, \mu\}$$

The parameter values are chosen with a quarter as the implicit unit of time.

Table 8 summarizes the value of the fixed parameters based on data or micro studies.

The real interest rate in Colombia in 2003 was approximately 7.8 percent,[72]so r is chosen to match the corresponding quarterly rate. Price indexes of tradable and non-tradable goods are used as proxies for the sectoral prices P_F and P_I, respectively.

The parameters μ_y, σ_y^2 are chosen to coincide with the corresponding empirical moments.[73]

I choose the rate of arrival of the productivity shock λ equal to 0.04 times per quarter. I use our micro data to test if the estimate falls within an admissible range. Using the separation rates of the formal and informal sectors, h_{FF} and h_{II}, I can express λ as follows:

$$\lambda = \frac{h_{FF}}{H_F(R_{FU}(y) \mid y)}$$

$$\lambda = \frac{h_{II}}{H_I(R_{IU}(y) \mid y)}$$

Using duration data, I estimate that a formal-sector job lasts on average 6.89 years (27.5 quarters) while an informal-sector job lasts 7.69 years (30.77 quarters). Assuming that employment duration follows an exponential distribution, I estimate formal and informal-sector mean separation rates of 0.036 and 0.032, respectively. Given that $0 \leq H_F(R_{FU}(y) \mid y) \leq 1$ and $0 \leq H_I(R_{IU}(y) \mid y) \leq 1$, I have that $0.032 \leq \lambda < \infty$ and $0.036 \leq \lambda < \infty$, , so assuming 0.04 shocks per quarter is within an admissible range in the model.[74]

The opportunity cost of leisure, b, is set equal to 1. In Colombia there were no unemployment benefits in this period, so one must think about what the monetary compensation for the unemployed should be, relative to the average wage, or the average 'replacement rate'. According to HKV2005, such a rate is 0.7 for European countries where benefits are relatively high, and 0.2

[72]Source: IMF, International Financial Statistics.

[73] I use educational attainment as a proxy for worker types.

[74] In ANV2009 λ is set to 0.5 using one year as a unit of time, while in Hornstein, Krusell and Violante (2005) (hereafter HKV2005) λ is set to 0.1 using one quarter as a unit of time.

for the U.S (at most). In Shimer (2005) the rate is about 0.4, since $b = 0.4$ and the average wage is close to 1. For the Colombian case, it may be reasonable to assume that this ratio would be close to 0.2. Considering that the average wage in the model is close to 6.75, the chosen b gives us a replacement rate of 0.15.

Following standard results in the literature, β is set equal to 0.5, so shares are split equally between workers and firms.[75]

The parameters of the matching function are set in the following way. I assume a standard Cobb-Douglas function given by $m(\theta)=A\theta^{1-\alpha_m}$, with $A = 2.0$ and $\alpha_m = 0.5$. Since there are no data on vacancies for Colombia, I follow standard results for the U.S. from Mortensen and Nagypal (2007) and Brugemann (2008). The first paper estimates this elasticity at 0.45 and the latter between 0.54 and 0.63.[76] Also, job-finding rates in the U.S. are estimated to be 0.45 per month, or 1.35 per quarter. I assume job-finding rates in Colombia in the formal sector to be closer to 1.5 per quarter.[77] Assuming a reasonable labor market tightness, θ, ranging between 0.5 and 1, gives us a technological parameter A ranging between 1.5 and 2.12.

The policy parameters δ_1, δ_2 and s chosen correspond to the estimated 2003 values: $s = 0$, $\delta_1 = 0.534$, $\delta_2 = 0.0837$, and $\delta_3 = 0.118$.[78]

With regards to workers' preferences, in the benchmark case it is assumed that workers do not value social security services ($\tau = 0$) as in a model without valuations, but they value social assistance benefits partially ($\mu = 0.5$), so a change in δ_3 affects workers' behavior.[79]

Let Ω_2 be defined as follows:

$$\Omega_2 = \{c, \alpha, B_F, B_I, \sigma_F, \sigma_I\}$$

I want to match seven empirical targets:[80] the unemployment rate, formal and informal-sector employment rates, formal and informal-sector employment durations, the differences in sectoral mean log-wages, and the ratio between the log-wage variance in the informal sector relative to the formal sector.

I simulate the model at the given parameter vector and calculate the moments of interest from

[75] Here, the Hosios condition that requires $\beta = \alpha_m$ is satisfied, but in this environment this condition does not imply efficiency search because there is a 'composition' externality that is not internalized. When a firm hires a type-y worker, it changes the mix of types in the pool from which other firms are hiring.

[76] Alternatively I could have set the value of the elasticity closer to the labor share, but there are not reliable estimates for Colombia. Shimer (2005) estimates this elasticity at 0.72.

[77] Notice that job-finding rates in the FS cannot be identified directly using data on unemployment duration because the hazard rate out of unemployment depends not only on $m(\theta)$ but also on the reservation productivity rules.

[78] To get s, severance in the formal sector is one monthly wage per year plus 1 percent interest (or 8.33 percent plus 1 percent interest=9.33 percent of average wage), but since it's paid every year and not when employment terminates, it's included in the total figure of δ_1. To get δ_3, I need to calculate the expected value of the services received in the subsidized regime as a percent of the wage. I multiplied the value of the services offered in the contributive regime (I assume their values are the same) by the corresponding probability of getting those services, so $\delta_3 = P_i(\delta_1 + \delta_2)$, where P_i is the proportion of IS workers who receive subsidized health in the data. See detailed employee and employer NWL costs in Tables 12 and 13.

[79] I will analyze in the policy experiments how results change with changes in valuations.

[80] For the model to be identified, at least five empirical moments are needed.

the simulated data. Then, I compare these moments to the empirical moments estimated using the real data.[81]

Then, for the construction of our simulation-based estimator, I implement the following two-step algorithm:[82]

1. Set guesses for the parameters of the productivity distributions $\{B_F, B_I, \sigma_F, \sigma_I\}$. Conditional on these guesses, estimate $\{c, \alpha\}$ to match u, n_f and n_I.

2. Conditional on $\{u$, n_f and $n_I\}$ obtained in the first step, estimate $\{B_F, B_I, \sigma_F, \sigma_I\}$ to match $\mu_{ln(W_F)} - \mu_{ln(W_I)}$, $\sigma_{ln(W_F)}/\sigma_{ln(W_I)}$, t_{e_F} and t_{e_I}. Iterate until convergence.

Table 9 shows the estimated parameters.

I estimate $\widehat{B_F}$ equal to 0.62 and $\widehat{B_I}$ equal to 0.41, so the mean productivity varies linearly with education in both sectors, but the conditional mean of the match-specific productivity is less sensitive to y in the informal than in the formal sector.

I estimate $\hat{c}$ equal to 2.50. This implies that the cost of posting a vacancy as a proportion of the model mean wage (vacancy ratio) is 37.04 percent, close to previous results in the literature.[83]

I estimate $\hat{\alpha}$ equal to 2.91 jobs per quarter in the informal sector, higher than the estimated rate of arrival in the formal sector, $\widehat{m(\theta)}$ equal to 1.67. These estimates seem reasonable given other results in the literature.[84]

The implied replacement ratio in the model is 0.148, reasonable considering there were no unemployment benefits in Colombia in the second quarter of 2003, and the estimated wage share is 0.63, within a reasonable range in the literature.[85]

The model can approximately match the aggregate unemployment rate, formal and informal-sector employment rates, mean employment duration in the formal sector, and the relative dispersion in log-wages within the informal sector relative to the formal. The model underestimates relative mean log-wages. This can be explained by the fact that in the model there is no minimum wage. Thus, if the moments of the wage distribution that the model produces were compared to the moments derived from a counterfactual distribution of wages in the data, absent minimum wages, the size of the bias would be smaller. The model also overestimates mean unemployment duration and produces shorter mean employment duration in the informal sector.

Table 10 presents a comparison between the simulated statistics and the data-based statistics.

Figures 13- 22 show the equilibrium objects and simulated distributions for the benchmark case.

Figure 13 shows the analytical density of worker types, y, assumed to be lognormal, compared

[81] To simulate from the distributions $f_F(y)$ and $f_I(y)$, samples are drawn using quantile transform since the analytical expressions of the densities are known, but not the underlying population parameters. After sampling worker types in each sector, one can simulate productivities (conditional on types) given the population parameters B_F and B_I, and then use the wage functions to draw wages. Finally, Kernel densities can be estimated using the simulated data and the simulation-based statistics of interest can be computed.

[82] This two-step algorithm is similar to the one used in Bradley, Postel-Vinay and Turon (2017).

[83] In Shimer (2005), the vacancy ratio is around 20 percent. In HKV(2005), the vacancy ratio is 32.4 percent.

[84] In Shimer(2005), job-finding rates are 1.35 per quarter.

[85] Wage share is the ratio of mean wages to mean productivities. In Shimer (2005), the wage share is 0.97.

with the non-parametric estimate of $F(y)$ using sample fractions at each educational level. The figure shows that the lognormal-assumption fits the data relatively well.

Figure 14 shows the steady state distribution of types across sectors, where the formal sector distribution is shifted to the right compared to the one in the informal sector, consistent with the empirical distribution. These densities are 'contaminated', i.e. only incorporate a restricted pool of workers in each sector.

Figures 15 and 16 show monotonic unemployment and hazard rates out of unemployment by type. The average unemployment rate and unemployment duration for workers with 'high' human capital levels are much lower than for those who are uneducated, given that these workers tend to accept both formal and informal-sector jobs.

Figure 17 presents the reservation productivity schedules. The figure shows that $R_{UF}(y) > R_{UI}(y)$ for all y, and both are strictly increasing in y. Conditional on type, workers in the formal sector are pickier and less likely to start a match (and more likely to discontinue a match[86]) than workers in the informal sector. 'High' type workers are pickier than 'low' type workers since they have more outside opportunities, an effect that for this parameterization is stronger than the 'labor hoarding' effect.

Figure 18 shows monotone formal and informal-sector employment rates. 'Low' type workers are more likely to be informal while 'high' type workers are more likely to be formal. I also observe this monotonicity empirically.

Figure 19 shows mean job durations, disaggregated by sector. Figures 20, 21 and 22 show simulated steady state distributions of types, productivities and wages shifted to the right in the formal sector.[87]

4.4 Results

4.4.1 Labor Tax Experiments

In this set of experiments I simulate an increase of 4.46 percent and 7.53 percent in employer and employee NWL costs, δ_1 and δ_2, respectively.[88] Three scenarios are analyzed under different workers' valuations of social security services.

Scenario 1: No valuation of SS services ($\tau = 0$)

If workers do not value SS contributions ($\tau = 0$), an increase in δ_2 is perceived as a pure net cost from the worker's perspective, while an increase in δ_1 will not directly affect the worker's value function.

A higher value of δ_1 makes vacancy creation less appealing for the formal-sector firm, decreasing θ from 0.67 to 0.62. An increase in δ_2 makes the formal-sector match less attractive from the worker's perspective. Increasing these NWLC shifts the reservation productivity $R_{FU}(y)$ up, so

[86] Here, $R_{FU}(y) = R_{UF}(y)$ since it is assumed in the simulations that $s = 0$.

[87] Notice how the simulated distribution of types in Figure 20 is a very good approximation to the analytical density shown in Figure 14.

[88] This is the actual increase from 2003 to 2012.

there is a higher probability of discontinuing the formal-sector match, reducing employment duration in the formal sector slightly (less than a month).

Because of the flows in the model from the formal to the informal sector, there is also a positive impact on vacancy creation in the informal sector. Formal-sector workers are more likely to end up in unemployment, so the expected benefits of an informal-sector firm hiring a formal-sector worker affected by a 'bad' shock are larger. Vacancy creation in the informal sector is therefore more attractive: ϑ goes up from 0.30 to 0.32. In other words, the informal-sector acts as a 'buffer' sector, absorbing some displaced workers from the other sector.

Given that there is more job destruction and less job creation in the formal sector, the formal-sector aggregate employment rate decreases from 29.7 to 27.5 percent, but the level of absorption in the informal sector is quite high, since the informal-sector aggregate employment rate increases from 52.2 to 53.9 percent. As a result, the overall unemployment rate increases slightly from 17.8 to 18.2 percent.[89]

This policy increases unemployment duration slightly (less than a month), and decreases the hazard out of unemployment by shifting up the reservation productivity schedule, $R_{UF}(y)$.

Figures 23, 24 and 25 summarize the impact of this policy on the distributions of worker types, productivities and wages. The distribution of types in both sectors shifts to the right and is more dispersed with this policy (due to the higher incidence of unemployment among the unskilled workers), but the quantitative impact is very small. Given the compositional effects of this policy and the upward shifts of the reservation productivity schedule, the density of productivities in the formal sector also shifts slightly to the right, causing an average formal-sector productivity rise. Also, when δ_1 increases, the benefits of the formal-sector firm negotiating with the worker are reduced, improving the firm's bargaining power in the negotiation, and causing a reduction in the formal-sector wage. From the worker's perspective, a higher δ_2 implies less benefits from the negotiation agreement with the firm, improved bargaining power, and therefore a higher formal sector wage.

There are also some additional effects that need to be considered, since a higher reservation productivity $R_{UF}(y)$ implies a lower continuation value in the wage negotiation, and therefore a lower formal-sector wage. The flow value of unemployment $U(y)$ is also affected in equilibrium, affecting the continuation value of the negotiation in both sectors.

Finally, the average formal-sector wage increases very little when considering all equilibrium effects, so the log wage gap is barely affected (See Figure 25.) Total log-wage variance is scarcely affected under this scenario.

In conclusion, the distributional impact of this policy when workers do not value SS contributions is minimal.

Scenario 2: Medium valuation of SS services ($\tau = 0.5$)

When workers value social security services somewhat, $\tau > 0$, qualitative results change since workers may perceive this policy either as a policy that reduces employee NWL costs (or a reduction

[89]These statistics cannot be compared one-to-one with the statistics presented in Section 2 since they are valid for a restricted subsample, not for all urban workers.

in the payroll tax) or as a net subsidy from the government.

In this case, formal-sector firms are affected by higher NWL costs (higher δ_1), and therefore, they are more willing to end the match. Under this parameterization, the parameter $\hat{\delta}_2$ is actually reduced (from 8.3 to -23.3 percent), which means that from the workers' perspective, this policy is perceived as a subsidy that increases formal-sector wages by 23 percent. Workers are more willing to continue in a formal-sector match, counteracting the initial upward impact on the reservation productivity. The latter effect dominates, so the probability of discontinuing the formal-sector match goes down, increasing job duration in the formal sector by about 2 months.

From the job creation side, workers are also more willing to start a formal-sector match (a lower $R_{UF}(y)$), affecting job creation positively in the formal sector (θ goes up from 0.67 to 1.69). There is less vacancy creation in the informal sector (ϑ goes down from 0.30 to 0.10).

Overall, this policy will significantly shift employment from the informal to the formal: the informal-sector aggregate employment rate decreases from 52.2 to 25.7 percent, and the formal-sector aggregate employment rate increases from 29.6 to 59.9 percent. The higher labor market tightness and expected formal sector job duration imply a significant reduction in the overall unemployment rate from 17.8 to 14.1 percentage points. The hazard out of unemployment increases substantially, decreasing average unemployment duration from 69 to 53 weeks.

The distributions of worker types in both sectors shift to the left slightly (due to a drop in the unemployment rate among the unskilled workers), and become more compressed in the informal sector (See Figure 26). Given these compositional changes and the downward shift in the formal-sector reservation productivity, average formal-sector productivity decreases significantly relative to informal-sector productivity (see Figure 27). The strong negative impact on productivity drives formal-sector wages down, producing a significant fall in the log-wage gap (0.26 log wage points), and a significant reduction in between-sector log-wage variance. Even if this policy makes the distribution of wages within each sector more dispersed, the impact of the log-wage gap dominates, and as a result, total variance of log-wages falls substantially by 33.7 percent (see Figure 28).

Scenario 3: High valuation of SS services ($\tau = 1.0$)

If workers value these contributions highly ($\tau = 1$), an increase in δ_2, will not affect the worker's flow utility. Nonetheless, an increase in δ_1 will increase the flow income in the formal sector, since workers associate this increase with more valuable and efficient services.

In this case, aggregate and compositional effects are of of higher magnitude. The informality rate is reduced to 11.7 percent, the formal-sector employment rate rises to 75.3 percent, and the distributions of worker types in both sectors shift to the left. The distributional impact is also quite sizable. This 'subsidy' to formal-sector workers shifts the density of formal-sector productivity to the left and compress it, via the downward shift in reservation productivity. The opposite happens in the informal-sector productivity density. Therefore, the log-wage gap is narrowed and even becomes negative (see Figures 29, 30 and 31).

Overall log-wage inequality is substantially reduced by 28 percent, driven by the important decrease in between-sector variance.

4.4.2 Subsidized Health Experiments

I simulate an expansion in publicly-funded health insurance programs to informal sector workers. While in the benchmark case only 19.2 percent of informal workers have access to subsidized health, here I simulate an increase in coverage up to 50 percent of informal workers, that is an increase in δ_3 by 159.7 percent.[90] Two scenarios are made for medium and full valuation of these services.

Scenario 1: Medium valuation of SA services ($\mu = 0.5$)

An expansion of subsidized health (increase in δ_3) has a relatively small positive impact on the size of the informal sector (from 52 to 54 percent),[91]even if the simulated percent change in δ_3 is big (159.75 percent). This is explained by the fact that the initial size of δ_3 is quite small, given that a very small percentage of informal-sector workers were covered by the subsidy in the second quarter of 2003.

The job destruction curve in the informal sector shifts down (a lower reservation productivity $R_{IU}(y)$ for each labor market tightness ϑ), since workers have more benefits when continuing employment in the informal sector, and therefore are less likely to discontinue the match. This suggests an initial expansion of vacancies in this sector.

Nonetheless, the assumption that formal-sector workers may move to the informal sector in case of a 'bad' shock introduces additional equilibrium effects which may counteract the initial positive effect on informal-sector vacancies. If formal-sector workers have a 'bad' productivity shock and have the option to move to the informal sector, they will receive more continuation value (benefits) from the current formal-sector match when δ_3 rises. This shifts $R_{FU}(y)$ downward, so workers in the formal sector are less likely to end the match and more likely to start a formal sector match (lower $R_{UF}(y)$ as well). If workers in the formal sector are less likely to end the match, the expected benefits of the informal-sector hiring formal-sector workers affected by a 'bad' shock are lower, negatively affecting vacancy creation in the informal sector and counteracting the initial positive effect (ϑ decreases slightly constant).

The health subsidy increases job duration in the informal-sector slightly by shifting down the reservation productivity schedule, and increases the informal-sector employment rate from 52.2 to 54.1 percent. By making formal-sector vacancy creation less attractive, θ is reduced from 0.67 to 0.51.

Given that labor market tightness is reduced significantly, the aggregate unemployment rate increases from 17.8 to 18.1 percent, and the employment rate in the formal sector declines from 29.6 to 27.5 percent.

Both distributions of worker types are shifted to the right (due to a higher incidence of unemployment among the unskilled workers), but these compositional effects are small. Mean productivity

[90] There is no good proxy for δ_3 in the data. Therefore, to approximate δ_3 in the benchmark case I assume that $\delta_3 = p_s(\delta_1 + \delta_2)$, where p_s is the proportion of IS workers who receive subsidized health. The value of the SA services is the same as the value of SS services (0.617 in benchmark case), adjusted by the probability of receiving those services. This generates a change in δ_3 from 0.1185 (0.192*0.617) to 0.3085 (0.5*0.617).

[91] This is more or less consistent with previous results found in Camacho, Conover, Hoyos (2013).

in the informal sector falls slightly, driven by the downward shift in the reservation productivity (see Figure 20). Consequently, the average informal-sector wage falls relative to the formal, slightly increasing the log-wage gap.

The size of these distributional impacts is also quite small (see Figures 32, 33 and 34). Therefore, the effect of this policy on overall log-wage inequality when workers have partial valuation of these services is minimal.

Scenario 2: Full valuation of SA services ($\mu = 1.0$)

When workers view the health subsidies as more valuable, the qualitative impacts are similar to the previous case. Equilibrium effects are of smaller magnitude, so formal-sector reservation productivity rises, leading to a stronger positive-effect on formal-sector productivity. As a result, the log-wage gap increases more than in the previous case.

This policy increases unemployment duration significantly by 7 weeks, and decreases the hazard rate of unemployment by shifting up the reservation productivity schedules, $R_{UF}(y)$. Distributional effects do not vary significantly relative to the previous case (see Figures 35, 36 and 37).

4.4.3 Relative Price Experiments

I simulate a drop in the price of tradables relative to non-tradables by 5 and 10 percent.[92]

When the formal sector is less profitable due to trade policies or other factors that reduce relative prices (formal relative to informal), there is more job destruction and less job creation in the formal sector, increasing reservation productivity $R_{UF}(y)$ and making vacancy creation in the formal sector less attractive.

In the informal sector the impact is quite the opposite, since a lower $R_{IU}(y)$ induces more job creation. A fraction of the workers displaced in the formal sector will be absorbed in the informal sector, and the rest will join the pool of unemployed. The rise in the number of job seekers in the informal sector (unemployed and formal-sector workers affected by bad shocks) is more than offset by an increase in vacancy creation in the informal sector, so ϑ rises. The reduction in formal-sector vacancies as well as the higher number of unemployed job seekers make θ fall.

A reduction in relative prices has similar qualitative aggregate effects as the labor taxes policy with $\tau = 0$: there is a shift of resources from formality to informality, but in this case, many of the displaced workers will not be absorbed by the informal sector, inducing a larger rise in overall unemployment. A lower hazard out of unemployment implies an increase in mean unemployment duration, while job duration in the formal-sector decreases and in the informal sector increases. When prices drop significantly by 10%, mean unemployment duration can rise up to 9 weeks.

These aggregate impacts are more sizable than in the case of labor taxes with $\tau = 0$ since there are two effects that come into play: the direct effect of the change in prices on worker and firm surpluses, as well as the equilibrium impact due to changes in reservation productivities and market tightness.

With regard to compositional effects, both distributions of worker types shift slightly to the right

[92] Real appreciations of this magnitude occurred after 2003.

(as in the previous experiment, the policy leads to a higher incidence of unemployment among the unskilled workers). With regard to distributional effects, the density of formal-sector productivity shifts to the right, reflecting the upward shift in the reservation productivity, while in the informal-sector, it shifts to the left.

Two forces may affect log wage inequality in opposite ways: a lower relative price of formal-sector goods entails a lower wage rate in the bargaining negotiation, but the rightward changes in the distribution of types and productivities push the average formal-sector wage up. The first effect dominates so mean log wages in the formal sector decrease.

Despite the higher informal-sector good price, the mean wage in the informal sector drops as well as a result of the downward shift in productivities, so the overall impact on the log-wage gap is quite small. Both wage distributions shift to the left slightly (see Figures 38-43). Overall wage inequality is not affected significantly.

Table 11 summarizes the results of the simulated experiments.

5 Concluding Remarks

In this paper I build a search and matching model to understand the impact of labor market policies and social assistance programs on steady state unemployment, informal sector size and wage distribution in a small open economy with search frictions and idiosyncratic productivity shocks. I solve the model numerically and estimate the structural parameters using Colombian household-level data. The model accounts for the division of the labor force among unemployment, informal and formal sector employment, selected moments of the earning distribution, and formal-sector employment duration. I simulate the model and perform a set of policy experiments consistent with the policy reforms implemented in this developing economy in the last two decades.

My contributions to the literature are on two fronts: within the literature of reforms, I provide a new perspective on the impact of the reforms using a structural model; within the search and matching frictions literature with an informal sector, I generalize previous settings by building the informal sector as a 'disadvantaged' sector of a dualistic labor market (consistent with empirical evidence), and add an extra dimension of heterogeneity to fit better the wage distribution.

An expansion of public health insurance to informal sector workers, high payroll taxation and changes in relative prices (tradables vs non-tradables) may affect the division of the labor force into unemployment, employment in the formal and informal sectors (aggregate effects), the mix of workers in the two sectors (compositional effects), and the distribution of productivities and wages (distributional effects). From a policy perspective, it is critical to quantify these effects.

The counterfactual experiments performed suggest that the valuation of social security services and subsidized health is crucial.

Changes in labor taxes have small effects if workers do not value social security services, but may have quite sizable aggregate, compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient services. This perception of labor taxes as net transfers may actually induce a shift of resources from the informal to the formal sector and reduce unemployment, as well as improve overall income inequality by introducing important reductions in the log-wage gap among sectors. The higher the valuation of SS services, the more progressive these labor market policies become.

Social assistance programs that expand subsidized health to informal sector workers may only have mild effects if workers only partially value these services, even if the simulated increase in coverage to workers unregistered in this subsidized regime is quite significant. On the other hand, if workers value these services significantly (i.e. they perceive that the quality is comparable to the benefits offered in the contributive regime), expanding subsidized health may induce an increase in the unemployment rate by 1.5 percentage points, and an increase in unemployment duration of approximately 6 weeks.

Finally, changes in relative prices that negatively affect the relative profitability of the formal sector have quite sizable aggregate effects, producing more long run unemployment and informality. They can also increase the spell of unemployment by 9 weeks. The compositional and distributional effects are modest.

Tables

Table 1: Descriptive Statistics for Employed

	All Employed	Informal	Formal	I/F Ratio
Sample Size	16,405	10,446	5,959	1.75
Population	7,974,924	4,799,786	3,175,138	1.51
E(lnW_s)	-0.73	-1.11	-0.48	2.30
SD(lnW_s)	2.01	1.85	2.06	0.90
E(t_{es})	88.51	92.33	82.77	1.12
SD(t_{es})	106.22	113.70	93.60	1.21
E(t_{nes})	6.90	7.34	6.33	1.16
SD(t_{nes})	13.13	13.63	12.45	1.09
E(YS_s)	10.31	9.13	12.15	0.75
SD(YS_s)	4.17	3.82	4.20	0.91

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. YS_s: years of schooling. Employment and non-employment duration in months. Informality using SP(health and pension) definition. Log of Hourly Wages in current dollars of 2003. I/F ratio is different in the sample than in the population because of sample weights.

Table 2: Decomposition of Variance in Log Hourly Wages

	Informal Sector
Mean of Log Wages (M_1)	-1.11
Variance of Log Wages (σ_1)	3.42
Proportion of Employed in Sector (P_1)	0.39
	Formal Sector
Mean of Log Wages (M_2)	-0.48
Variance of Log Wages (σ_2)	4.27
Proportion of Employed in Sector (P_2)	0.61
	Economywide
Mean of Log Wages ($P_1M_1 + P_2M_2$)	-0.73
Sum of Within-Sector Variance ($P_1\sigma_1 + P_2\sigma_2$)	3.94
Between-Sector-Variance $P_1P_2(M_1 - M_2)^2$	0.09
Total Variance	4.04

Author's calculations based on ENH, Second quarter of 2003, 13 Metropolitan areas. All statistics weighted using expansion factors. Informality using SP (health and pension) definition. Log of nominal hourly wages in current 2003 dollars (Exchange Rate =2877.65 Pesos per dollar - Source: World Bank)

Table 3: Descriptive Statistics for Unemployed

	All Unemployed
Sample Size	3,606
Population	1,810,468
$P(s \in U)$	0.185
$E(t_{us})$	51.53
$SD(t_{us})$	53.15

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. Unemployment duration in weeks

Table 4: Transitions across Labor Market States (Population Values)

From \To	Unemployment	Formal Sector	Informal Sector
Unemployment	1,435,620	106,308	141,453
Inactive	124,238	196,693	261,717
Formal Sector	101,282	2,590,524	192,138
Informal Sector	149,328	191,894	4,294,197

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. By adding the number of rows in each column I recover the total number of individuals in each labor market state in the second quarter of 2013.

Table 5: Annual Discrete Transition Matrix

From \To	Unemployment	Formal Sector	Informal Sector	Other	Sum
Unemployment	0.72	0.05	0.07	0.16	1.0
	(0.004)	(0.0026)	(0.0020)	(0.0030)	
Other	0.04	0.06	0.08	0.82	1.0
	(0.002)	(0.0042)	(0.0051)	(0.0010)	
Formal Sector	0.03	0.81	0.06	0.10	1.0
	(0.0021)	(0.0030)	(0.0010)	(0.005)	
Informal Sector	0.03	0.04	0.93	0.00	1.0
	(0.0024)	(0.0018)	(0.0022)	(0.004)	

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

Table 6: Estimated Hazards

	Unemployment	Formal Sector	Informal Sector
Estimated Hazards	0.28	0.19	0.07
	(0.0041)	(0.0030)	(0.0022)

Author's calculations based on ECH, Second quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

Table 7: Estimates of Employment and Unemployment Duration, by Sector

		Unemployment (Weeks)	Formal Sector (Months)	Informal Sector (Months)
Estimate using hazards		183		
		(0.0041)		
	FSO Definition		62	167
			(0.0030)	(0.0022)
Estimate from Duration Data		52		
		(0.903)		
	FSO Definition		69	100
			(1.16)	(1.11)
	SP Definition		82	92
			(1.21)	(92.33)

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Standard errors in parentheses

Table 8: Fixed Parameters -Based on data or previous micro studies

	Description	Value
PARAMETERS		
r	interest rate	0.02
P_F	price formal-sector good	0.84
P_I	price informal-sector good	0.76
μ_y	mean, worker types	10.31
σ_y^2	variance, worker types	17.38
λ	rate of arrival productivity shock	0.04
b	opportunity cost of leisure	1.0
β	worker's Nash bargaining power	0.5
A	technological parameter, matching function	2.0
α_m	elasticity matching function	0.5
PUBLIC POLICY PARAMETERS		
δ_1	employer NWL cost as percent of FS wage	0.534
δ_2	employee NWL cost as percent of FS wage	0.083
δ_3	subsidized health as percent of IS wage	0.118
s	severance cost	0
VALUATION PARAMETERS		
τ	FS worker, valuation of social security services	0
μ	IS worker, valuation of social assistance services	0.5

Table 9: Estimated Parameters (Moment Simulation)

	Description	Value
PARAMETERS		
c	cost of posting a vacancy	2.50
α	rate of arrival IS opportunities	2.91
B_F	FS idiosyncratic productivity, log mean parameter	0.62
B_I	IS idiosyncratic productivity, log mean parameter	0.41
σ_F	FS idiosyncratic and match-specific productivity, log standard deviation	0.65
σ_I	IS idiosyncratic and match-specific productivity, log standard deviation	0.68

Table 10: Estimation: Data-based vs. Simulated Statistics

Variable	Model	Data
AGGREGATE UNEMPLOYMENT AND EMPLOYMENT RATES		
u	17.86 (0.0022)	18.50 (0.0027)
n_F	29.66 (0.0152)	32.44 (0.0033)
n_I	52.23 (0.0174)	49.05 (0.0035)
LABOR MARKET TIGHTNESS		
θ	0.67 (0.1048)	n.a.
ϑ	0.30 (0.0043)	n.a.
MEAN EMPLOYMENT AND UNEMPLOYMENT DURATIONS		
t_U	68.6 (1.1401)	51.5 (0.9032)
t_{e_F}	6.52 (0.0151)	6.89 (0 .1011)
t_{e_I}	6.52 (0.0125)	7.69 (0.0929)
MEASURES OF WAGE DISPERSION		
$\mu_{ln(W_F)} - \mu_{ln(W_I)}$	0.40 (0.0045)	0.63 (0.0430)
$\sigma_{ln(W_I)}/\sigma_{ln(W_F)}$	0.88 (0.0167)	0.89 (.0126)

[a] Unemployment and employment rates in percentage points. Unemployment duration in weeks. Mean employment and unemployment durations in years. Durations from data are estimated using observed durations, not estimated hazards. Standard errors in parenthesis. The standard errors of the simulated statistics are constructed using bootstrapping (thirty bootstrap replications).

Table 11: Compositional and Distributional Effects of Labor Market Policies, Social Assistance and Relative Prices

	Valuations	Aggregate Effects			LM Tightness		Distributional Effects				Mean Durations [b]		
	τ, μ	u	n_F	n_I	θ	ϑ	$\mu_{lnW_F} - \mu_{lnW_I}$	σ_{WS}	σ_{BS}	σ	t_u	t_{n_F}	t_{n_I}
Benchmarck [a]	$\tau = 0$, $\mu = 0.5$	17.86	29.66	52.23	0.67	0.30	0.40	0.052	0.043	0.089	69	6.52	6.52
Δ Labor Taxes[c]													
$\delta_1 \uparrow 4.46\%$, $\delta_2 \uparrow 7.53\%$													
	$\tau = 0$	18.28	27.51	53.94	0.62	0.32	0.41	0.053	0.037	0.090	70	6.50	6.52
	$\tau = 0.5$	14.11	59.90	25.73	1.69	0.10	0.14	0.057	0.004	0.059	53	6.67	6.42
	$\tau = 1.0$	12.68	75.35	11.71	2.46	0.04	-0.06	0.064	0.000	0.064	47	6.73	6.34
Δ SA [d]													
$\delta_3 \uparrow$ by 159.74%													
	$\mu = 0.5$	18.12	27.51	54.11	0.51	0.29	0.42	0.052	0.039	0.091	70	6.53	6.53
	$\mu = 1$	19.49	29.74	50.51	0.717	0.31	0.44	0.054	0.044	0.098	76	6.48	6.49
Δ Relative Prices[e]													
$P_F/P_I \downarrow$ by 5%		18.69	22.68	58.37	0.43	0.37	0.40	0.053	0.031	0.085	73	6.49	6.54
$P_F/P_I \downarrow$ by 10%		19.76	18.70	61.28	0.39	0.45	0.40	0.056	0.028	0.085	78	6.45	6.53

[a] Unemployment and employment rates in percentage points. Simulation results are robust to changes in initial conditions (guessed initial vectors for approximating steady state equilibrium) and multiple repetitions of the sampling process.
[b] Unemployment duration in weeks, employment duration in years.
[c] In these set of experiments $\mu = 0.5$.
[d] In these set of experiments $\tau = 0$.
[e] In these set of experiments $\tau = 0$, $\mu = 0.5$

Table 12: Detailed Employee Non-Wage Labor Costs (as percent of Wage)-Statutory Values (1984-2013)

	1984	1985-1990	Post90 Reform			Post93 Reform			Post2003 Reform			Post2007 Reform		Post2012 Reform
	1984	1985 – 1990	1991	1992	1993	1994	1995	1996-2003	2004	2005	2006	2007	2008-2012	2013
SOCIAL SECURITY														
Pension	1.50	2.17	2.17	2.17	2.17	2.875	3.125	3.375	3.625	3.75	3.875	3.875	4.0	4.0
Health	2.33	2.33	2.33	2.33	2.33	4.00	4.00	4.00	4.00	4.00	4.00	4.0	4.0	4.0
Subtotal Social Security	3.83	4.50	4.50	4.50	4.50	6.875	7.125	7.375	7.625	7.75	7.875	7.875	8.00	8.00
OTHERS														
Solidarity Fund		0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
TOTAL														
EMPLOYEE NWLC	3.83	4.50	4.50	5.50	5.50	7.875	8.125	8.375	8.625	8.75	8.875	8.875	9.00	9.00
(AS percent OF WAGE)														

Author's Calculations based on statutory values established in the Labor Codes. See Law 100 of 1993, Law 797 of 2003, Law 1122 of 2007 and Law 1607 of 2012.

Table 13: Detailed Employer Non-Wage Labor Costs (as percent of Wage)-Statutory Values (1984-2013)

	1984	1985-1990	Post90 Reform			Post93 Reform			Post2003 Reform			Post2007 Reform		Post2012Reform
	1984	1985 – 1990	1991	1992	1993	1994	1995	1996-2003	2004	2005	2006	2007	2008-2012	2013
SOCIAL SECURITY														
Pension	3.00	4.33	4.33	4.33	4.33	8.63	9.38	10.12	10.87	11.25	11.62	11.63	12.00	12.00
Health	4.67	4.67	4.67	4.67	4.67	8.00	8.00	8.00	8.00	8.00	8.00	8.50	8.50	8.50
Work Injury	2.0	2.00	2.00	2.00	2.00	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.4
Subtotal Social Security	9.67	11.00	11.00	11.00	11.00	19.03	19.78	20.53	21.28	21.65	22.03	22.53	22.90	22.90
PARA-STATAL														
Training	2.0	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.00
In-Kind Tranfers	2.0	2.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.00
Family Allowances	4.0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Subtotal Para-statal	8.0	8.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	4.0
OTHERS														
Mandatory Bonuses	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Paid Vacations	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Severance Pay	13.5	13.5	12.8	12.1	11.4	10.70	10.0	9.30	9.30	9.30	9.30	9.30	9.30	9.30
Subtotal Others	28.08	28.08	27.38	26.68	25.98	25.28	24.58	23.88	23.88	23.88	23.88	23.88	23.88	23.88
TOTAL														
EMPLOYER NWLC	45.75	47.08	47.38	46.68	45.98	53.31	53.36	53.41	54.16	54.53	54.91	55.41	55.78	50.78
(AS percent OF WAGE)														

Author's Calculations based on statutory values established in the Labor Codes. See Law 50 of 1990, Law 100 of 1993, Law 797 of 2003, Law 1122 of 2007, and Law 1607 of 2012.
Work injury compensations vary with the degree of risk in the occupation, ranging from degree 1 (lowest risk) to degree 5 (highest risk). Here, I show the values of a level 3 risk.
Severance pay was paid upon employment termination prior to the 1990 Labor Reform, but it was turned into a payroll tax afterwards since firm have to deposit payments on a monthly basis in a financial account.
Training refers to SENA, In-kind Transfers to ICBF, and Family Allowances to Cajas de Compensacion Familiar.
The reduction in parastatal contributions in 2012 only applied to workers earning less than 10x the minimum wage.

Figures

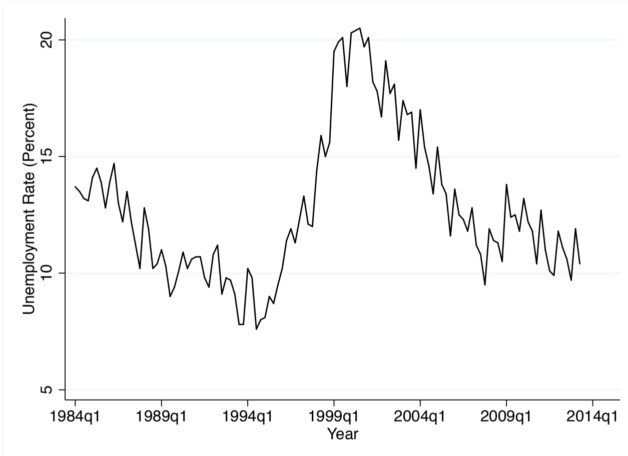

Figure 1: Evolution of Unemployment Rate
All urban workers. Seven Metropolitan areas: Bucaramanga, Barranquilla, Bogota, Cali, Medellin, Manizales and Pasto.
Source: *Colombian Central Bank*

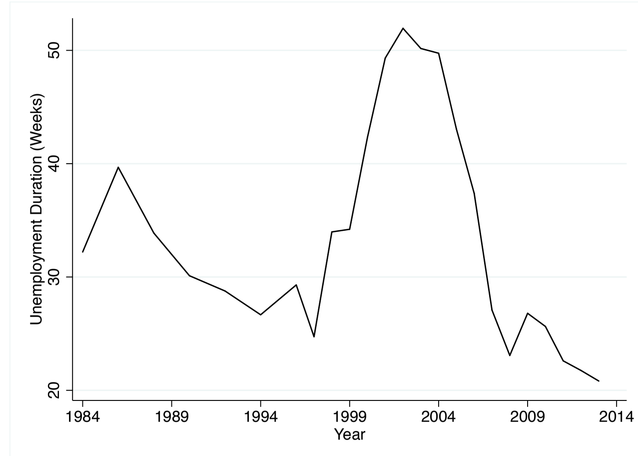

Figure 2: Evolution of Mean Unemployment Duration
Author's calculation based on Colombian Household Surveys

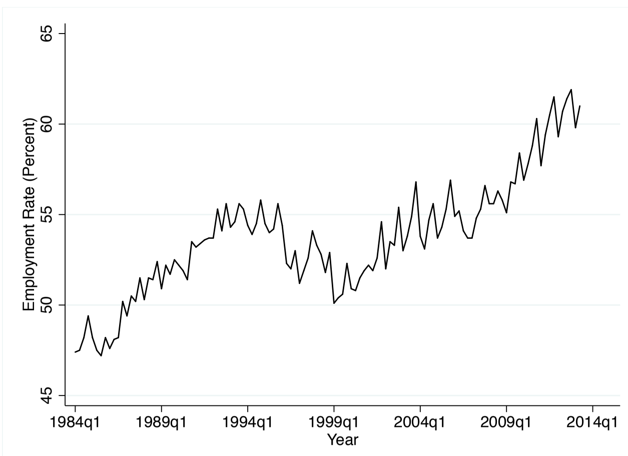

Figure 3: Evolution of Employment Rate
All urban workers. Seven Metropolitan areas: Bucaramanga, Barranquilla, Bogota, Cali, Medellin, Manizales and Pasto.
Source: *Colombian Central Bank*

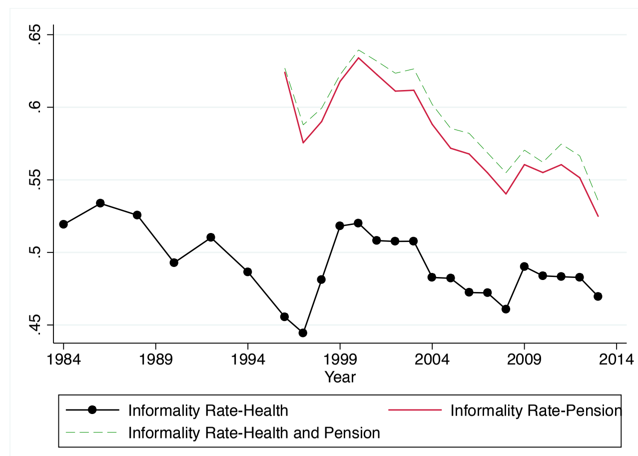

Figure 4: Evolution of Informality Rates, by definition of Informality
Author's calculation based on Colombian Household Surveys. There is no information on pension contributions before 1996.

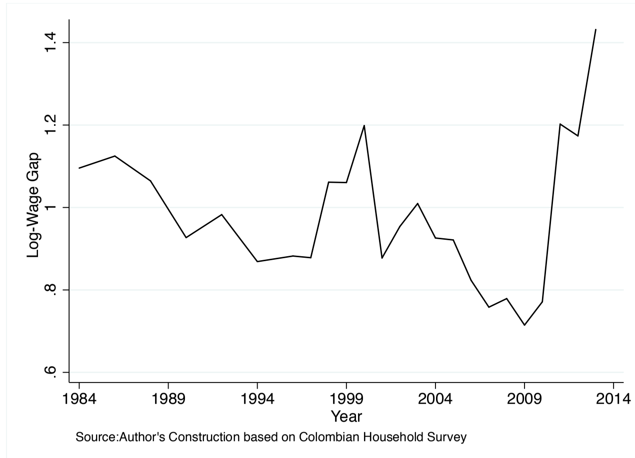

Figure 5: Evolution of Log Wage Gap
Log Wage Gap defined as $\overline{lnw_F} - \overline{lnw_I}$. Informality based on health criterion. Log of hourly wages in constant dollars of 2008. *Source: Author's calculation based on Colombian Household Surveys*

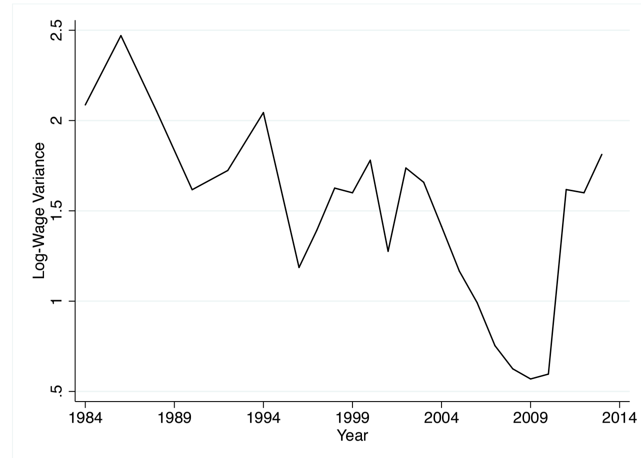

Figure 6: Evolution of Variance of Log Hourly Wages
Log of hourly wages in constant dollars of 2008. *Source: Author's calculation based on Colombian Household Surveys*

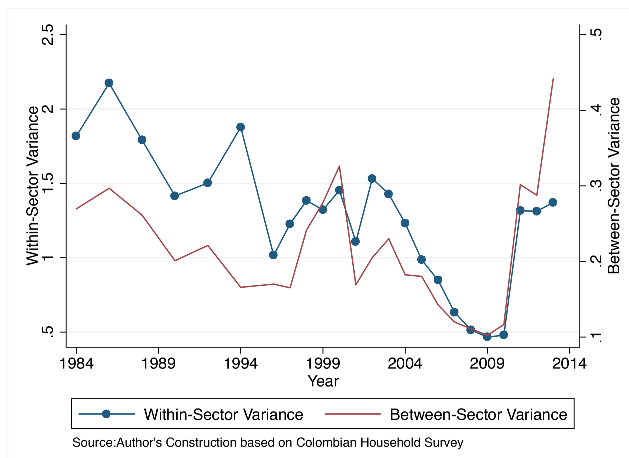

Figure 7: Decomposition of Variance of Log Hourly Wages
Log of hourly wages in constant dollars of 2008. *Source: Author's calculation based on Colombian Household Surveys*

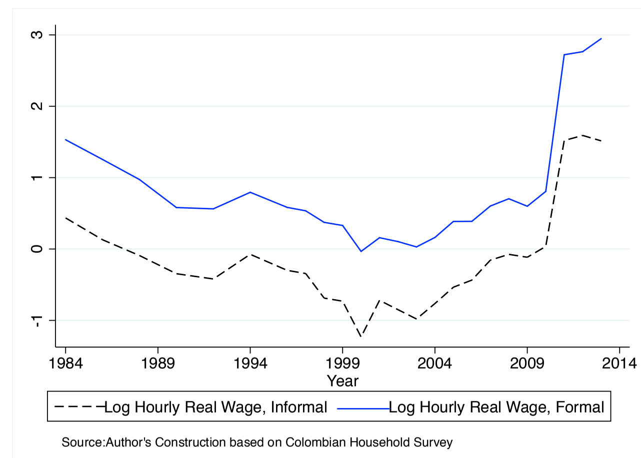

Figure 8: Evolution of Mean of Log Hourly Wages
Log of hourly wages in constant dollars of 2008. *Source: Author's calculation based on Colombian Household Surveys*

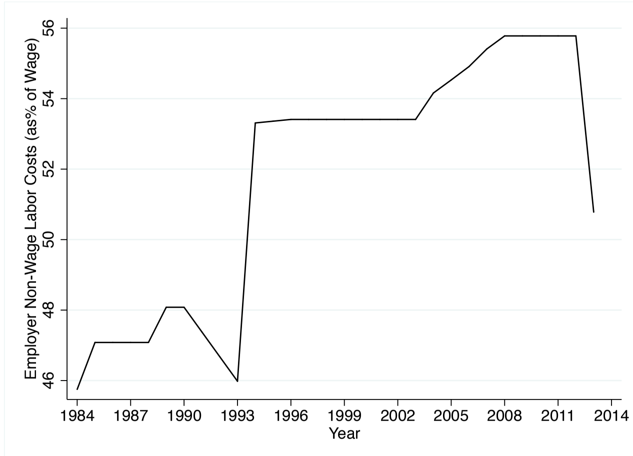

Figure 9: Evolution of Employer Non-Wage Labor Costs (as % of Wage)
See Table 13 for a detailed construction of these costs. *Author's Calculations based on Statutory Values*

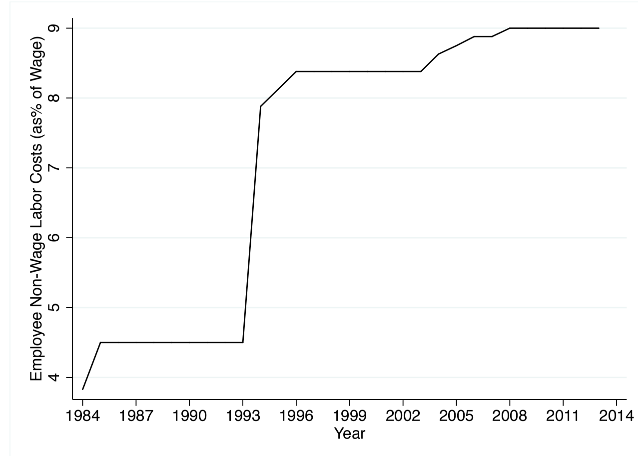

Figure 10: Evolution of Employee Non-Wage Labor Costs (as % of Wage)
See Table 12 for a detailed construction of these costs. *Author's Calculations based on Statutory Values*

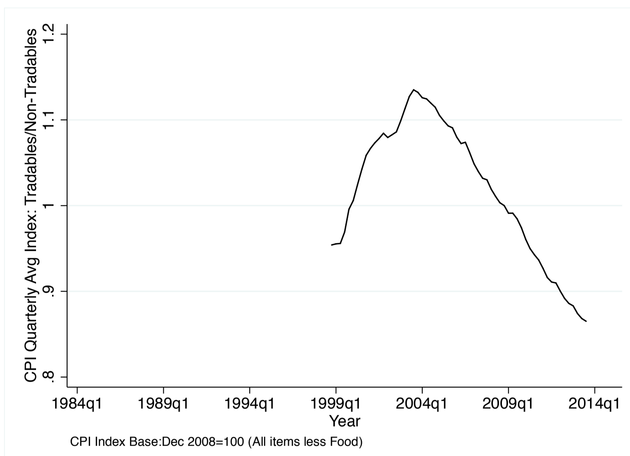

Figure 11: Evolution of Relative Prices
CPI Quarterly Average Index- Tradables relative to Non-Tradables. No data availability before 1998 *Source: Colombian Central Bank*

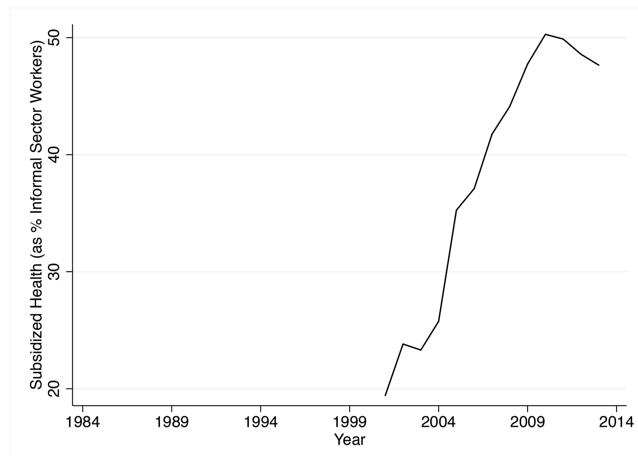

Figure 12: Evolution of Subsidized Health
Percentage of Informal-Sector Workers who have access to the Subsidized Regime- No data availability in the surveys before 2001. *Author's Calculations based on Colombian Household Surveys*

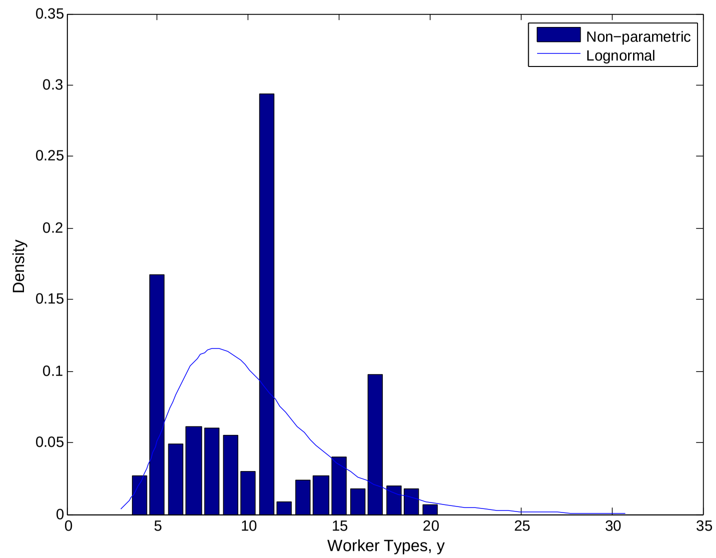

Figure 13: Analytical Density of Worker Types: Data vs Lognormal

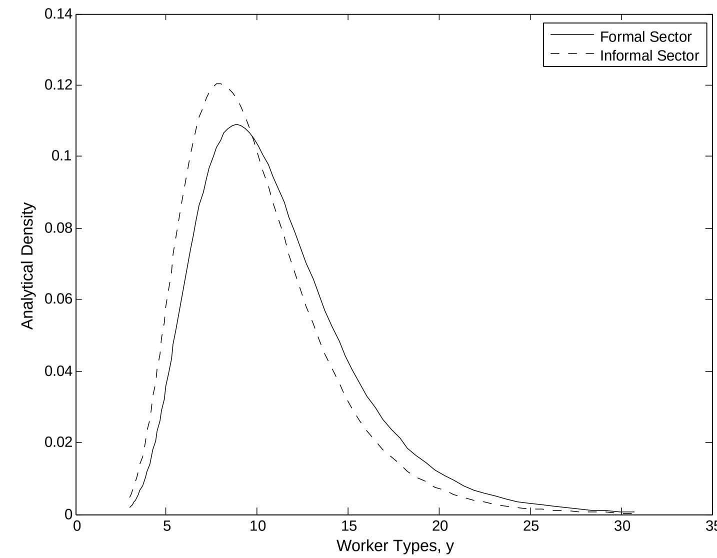

Figure 14: Analytical Density of Worker Types, Formal and Informal Sector

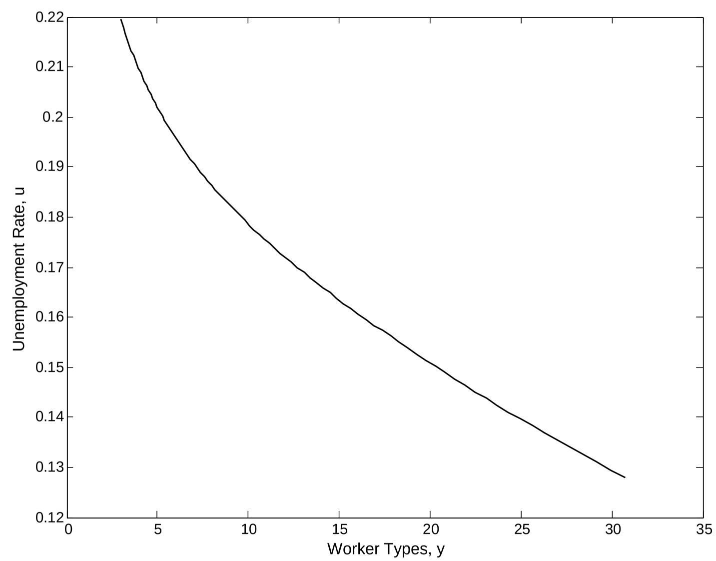

Figure 15: Unemployment Rate

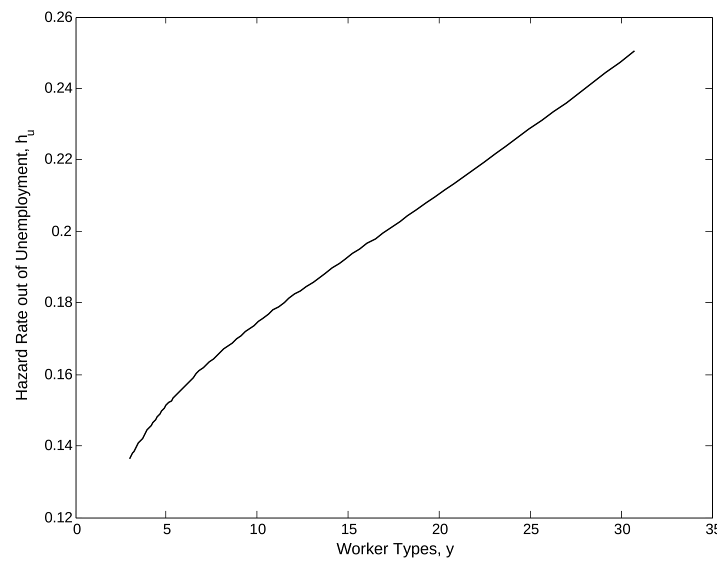

Figure 16: Hazard Rate out of Unemployment

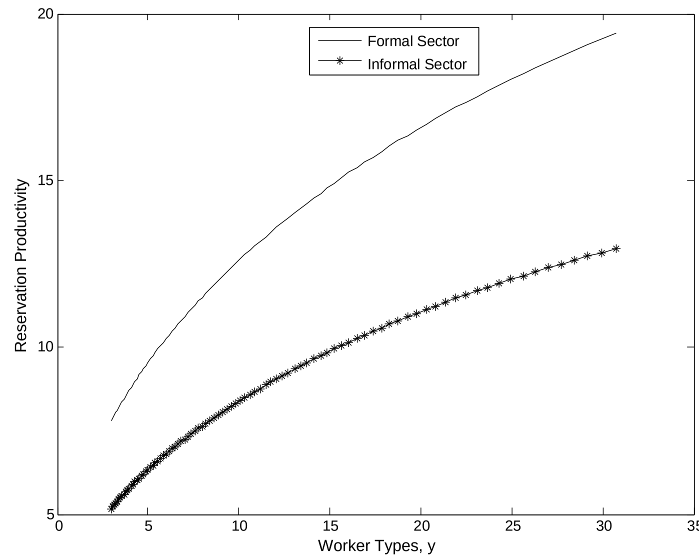

Figure 17: Reservation Productivity, Formal and Informal Sector

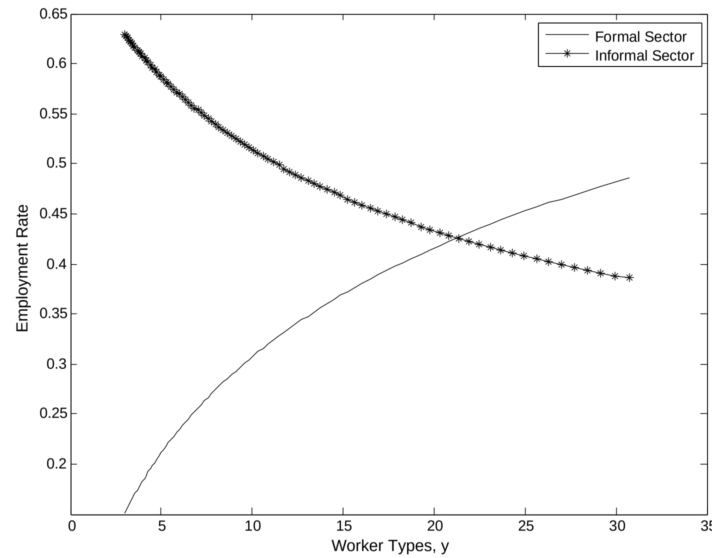

Figure 18: Employment Rate, Formal and Informal Sector

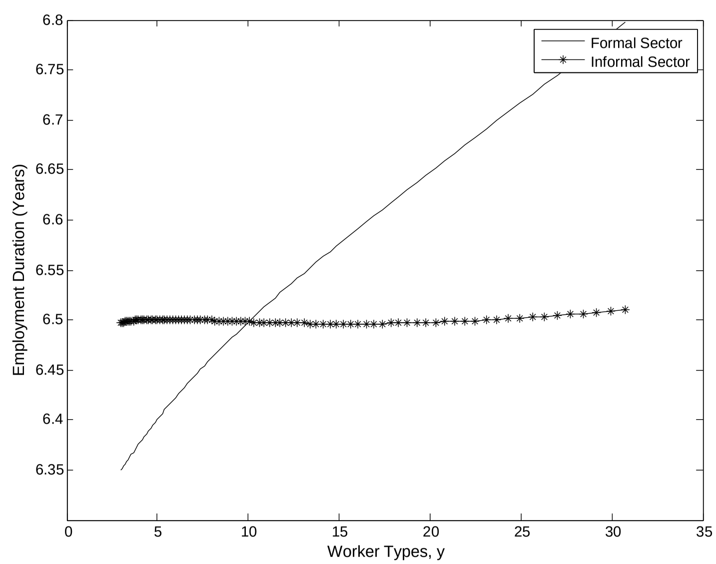

Figure 19: Job duration (Years), by Sector

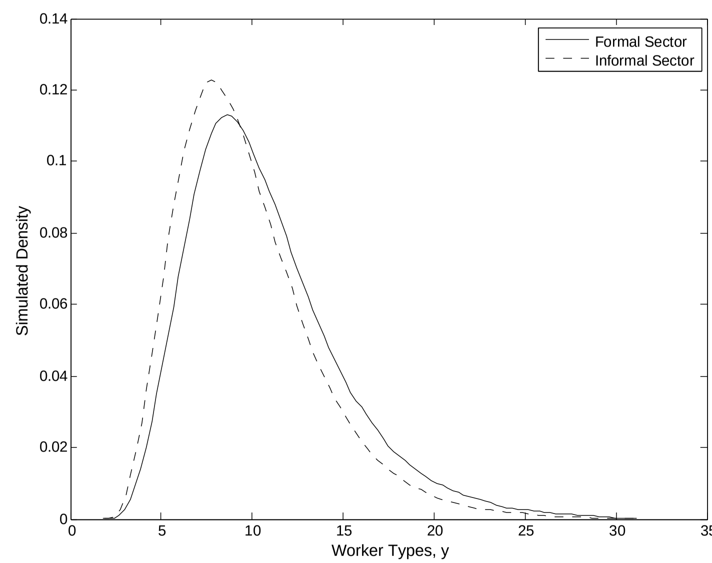

Figure 20: Kernel Density Simulated Distribution of Types, by Sector

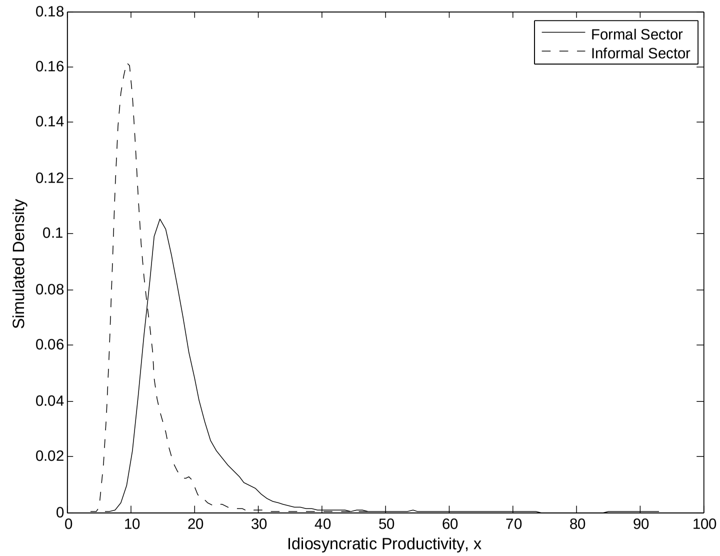

Figure 21: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector

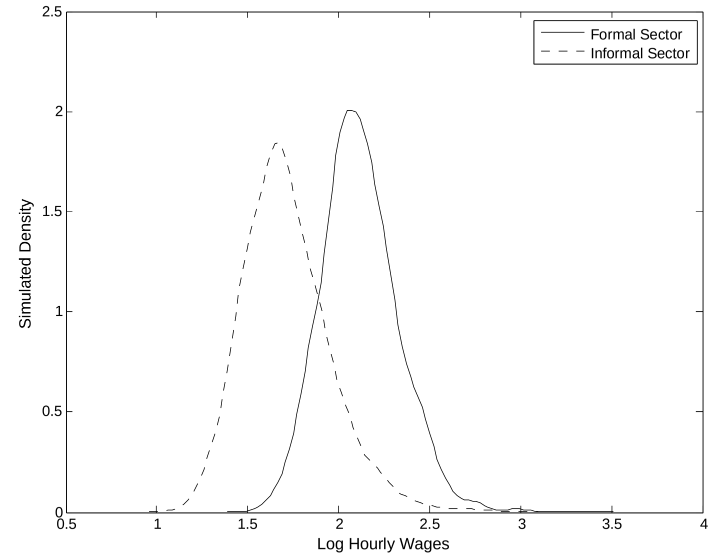

Figure 22: Kernel Density Simulated Distribution of Log Wages, by Sector

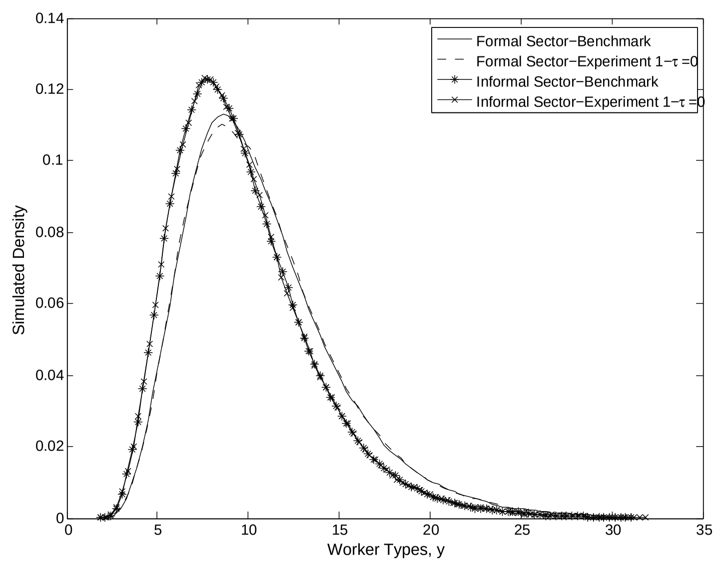

Figure 23: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 1, ($\tau = 0$)

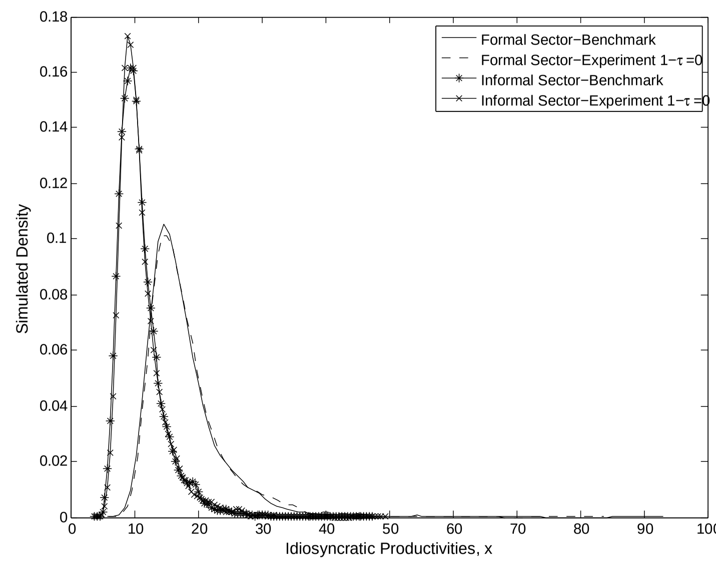

Figure 24: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 1($\tau = 0$)

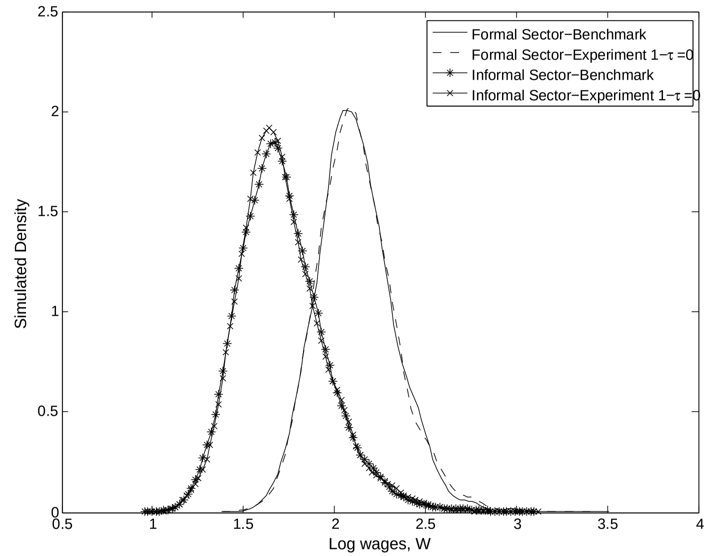

Figure 25: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 1 ($\tau = 0$)

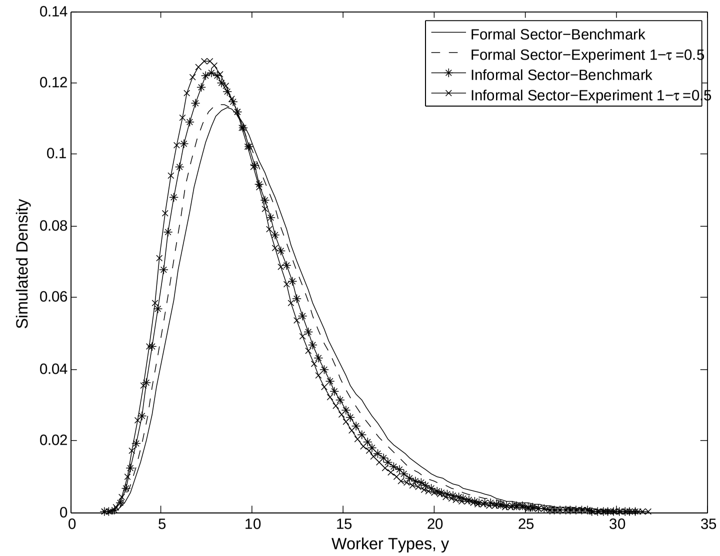

Figure 26: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 1, ($\tau = 0.5$)

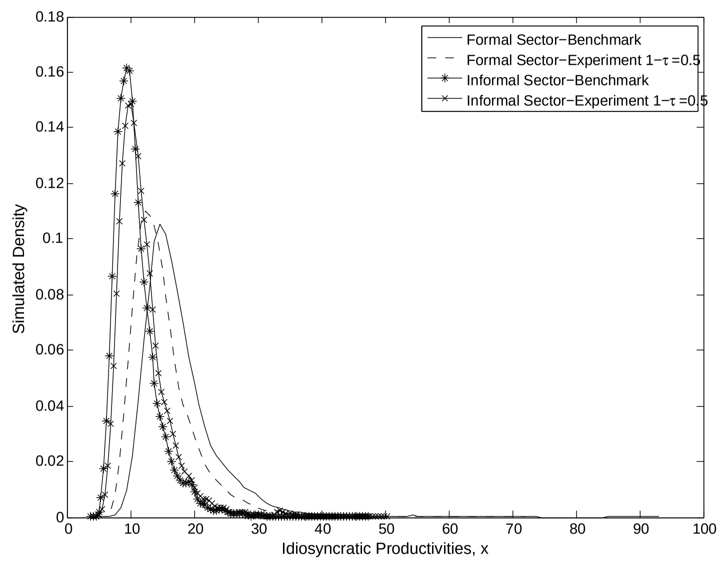

Figure 27: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 1 ($\tau = 0.5$)

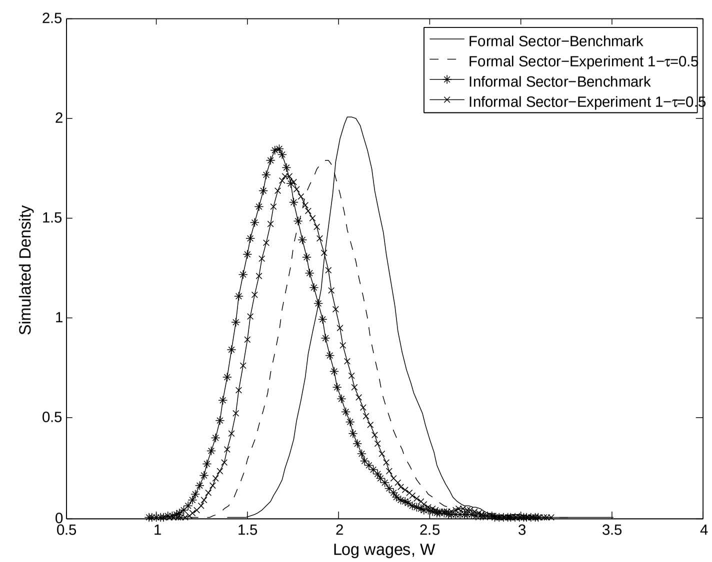

Figure 28: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 1 ($\tau = 0.5$)

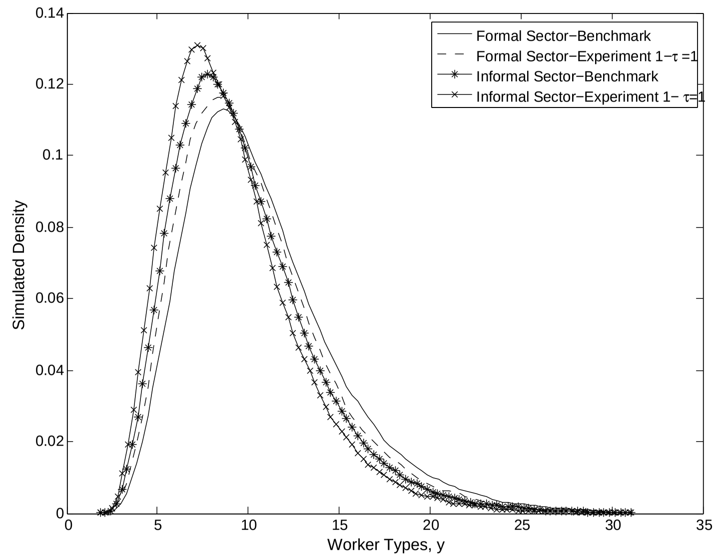

Figure 29: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 1, ($\tau = 1$)

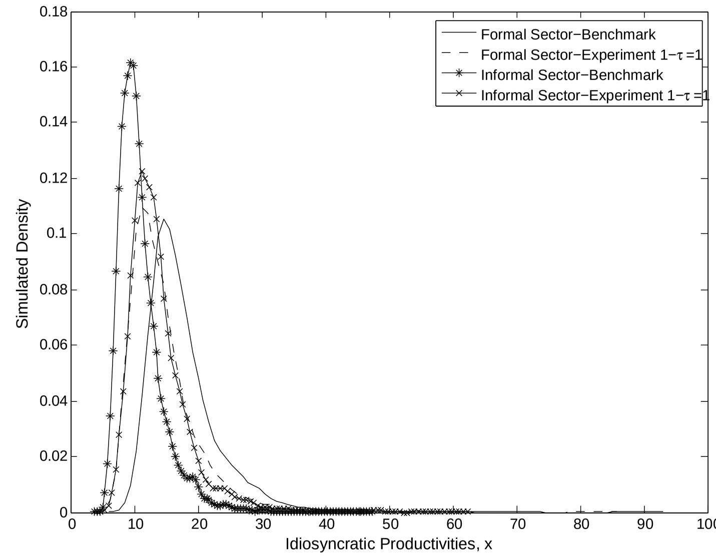

Figure 30: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 1 ($\tau = 1$)

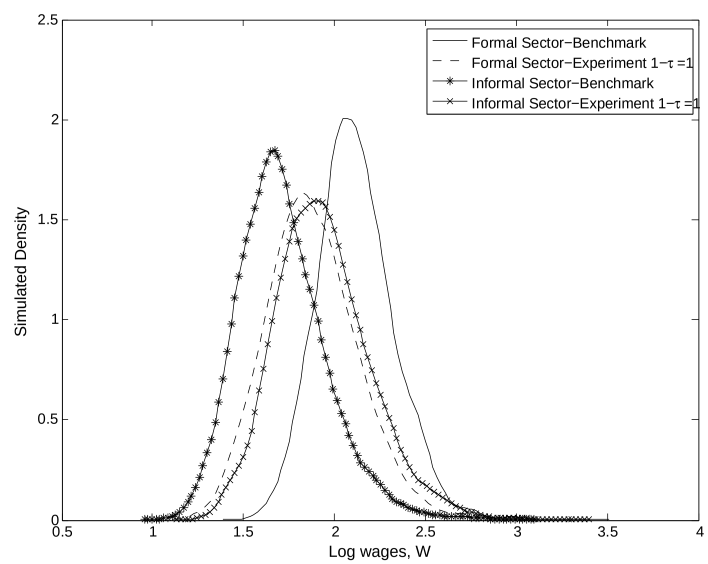

Figure 31: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 1 ($\tau = 1$)

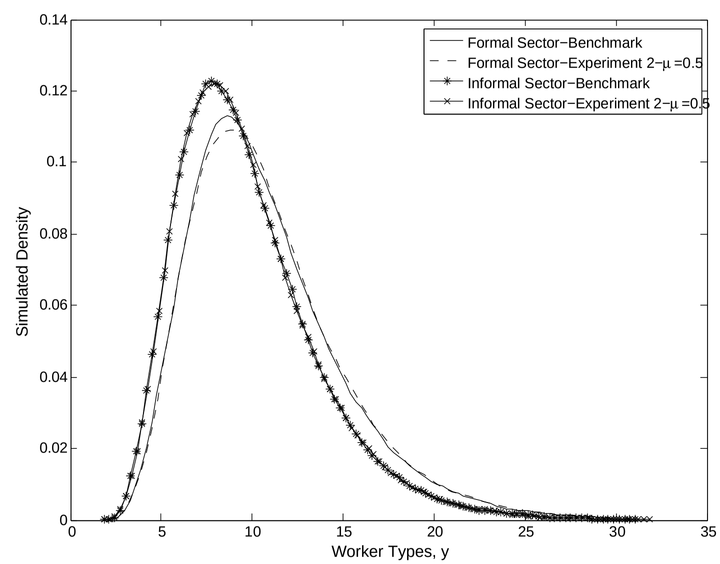

Figure 32: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 2 ($\mu = 0.5$)

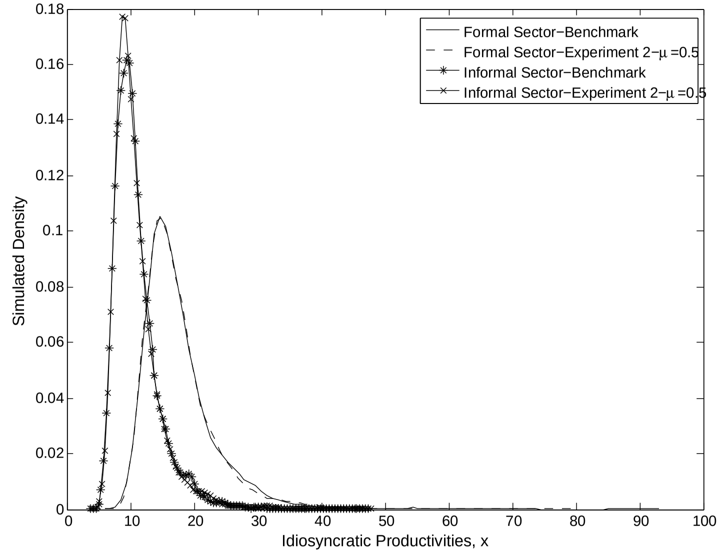

Figure 33: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 2 ($\mu = 0.5$)

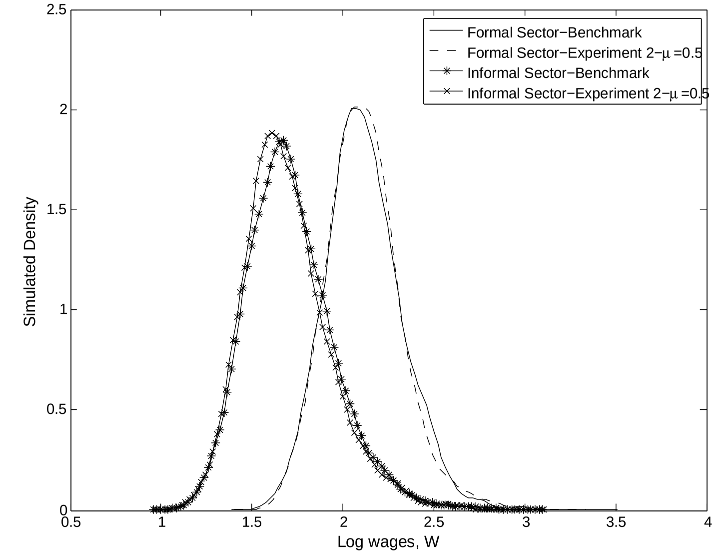

Figure 34: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 2 ($\mu = 0.5$)

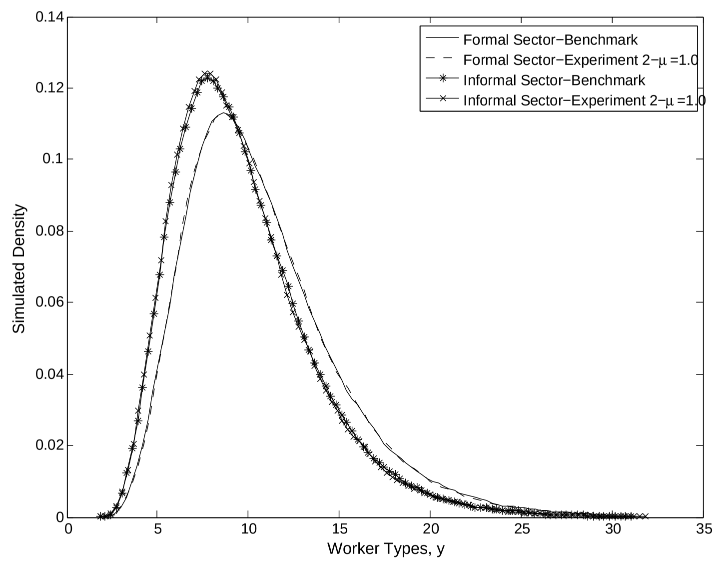

Figure 35: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 2 ($\mu = 1$)

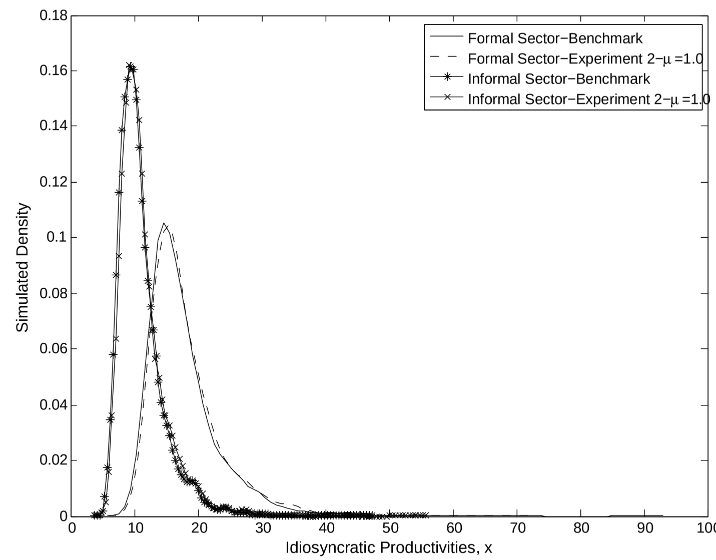

Figure 36: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 2 ($\mu = 1$)

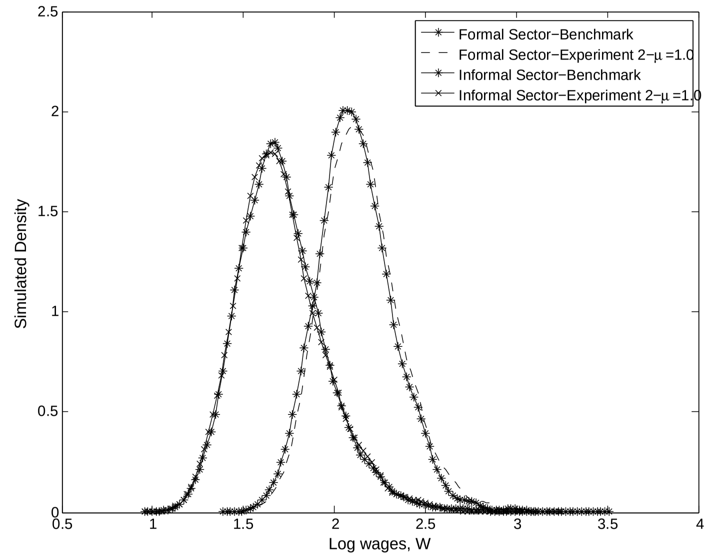

Figure 37: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 2 ($\mu = 1$)

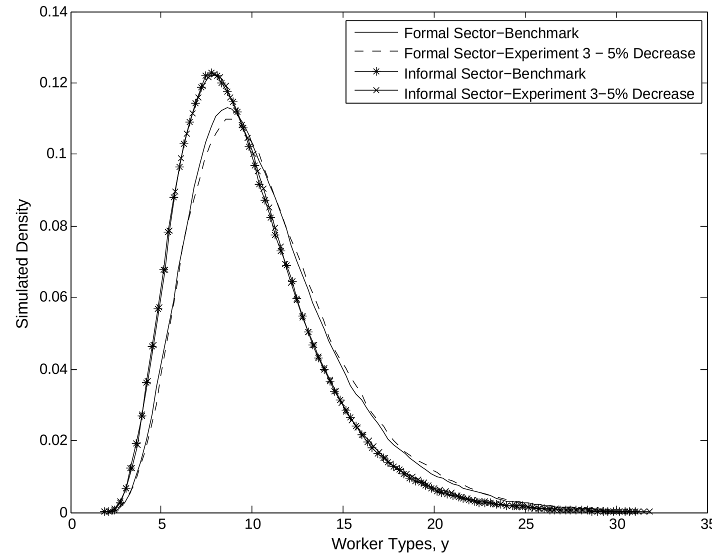

Figure 38: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 3 (5% $\Downarrow$)

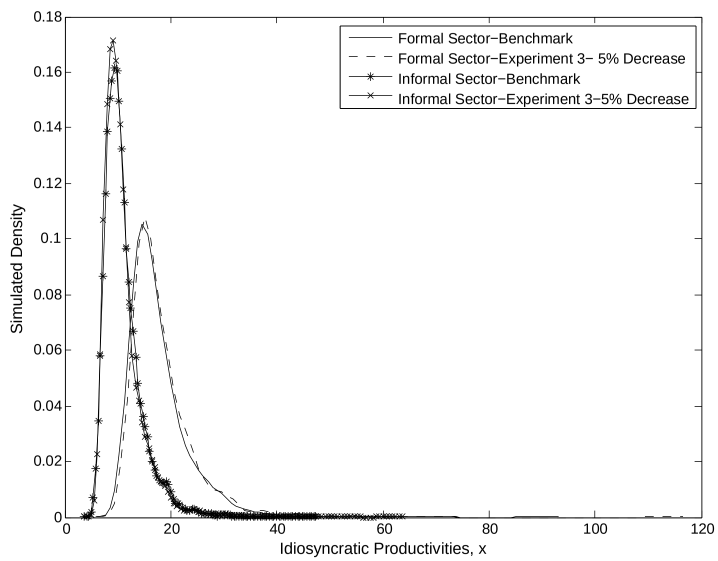

Figure 39: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 3 (5% $\Downarrow$)

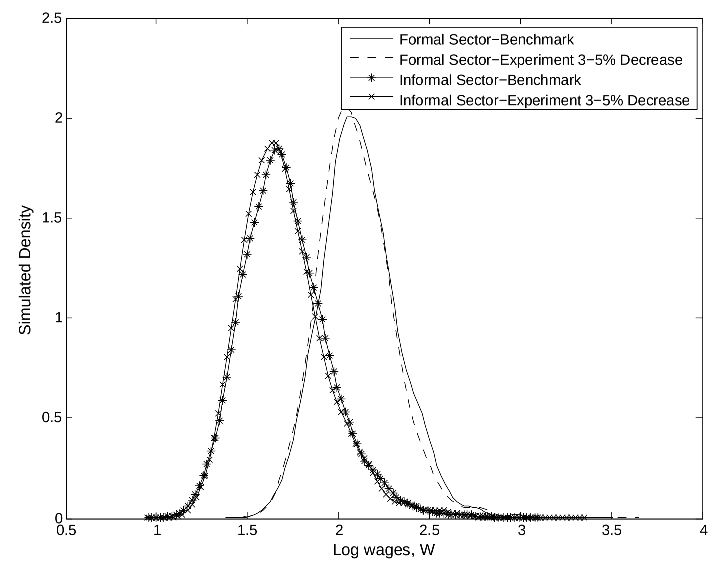

Figure 40: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 3(5% $\Downarrow$)

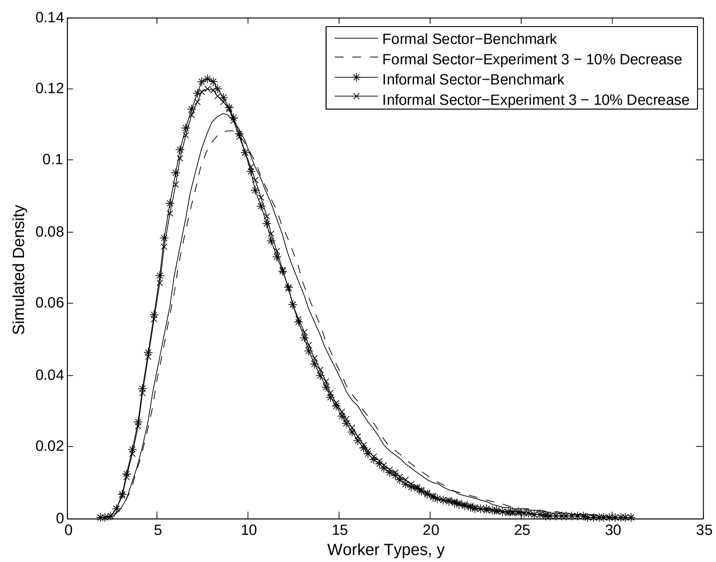

Figure 41: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 3 (10% $\Downarrow$)

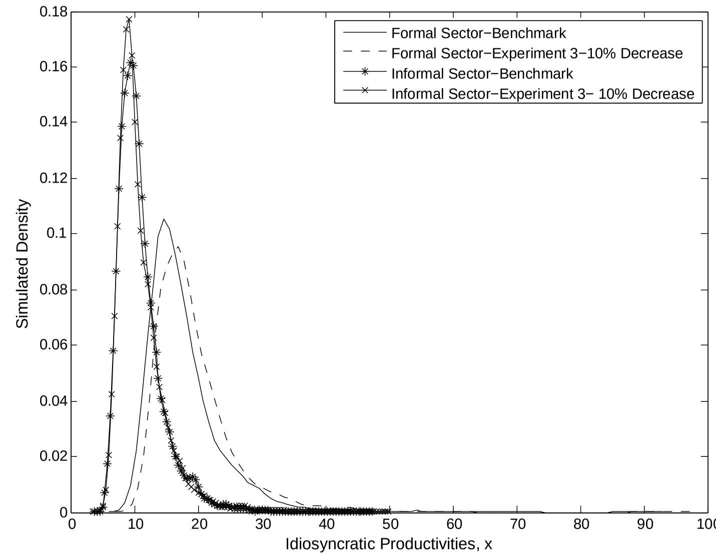

Figure 42: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 3 (10% $\Downarrow$)

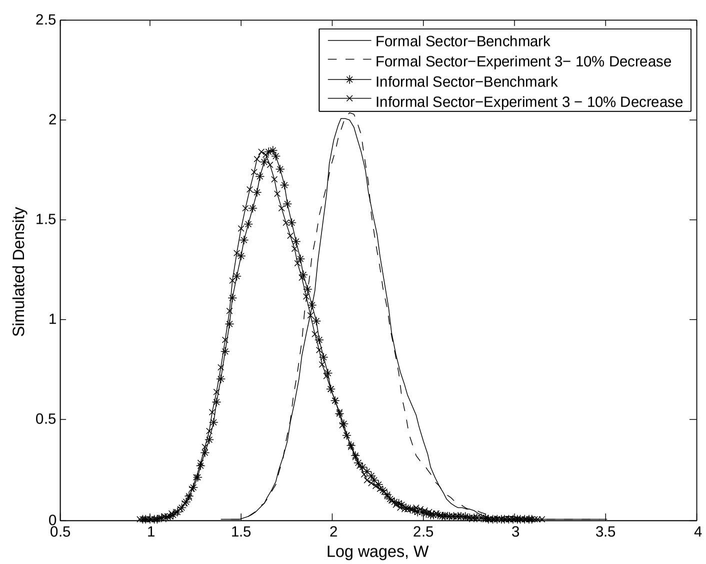

Figure 43: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 3(10% $\Downarrow$)

6 Appendix

6.1 Total Surplus, Worker and Firm Surplus

Since most equilibrium objects are expressed as functions of workers' or firms' surpluses, it is useful to derive expressions for these surpluses as functions of the reservation productivities.

6.1.1 Formal Sector

In order to get the firm surplus in the formal sector as a function of the reservation productivity $R_{UF}(y)$, I use the value function $J_F(y',y)$ and the wage equation $w_F(y',y)$. Then, I use the Nash sharing rule to obtain total surplus and worker surplus.

The total surplus, worker and firm surplus for a worker of type y and productivity y' in the formal sector are, respectively, given by[93]

$$S_F(y',y) = \frac{p_F\left(1-\hat{\delta}_2\right)(y'-R_{UF}(y))}{(1+\delta_1)(r+\lambda)}$$

$$N_F(y',y) - U(y) = \frac{\beta p_F\left(1-\hat{\delta}_2\right)(y'-R_{UF}(y))}{(1+\delta_1)(r+\lambda)}$$

$$J_F(y',y) - V_F = \frac{(1-\beta)p_F\,(y'-R_{UF}(y))}{(r+\lambda)}$$

The total surplus, workers' and firms' surplus for a worker of type y and productivity x in the formal sector are, respectively, given by:

$$S_F(x,y) = \frac{p_F\left(1-\hat{\delta}_2\right)(x-R_{FU}(y))}{(1+\delta_1)(r+\lambda)}$$

$$N_F(x,y) - U(y) = \frac{\beta p_F\left(1-\hat{\delta}_2\right)(x-R_{FU}(y))}{(1+\delta_1)(r+\lambda)}$$

$$J_F(x,y) - V_F = \frac{(1-\beta)p_F\,(x-R_{FU}(y))}{(r+\lambda)} - s$$

6.1.2 Informal Sector

The corresponding surpluses for a worker of type y and current productivity y' in the informal sector are given by:

$$S_I(y',y) = \frac{p_I\left(1+\hat{\delta}_3\right)(y'-R_{IU}(y))}{r+\lambda}$$

[93]Notice that workers always get a fixed proportion of the total surplus, so $N_F(y',y) - U(y) = \beta S_F(y',y)$. Substituting this expression in the Nash sharing rule I get $J_F(y',y) - V_F = \frac{(1-\beta)(1+\delta_1)}{(1-\hat{\delta}_2)} S_F(y',y)$. So the total surplus $S_F(y',y) = N_F(y',y) - U(y) + \frac{(1-\hat{\delta}_2)}{(1+\delta_1)} J_F(y',y)$.

$$N_I(y',y) - U(y) = \frac{\beta p_I \left(1+\hat{\delta}_3\right)(y' - R_{IU}(y))}{r+\lambda}$$

$$J_I(y',y) - V_I = \frac{(1-\beta)p_I\,(y' - R_{IU}(y))}{r+\lambda}$$

Therefore, in both sectors, the surpluses depend positively on the market value of the gap between current and minimum productivity at a particular match, and negatively on the rate of arrival of the productivity shock and the interest rate. Higher λ implies a higher turnover rate, and higher r implies that future returns are discounted at a higher rate.

6.2 Derivation of Steady State Productivity Distributions

I want to know how policies affect the marginal distributions of types and productivities in each sector, which also affect the sectoral wage distributions. This section follows closely ANV(2009).

6.2.1 Formal Sector

First, I want to compute the joint steady state distribution of current productivity and worker types across workers in the formal sector, $f_F(x,y)$.

$$f_F(x,y) = f_F(x \mid y) f_F(y)$$

I use Bayes rule to compute $f_F(y)$ as follows:

$$f_F(y) = \frac{n_F(y)f(y)}{\int_0^\infty n_F(y)f(y)dy}$$

Now I need $f_F(x \mid y)$.

Let N be the number of shocks that the worker has experienced to date (in the current spell of unemployment in the FS). If $N=0$ current productivity x equals match-specific productivity y' with prob 1. If $N>0$, x is a draw from a truncated density $h_F(x \mid y)/(1 - H_F(R_{FU}(y) \mid y))$, for $R_{FU}(y) \leq x \leq \infty$.

So I have:

$$P(N=0) = P(x = y' \mid y) \quad \text{for } x{=}y'.$$

$$f_F(x \mid y) = \frac{h_F(x \mid y)}{(1 - H_F(R_{FU}(y) \mid y))}(1 - P(N=0)) \quad \text{for } R_{FU}(y) \leq x \leq \infty \text{ and } x \neq y'$$

Let t be elapsed duration of employment in the current formal-sector job. Let N_t be the number of shocks that the worker has experienced to date t. Let's assume that:

$$t \sim exp\left(\lambda H_F(R_{FU}(y) \mid y)\right)$$

$$N_t \sim Poisson\left(\lambda(1 - H_F(R_{FU}(y) \mid y))t\right)$$

So I have:

$$P(N_t = 0) = exp\,(-\lambda(1 - H_F(R_{FU}(y) \mid y))t)$$

$$P[N = 0] = \int_0^{\infty} exp\,(-\lambda(1 - H_F(R_{FU}(y) \mid y))t)\,\lambda\,(1 - H_F(R_{FU}(y) \mid y))\,exp\,(-\lambda H_F(R_{FU}(y) \mid y)t)\,dt$$

So I get:

$$P[N = 0] = H_F(R_{FU}(y) \mid y))$$

The density of current productivity x given worker type y in the formal sector is given by:

$$P(x = y' \mid y) = H_F(R_{FU}(y) \mid y)) \quad \text{for } x{=}y'.$$

$$f_F(x \mid y) = h_F(x \mid y) \quad \text{for } R_{FU}(y) \leq x \leq \infty \text{ and } x \neq y'$$

I can now compute the steady state joint distribution of types and productivity in the formal sector, and the marginal distribution of current productivity in the formal sector as follows:

$$f_F(x, y) = f_F(x \mid y) f_F(y)$$

$$f_F(x) = \int f_F(x, y) dy$$

6.2.2 Informal Sector

Doing the analogous exercise for the informal sector I get:

$$f_I(x) = \int f_I(x, y) dy$$

$$f_I(x, y) = f_I(x \mid y) f_I(y)$$

where $f_I(y)$ and $f_I(x \mid y)$ are as follows:

$$f_I(y) = \frac{n_I(y) f(y)}{\int_0^{\infty} n_I(y) f(y) dy}$$

$$P(x = y' \mid y) = H_I(R_{IU}(y) \mid y)) \quad \text{for } x{=}y'.$$

$$f_I(x \mid y) = h_I(x \mid y) \quad \text{for } R_{IU}(y) \leq x \leq \infty \text{ and } x \neq y'$$

6.3 Derivation of Steady State Wage Distributions

To compute the impact of policies on wage inequality in both sectors, I need to derive the distribution of wages across formal and informal sector employment, $m_F(w)$ and $m_I(w)$.

6.3.1 Formal Sector

There are two types of workers currently employed in the formal sector: the workers that have not received any shock whose current productivity is match-specific ($x = y'$ with probability 1), and those who received a shock and continue in the match ($x \geq R_{FU}(y)$).

This suggests that the distribution of wages in formal-sector employment (conditional on y) consists of a smooth density for $w_F \in [w_F^s[R_{FU}(y), y), w_F^s(\infty, y)]$, and a mass point at $w_F = w_F(y', y)$.

So with P(N=0) the worker of type y receives:

$$w_F(y', y) = \beta \left[\frac{p_F y'}{1+\delta_1} - \frac{\lambda s}{(1+\delta_1)}\right] + (1-\beta)\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} [N_I(x,y) - U(y)]\, dH_I(x \mid y)}{(1-\hat{\delta}_2)}\right]$$

and with $[1 - P(N = 0)]$ the worker of type y receives:

$$w_F^s(x, y) = \beta \left[\frac{p_F x}{1+\delta_1} + \frac{rs}{(1+\delta_1)}\right] + (1-\beta)\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} [N_I(x',y) - U(y)]\, dH_I(x' \mid y)}{(1-\hat{\delta}_2)}\right]$$

Let's first calculate calculate $m_F(w/y)$.

To compute the conditional density of a transformed variable (productivity as a function of wages conditional on y), I know that: $m_F(w/y) = h_F[x = S(w_F^s, y) \mid y)]\frac{dS(w,y)}{dw}$.

Inverting (81) I get $x = S(w_F^s, y)$ as follows:

$$x \equiv S(w_F^s, y) = \left[\frac{1+\delta_1}{\beta P_F}\right] w_F^s - \frac{rs}{P_F}$$

$$-\frac{(1-\beta)(1+\delta_1)}{\beta P_F}\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} [N_I(x',y) - U(y)]\, dH_I(x' \mid y)}{(1-\hat{\delta}_2)}\right]$$

$$\frac{dx}{dw} = \frac{dS(w_F^s, y)}{dw} = \frac{1+\delta_1}{\beta P_F}$$

The conditional distribution of wages in formal-sector employment is:

$$P[w_F = w_F(y', y)] = H_F(R_{FU}(y) \mid y) \quad \text{for } w_F = w_F(y', y).$$

$$m_F(w/y) = \left[\frac{1+\delta_1}{\beta P_F}\right] h_F(x \equiv S(w_F^s, y) \mid y) \quad \text{for } w_F \in [w_F^s[R_{FU}(y), y), w_F^s(\infty, y)] \text{ and } w_F \neq w_F(y', y)$$

I finally compute can compute $m_F(w)$ by using:

$$m_F(w) = \int m_F(w/y) f_F(y) dy$$

6.3.2 Informal Sector

Doing the analogous exercise for the informal sector I can get $m_I(w)$ as follows:

$$m_I(w) = \int m_I(w/y) f_I(y) dy$$

where $m_I(w/y)$ is the steady state conditional distribution of wages in IS employment and $f_I(y)$ is steady state density of types among IS employment.

The mapping from wages to productivity, conditional on y is:

$$x \equiv S(w_I, y) = \left[\frac{w_I}{\beta P_I}\right] - \left[\frac{(1-\beta)}{\beta P_I}\right]\left[\frac{rU(y)}{(1+\hat{\delta}_3)}\right]$$

$$\frac{dx}{dw} = \frac{dS(w_I, y)}{dw} = \frac{1}{\beta P_I}$$

After receiving a shock, the worker continues in the match and receives a salary only if $x \geq R_{IU(y)}$. The conditional distribution of wages in formal-sector employment is:

$$P[w_I = w_I(y', y)] = H_I(R_{IU}(y) \mid y) \quad \text{for } w_I = w_I(y', y).$$

$$m_I(w/y) = \left[\frac{1}{\beta P_I}\right] h_I(x \equiv S(w_I, y) \mid y) \quad \text{for } w_I \in [w_I^s[R_{IU}(y), y), w_I^s(\infty, y)] \text{ and } w_I \neq w_I(y', y)$$

6.4 Steady State Equilibrium

Definition 1 *Given a vector of parameters* $\{b, \alpha, \beta, b, c, \lambda\}$*, a vector of prices* $\{p_F, p_I, r\}$*, a vector of taxes and subsidies* $\{\delta_1, \delta_2, \delta_3, s\}$*, a vector of valuation of social security and social assistance services* $\{\tau, \mu\}$*, matching function* $m(\cdot)$*, and cumulative density functions* $F(y)$*,* $H_i(\cdot \mid y)$ *(for i=I, F) , a* ***Steady State Equilibrium with an Informal sector*** *is a vector formed by the unemployment rate* $u(y)$*, sector-i employment rates* $n_i(y)$*, the value of unemployment* $U(y)$*, the reservation productivities* $R_{UF}(y)$*,* $R_{FU}(y)$*,* $R_{UI}(y)$*,* $R_{IU}(y)$ *and* $R_{FI}(y)$*, sector-i wages* $w_i(y', y)$ *(for i=I, F) and* $w_i^s(x, y)$ *(for i=F), and labor market tightness in the formal and informal sectors* θ *and* ϑ*, such that:*

1. *The flow value of unemployment* $U(y)$ *that satisfies (39).*
2. *The reservation productivity schedule* $R_{FU}(y)$ *that satisfies (19).*
3. *The reservation productivity schedule* $R_{UF}(y)$ *that satisfies (21).*
4. *The reservation productivity schedule* $R_{IU}(y)$ *that satisfies (24).*
5. *The unemployment rate and the sector-i employment rates that satisfy (36), (37) and (38).*
6. *The labor market tightness parameter* θ *that satisfies (34).*
7. *The labor market tightness parameter* ϑ *that satisfies (35).*
8. *The formal-sector wages* $w_F(y', y)$ *and* $w_F^s(x, y)$ *that satisfy (16) and (17).*
9. *The informal-sector wages* $w_I(y', y)$ *that satisfies (18).*

Proof. Existence of Equilibrium

The Steady State Equilibrium with an Informal sector exists if there is a θ that satisfies the job creation condition in the formal sector, equation (34), since all the other equilibrium objects are uniquely determined by θ.

Given that the right-hand side of (34) is continuous in theta, a solution to (34) exists.

To establish uniqueness, I would need to show that the the right-hand side of (34) is strictly monotone. First, $m(\theta)/\theta$ is monotonically decreasing by assumption. Second, $R_{UF}(y)$ is monotonically increasing in θ (since higher θ means higher $U(y)$), so $J_F(y', y)$ is monotonically decreasing in θ. Finally, $u(y)$ should be decreasing in θ due to the dominant negative impact of formal-sector job creation on unemployment (as explained before), and the aggregate unemployment rate u should also be decreasing in θ. However, further assumptions on $H_i(y' \mid y)$ are required to prove that the ratio $u(y)/u$ is monotonically decreasing as well, so uniqueness is not guaranteed. ■

6.5 Computational Algorithm: Approximation of Steady State Equilibrium

Following the definition of Steady State Equilibrium, I write the following computational algorithm to approximate the steady state of the model numerically:

1. Guess values for θ^0. Start an outer loop. Guess values for $R_{UF}(y)^0$, $R_{UI}(y)^0$ and $R_{FU}(y)^0$. Start an inner loop, for fixed values of θ^0. Substitute these values in equation (30) to calculate $rU(y)$.

2. Given $rU(y)$ and $R_{FI}(y)^0$, iterate the Bellman equation (19) to find the fixed point on $R_{FU}(y)$. Call the solution $\hat{R_{FU}}(y)$.

3. Given $\hat{R_{FU}}(y)$, use equation (21) to calculate $\hat{R_{UF}}(y)$.

4. Given $U(y)$, iterate the Bellman equation (24) to find the fixed point on $R_{IU}(y)$.

Call the solution $\hat{R_{IU}}(y)$. Notice that $\hat{R_{IU}}(y)$=$\hat{R_{UI}}(y)$=$\hat{R_{FI}}(y)$.

If the following conditions are met:

$\| R_{UF}(y)^0 - \hat{R_{UF}}(y) \| < \epsilon_{UF}$, and

$\| R_{UI}(y)^0 - \hat{R_{UI}}(y) \| < \epsilon_{UI}$, and

$\| R_{FU}(y)^0 - \hat{R_{FU}}(y) \| < \epsilon_{FU}$, and

Then stop the inner loop. Otherwise update as follows:

$R_{UF}(y)^{new} = R_{UF}(y)^0 + \nu_{UF}(R_{UF}(y)^0 - \hat{R_{UF}}(y))$

$R_{UI}(y)^{new} = R_{UI}(y)^0 + \nu_{UI}(R_{UI}(y)^0 - \hat{R_{UI}}(y))$

$R_{FU}(y)^{new} = R_{FU}(y)^0 + \nu_{FU}(R_{FU}(y)^0 - \hat{R_{FU}}(y))$

where ϵ_{ij} and ν_{ij} are the tolerance levels and step sizes respectively[94].

5. Once convergence is reached in the inner loop, use $\hat{R_{UI}}(y)$, $\hat{R_{UF}}(y)$, and $\hat{R_{FU}}(y)$ and θ^0 in equations (27) - (29) to calculate $u(y)$, $n_F(y)$ and $n_I(y)$. Aggregate over y to get u, n_F and n_I.

6. Given $\hat{R_{UF}}(y)$ and $u(y)$, solve equation (25) to get the equilibrium labor market tightness parameter $\hat{\theta}$. If the following condition is met, $\mid \theta^0 - \hat{\theta} \mid < \epsilon_\theta$, then stop the outer loop. Otherwise

[94] The smaller the steps and the tolerance, the more accurate the results but the longer the computational time. I choose $\epsilon_{ij} = 10^{-3}$, ν_{ij}=0.05.

update as follows: $\theta^{new} = \theta^0 + \nu_\theta(\theta^0 - \hat{\theta})$.

7. Once convergence is reached in the outer loop, use equilibrium reservation productivities,$R_{FU}(y)$ and $R_{IU}(y)$, $u(y)$ and $n_F(y)$, to get the labor market tightness parameter ϑ that satisfies equation (26).

8. Given equilibrium $U(y)$, equilibrium reservation productivities $R_{FU}(y)$, $R_{FI}(y)$, $R_{IU}(y)$, get formal sector wages $w_F(y', y)$ and $w_F(x, y)$ that satisfy equations (16) and (17).

9. Given equilibrium $U(y)$, get informal-sector wage $w_I(y', y)$ that satisfies equation(18).

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