

Catastrophe Risk Pricing

An Empirical Analysis

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Abstract

The price of catastrophe risks is viewed by many to be too high and/or too volatile. Catastrophe risk practitioners point out that, contrary to standard insurance, such as automobile insurance, catastrophe re-insurance is exposed to infrequent but potentially very large losses. It thus requires keeping a large amount of capital in hand, generating a cost of capital to be added to the long-term expected loss. This paper pulls together data from about 250 catastrophe bonds issued on the capital markets to investigate how catastrophe risks are priced. The analysis reveals that catastrophe risk prices

are a function of the underlying peril, the expected loss, the wider capital market cycle, and the risk profile of the transaction. The market-based catastrophe risk price is estimated to be 2.69 times the expected loss over the long term, that is, the long-term average multiple is 2.69. When adjusted from the market cycle, the multiple is estimated at 2.33. Peak perils like US Wind are shown to have a much higher multiple than that of non-peak perils like Japan Wind, revealing the diversification of credit from the market.

This paper—a product of the Global Capital Market Non Banking Financial Institutions (GCMNB) Division, Finance and Private Sector Development Vice Presidency—is part of a larger effort in the department to support clients in the financial management of disaster risks. Policy Research Working Papers are also posted on the Web at <http://econ.worldbank.org>. The author may be contacted at omahul@worldbank.org.

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Catastrophe Risk Pricing: An Empirical Analysis

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Introduction

The price of catastrophe reinsurance is viewed by many to be too high and/or too volatile. Practitioners of catastrophe reinsurance point out that the cost of reinsurance is high because providing protection against large but infrequent events requires keeping large amounts of capital on hand. The return necessary to reward this extra capital is a cost of doing business and that cost is reflected in their protection product. Primary insurance, which serves small but more frequent events of loss, is priced on the basis of expected, or anticipated, loss plus a safety provision. That safety provision, for unanticipated loss, is generally small because actual losses tend to be close to expected losses due to the Law of Large Numbers. Thus the price of primary insurance is seen, with competition, as driving towards expected loss.

Reinsurance on the other hand, because it is concerned with infrequent but large losses, would expect the capital component of price to be much larger. It is likely to be multiples of expected loss instead of fractions of expected loss for primary insurance. Added to the concern about high price is the fact that the traditional reinsurance industry is somewhat opaque. Actual prices are difficult to obtain and analyze. Further, premiums are not typically decomposed into expected loss and capital load so it is hard to say whether the high price is justified or not.

The introduction of Insurance Linked Securities (ILS), popularly known as “cat bonds”, in the mid-1990s has now opened up some of the traditional market practices for closer scrutiny. An ILS issued by an insurer (or reinsurer) represents a reinsurance of a specific risk, transformed into bond form, so that it can be accepted in the broader capital markets. What was reinsurance is now a security and as such that security has a price that can be inspected and analyzed. To put this in perspective, about US\$22 billion of ILS have been issued since the mid-1990s and as of Q1 2008 some US\$13.5 billion in catastrophe securities was outstanding (i.e., still on-risk). The premiums associated with those outstandings were approximately US\$840 million. This new market is not a threat to major reinsurers whose premiums are several orders of magnitude larger, but it could represent a respectable syndicate at Lloyds and is larger than some of the Bermuda reinsurers, so it is not without consequence.

Figure 1 shows the distribution of perils for all the ILS issued since inception. Each peril is a classification of geographical zone and natural hazard – windstorm or earthquake – plus an all “other” category. The graphs show the distribution of perils by expected loss and by limit, respectively.

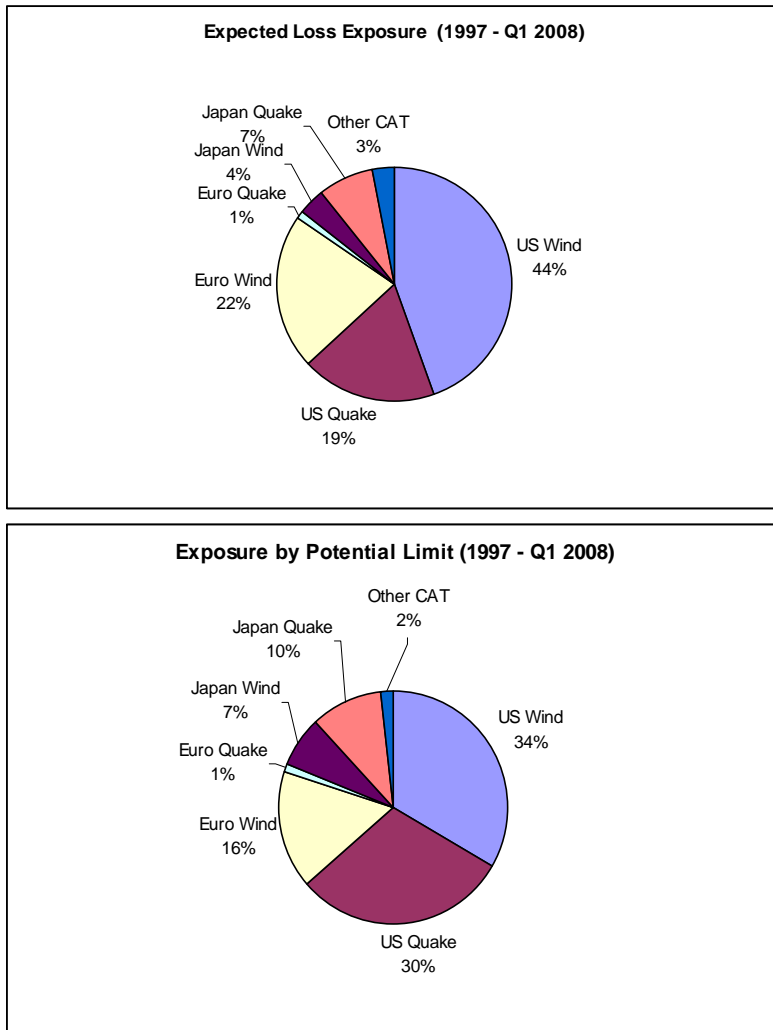
The U.S. market captures about 42% of the catastrophe reinsurance capacity, while it captures almost 60% of the coverage issued through ILS, as shown on Figure 2. European catastrophe risks capture 28% of the traditional catastrophe reinsurance capacity and only 17% of the ILS coverage.

This paper aims to analyze the pricing of catastrophe reinsurance and to identify key components that impact catastrophe risk prices, with a particular attention to the amount of capital that is necessary to protect against extreme risks. The paper examines the pricing of the catastrophe risks assumed by the ILS market and thereby we can make an inference to the whole catastrophe reinsurance market. Proceeding in this way, there are some obvious benefits and some pitfalls. The most obvious benefit is that prices are revealed in ways not available in the traditional reinsurance market. The equivalent of the reinsurance price or premium is the “spread” over LIBOR associated with an ILS at its issue². Further, each bond that is issued comes with a risk analysis that provides the investor with an independent view of that for which he is being compensated. This allows a linkage of price to risk, a most important capital component. This

² Note that while prices are available, they are the prices of “Private Placements”. The ILS market is still not a public market.

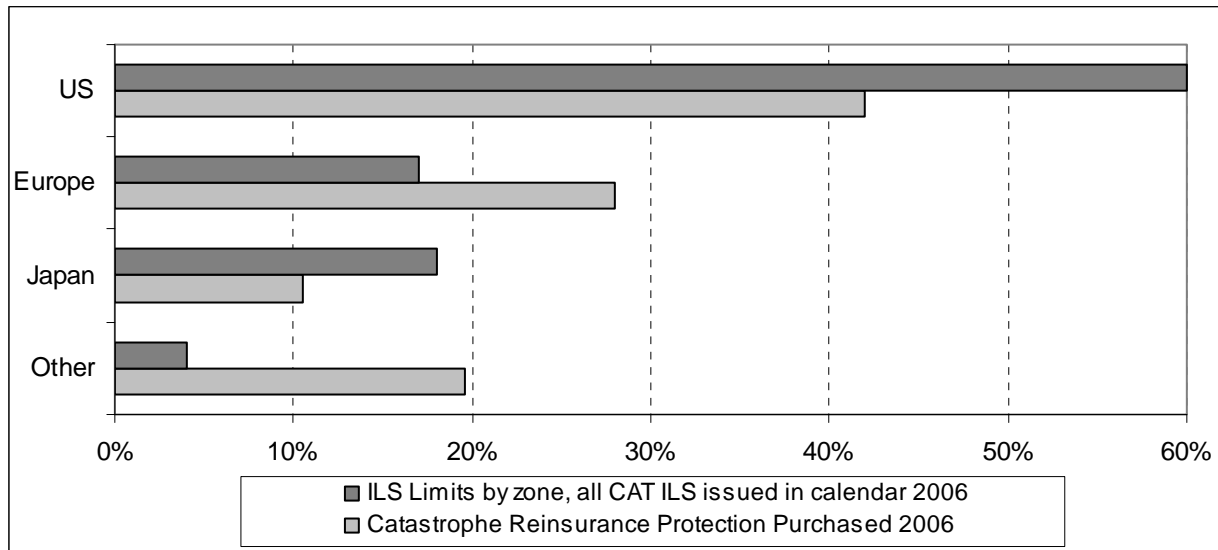
linkage is one that is unique to the ILS market and is usually only available to those few who participate in the market as broker or underwriter. Another advantage is that a secondary market in ILS trading has developed that allows insight into changes in price over time. This is unavailable in traditional markets where the philosophy has been to “write and ride” risk on an annual basis.

Figure 1. Distribution of Perils for Index-Linked Securities since Inception



Source: Lane Financial LLC (2008).

Figure 2. Fraction of Coverage in Catastrophe Reinsurance and ILS Markets, by Geographic Zone



Source: Lane Financial LLC (2008).

Among the pitfalls of the proposed approach is the extrapolation from the ILS market to the catastrophe reinsurance market as a whole. And while it is true that the risks covered by the ILS market are similar to those covered in the market as a whole, the ILS market is less rich and deep compared to the traditional reinsurance market. Finally, of course, the capital standards of the ILS market are quite different from the traditional market. All ILS are fully collateralized; traditional reinsurance is collateralized to some acceptable level of certainty (usually 1 in 250 years, i.e., 0.4%). It is acceptable, that is, to cedants, shareholders and rating agencies. ILS risk assumption is unleveraged; traditional market risk assumption is leveraged to some degree. Inferences can nevertheless be made concerning the risk profiles of the ILS market which are comparable to the market as a whole.

The paper proceeds in four parts as follows. Section 2 examines the issue prices of 247 ILS tranches and their respective risk statistics since 1997. Importantly, the risk statistics are broken down into component perils since peril diversity can be a determinant of capital use. Prices are then deflated by an index of general market prices to see if the cyclic effect of prices can be isolated.³

In Section 3, price relationships are examined without cyclic effects. Ideally this would be done by simultaneously examining all the ILS issued on a particular date. Given the small number of ILS this simultaneity of issue is a rarity⁴. However, an insight can be made with simultaneous evaluation, if not simultaneous issue. This is essentially the information that comes from the secondary market. Two cross section analyses are conducted at different points in the cycle: the most recent hard part in mid-2006 and the most recent part at the end of the first quarter of 2008. The ILS market is not the only part of the catastrophe market that is visible. Since the mid-1990s another vehicle that trades non-traditionally is the Industry Loss Warranty (ILW) market⁵. This market resides somewhere between the traditional market and the securities market in a series of private transactions. ILWs have become sufficiently commonplace for brokers to exist and provide potential participants with price “indication” sheets for a range of

³ See Annex 1 for a description of the data used in this analysis.

⁴ The closest the market ever came to a simultaneous issue of many securities at once was the Successor series of Swiss Re in June 2006.

⁵ The ILW market is also known interchangeably as the OLW market, meaning Original Market Loss Warranty.

coverages. Price “indications” are not transaction prices. Nevertheless, they convey useful information and we incorporate them as a further step in the cross section analysis to further isolate price relationships.

In Section 4, ILW are incorporated into the analysis because we are able to associate an independent estimate of the risk of each ILW with its price “indication”.⁶ A remodeling exercise is conducted to homogenize the risk statistics. While a particular ILS prospectus comes with a risk analysis from one of the major modeling companies (AIR Worldwide, RMS or EQECAT) it often only comes with summary statistics and they are not always directly comparable. The remodeling of nearly all ILS by one company allows two important features not previously available, namely a full risk profile for each ILS, and a set of profiles on the same basis. This set of data is used to compute alternative risk and/or capital measures such as Value at Risk and Tail Value at Risk and try to relate these to price.

Finally, the paper concludes with several observations about catastrophe risk pricing and capital allocation, which may be helpful to international financial institutions like the World Bank in structuring its efforts in this new arena. However, the exercise must be viewed as a best preliminary and the first in many such studies. The results are empirical and derived from unique sources and perspectives. As the market matures and progresses it is hoped that more telling insights can be derived as well as better theories of this important area. Hopefully the results herein will stimulate such activity.

Analysis of the Time Series of Original Issue Statistics

The simple linear model

Our objective is to explain ILS pricing and thereby make inference to the impact of capital on price. The pure transaction database available is the set of original issue prices for 247⁷ ILS issued since 1997. For each issue there is a known spread over LIBOR⁸ (the equivalent of reinsurance premium) and an associated expected loss⁹. So the simple linear model, Model 1, is:

$$\text{Premium Spread} = a + b * (\text{Expected Loss}) \quad (1)$$

If, in this expression, $a=0$ then *Spread* is proportional to *Expected Loss*. If also $b>1$ then *Spread* contains some “load” above *Expected Loss*. Often rationales for insurance pricing assume that

$$\text{Premium Spread} = \text{Function} (\text{Expected Loss}, \text{Load}, \text{Other Expenses})$$

In this expression the load is an amount over cost to generate the risk takers profit. More pointedly, load is to generate return on, or be the cost of, capital. On the ILS market, the investor is typically not confronted with expenses so our focus is on Expected Loss (EL) and Load. If Load is related to return on

⁶ The independent measure is provided courtesy of AIR Worldwide, one of the three independent modeling agencies that regularly provide modeling to ILS prospectuses.

⁷ The actual number of issues to Q1 2008 is actually higher, 251, but certain issues were excluded due to a lack of information or regularity of form.

⁸ The term “Premium Spread” is used to convey the fact that in an ILS the Spread over LIBOR is the equivalent of what Premium would be if the same risk were in reinsurance form. We also use it interchangeably to mean price.

⁹ Typically, the probability of attachment and exhaustion will also be available to investors, often with some other statistics on perils but we do not examine those here.

capital then for each individual ILS it is related to how much capital each ILS consumes and that is related to its riskiness.

The statistical results of the simple model are listed in Table 1 for the transactions since inception (1997-Q1 2008) and for the recent transactions (2003-Q1 2008). In rounded terms, the typical ILS issued between 1997 and Q1 2008 was priced according to a rule of 320 basis points plus double the EL. Or, alternatively, it was 2.7 times the expected loss. In both cases there is a high degree of explanation of price, as measured by the Multiple Correlation coefficient (84% and 90% respectively). Expected Loss is an important determinant of price, but the model may not be a good price predictor. It, nevertheless, shows that there is a load associated with price. In the simplest proportionate case it is the amount over and above expected loss. In this case, a further 1.7 times expected loss.

The ten year period is better in the sense that it contains two full market cycles, but it suffers from the disadvantage of containing a period of experimentation and infrequent issue in the late 1990s. Results from the most recent five year period consider a much more robust underlying market. Notwithstanding, similar orders of magnitude are seen in the coefficient estimates. The most recent period coefficient estimate implies that the multiple of Spread Premium to Expected Loss, called multiple, is 2.87. This may well do as an average estimate but the inspection of the data shows that the multiple varies considerably both over time, by peril and by level of expected loss. The simple model captures none of these.

Consider, for example, the issuance of the Swiss Re Arbor and Successor Series. In July 2003 Swiss Re issued Arbor I with a spread of 15.25% and an expected loss of 4.86%. Two years later, in June of 2005, they re-issued the same series, with the identical risk and peril characteristics, at 12.00%. In June 2006 they issued Successor IIA with a spread of 17.5% and an expected loss of 5.04%. One year after that they issued the same Successor deal at 14.00%. Just taking the multiple as a simple model it had gone from 3.1 to 2.5 back to 3.5 and down to 2.8 in a five year period. Any model like the simple liner model cannot capture a major cause of ILS price volatility. The expected loss in those four price observations was essentially constant.

Table 1. Catastrophe Pricing – Model (1)
Since Inception (1997-Q1 2008)

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	0.032049	0.002837	11.29511	4.31356E-24
Expected Loss	2.094608	0.084863	24.68215	2.16711E-68

Multiple R = 0.8445

No intercept

Intercept	0	#N/A	#N/A	#N/A
Expected Loss	2.694272	0.081477	33.06781	1.66551E-92

Multiple R = 0.9035

Recent Issues (2003-Q1 2008)

Intercept	0.027659	0.003359	8.23534	3.6431E-14
Expected Loss	2.360649	0.093134	25.34691	4.0013E-61

Multiple R = 0.8843

No intercept

Intercept	0	#N/A	#N/A	#N/A
Expected loss	2.866902	0.081927	34.9934	3.15972E-82

Multiple R = 0.9337

Measuring the long term cycle

The multiples or models were shifting due to the then current “hardness” or “softness” of the market. But that is almost tautological: price can only be explained if we know where we are in the cycle. If this is the case we need a measure of where we are in the cycle and a pricing model independent of the cycle. A satisfactory approach might be to try to strip out the cyclic effects and see if the cycle-adjusted price can be explained by expected loss. It might give a more stable relationship. The proposed approach herein is thus first to try to separate out cyclic effects and second to explain adjusted or separated spread by expected loss, then by components of expected loss, i.e., each peril separately.

One measure of cycles in price is demonstrated in Figure 3. It is an index composed of observations of price from various sources. From 1984 to 2000 it is an index of price movements calculated by the now defunct company, Paragon. To the best of our knowledge, it is the only long term index available. More recently Lane Financial LLC has established a new index¹⁰ based on observation of secondary market ILS prices, a pseudo constant expected loss series of original issues from Swiss Re, and observations of ILW prices. This last series combining several markets, the index is deemed to be more robust in the period since 2002. That leaves a gap from 2000 to 2002 where experts’ judgment has been used to complete the longer term view. The resulting index is displayed as the thick lines in Figure 3.

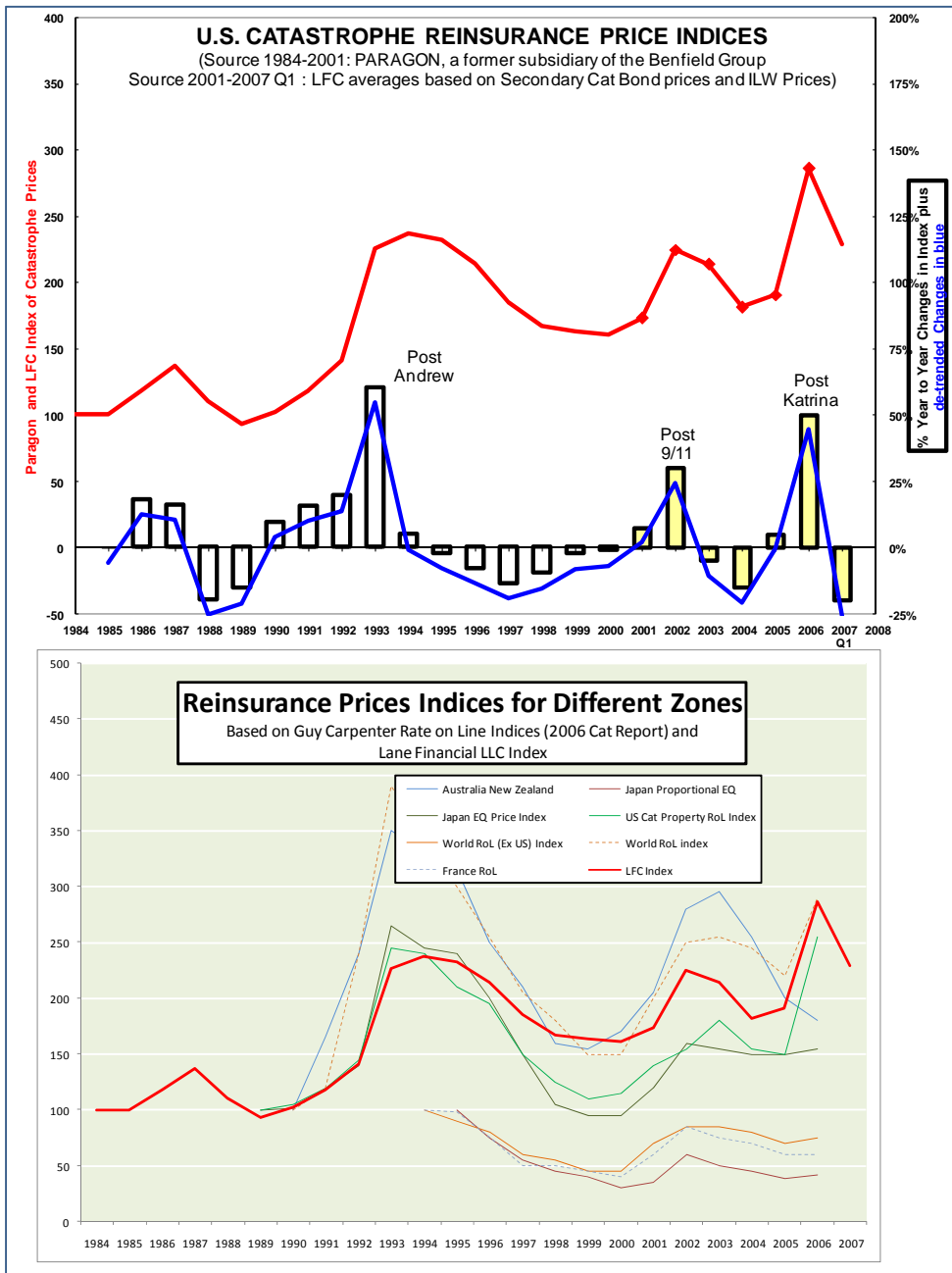
This index¹¹ is displayed in the lower panel of Figure 3 against the rate on line¹² reported by Guy Carpenter in 2006 for various zones or perils around the world. It seems to capture the essentials. The index exceeds 1 when the market is hard and is less than 1 when the market is soft. Over the period 1996-2008, the index varies from 0.76 (in 2000) to 1.65 (in mid-year 2006). This index is therefore used as the component to strip out cyclic effect from observed prices.

¹⁰ Both the longer term index and the shorter term version were presented in the 2007 Annual Review “That was the Year that was” published by Lane Financial LLC. See Annex 2 for the list of long term index of reinsurance prices.

¹¹ We have also cross-checked the analysis herein described using purely Paragon and secondary prices as an alternative cyclic indicator, i.e., without the 2000-2002 “guesstimates”, and using secondary prices in that gap. The main results do not change.

¹² The “rate on line” of reinsurance is analogous to premium for limit of coverage, i.e., just like the spread premium of an ILS. And while Guy Carpenter, and others in the traditional market, regularly report rates on line they do not report corresponding expected losses.

Figure 3. Catastrophe Reinsurance Cycles



Source: Lane Financial LLC (2008).

Adjusting for Cycle

There are many ways to adjust prices for cycle. Two simple approaches are considered; add another coefficient to the regression to represent the cycle level, or divide the actual spread by the index deflator. In other words, the simpler linear model displayed in Equation (1) is adjusted so that the relationship varies depending where one is in the cycle. Thus Model (2) is

$$(Premium\ Spread)_t = a + b * (Expected\ Loss) + c *(Cycle\ Level)_t \quad (2)$$

An alternative form is Model (3):

$$(Premium\ Spread)_t = (Cycle\ Level)_t * [a + b * (Expected\ Loss)] \quad (3)$$

Or, equivalently,

$$[(Premium\ Spread)_t / (Cycle\ Level)_t] = a + b * (Expected\ Loss) \quad (3b)$$

Some simple interpretations are available. In Model (2) when $a=0$ then the cycle component may be interpreted as a load factor which is unrelated to expected loss but which varies over time. It may be viewed as a loose estimate of capital load that depends on cycle: when the market is hard, that is, when the demand for capacity exceeds the supply, the cost of capital increases. Conversely, when the market is soft because of excess capacity the cost of capital decreases. Similarly, in Model (3) when $a=0$ the b may be interpreted as long term multiple, but the term $[(Cycle\ Level)_t * b]$ is the multiple expected to prevail at different points in time.

The results of fitting these models for the ten year and five year periods are displayed in Table 2 and Table 3. The degree of explanation provided by the models rises for the additive Model (2) but falls slightly for the adjusted Model (3). Second, over the five year horizon (lower panel), with a equal to zero the multiple correlation rises from 0.934 in Model (1) to 0.954 in Model (2), but falls to 0.922 in Model (3). Therefore, a simple additive load looks best.

To see the effect of the model on Spread Premiums consider Table 4. Various levels of expected loss and various levels of cycle index are estimated. Then using the five year Model (2) with $a=0$ (see Table 2 with recent issues) the spread levels are calculated. The Table clearly shows that the model picks up desirable features of varying Spread Level estimates by both expected loss level and cycle point: the multiple increases as the market becomes harder and the expected loss decreases.

Model (3) offers an interesting interpretation of the multiple. When $a=0$, Equation (3b) can be rewritten

$$(Premium\ Spread)_t / [(Cycle\ Level)_t * (Expected\ Loss)] = b \quad (3c)$$

The estimate parameter b can be interpreted as the cycle-adjusted multiple. Table 3 shows that the cycle-adjusted multiple is estimated at 2.31 over the period 1997 – Q1 2008, and at 2.37 over the period 2003 - Q1 2008.

Table 2. Catastrophe Risk Pricing – Model (2)
Since Inception (1997-Q1 2008)

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	0.00194012	0.009581203	0.202492	0.839700601
Expected Loss	2.051656829	0.084239213	24.35513	2.88432E-67
Cycle Index	0.033209231	0.010112302	3.284043	0.001173341

Multiple R = 0.8517

No intercept

Intercept	0	#N/A	#N/A	#N/A
Expected Loss	2.052184059	0.084034015	24.42087	1.38768E-67
Cycle Index	0.035168649	0.0029309	11.99927	2.11937E-26

Multiple R = 0.9404

Recent Issues (2003-Q1 2008)

Intercept	-0.00140586	0.010650567	-0.13199	0.895134073
Expected Loss	2.333766308	0.091786229	25.42611	3.76097E-61
Cycle Index	0.030405597	0.010596176	2.869488	0.004609275

Multiple R = 0.8898

No intercept

Intercept	0	#N/A	#N/A	#N/A
Expected Loss	2.332482952	0.091019005	25.62633	8.57089E-62
Cycle Index	0.02907543	0.003266839	8.900171	6.08881E-16

Multiple R = 0.9035

Table 3. Catastrophe Risk Pricing – Model (3)
Since Inception (1997-Q1 2008)

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	0.034679	0.002444	14.18696	9.43757E-34
Expected Loss	1.663705	0.073108	22.75669	2.3071E-62

Multiple R = 0.8239

No intercept

Intercept	0	#N/A	#N/A	#N/A
Expected Loss	2.31257	0.07682	30.10386	1.87423E-84

Multiple R = 0.8869

Recent Issues (2003-Q1 2008)

Intercept	0.027595	0.002911	9.478194	1.58295E-17
Expected Loss	1.86544	0.080734	23.10603	1.32376E-55

Multiple R = 0.8654

No intercept

Intercept	0	#N/A	#N/A	#N/A
Expected Loss	2.370522	0.074119	31.98266	3.65308E-76

Multiple R = 0.9222

Table 4. Indicated Levels of Premium Spreads from Model (2)

Expected Loss coefficient 2.33	Cycle Index Coefficient 2.91%	Model Indicated Premium Spread Levels	Multiples
Mid Market Cycle Levels			
0.50%	1.00	4.08%	8.15
1.00%	1.00	5.24%	5.24
2.00%	1.00	7.57%	3.79
5.00%	1.00	14.56%	2.91
Hard Market Cycle Levels			
0.50%	1.60	5.82%	11.64
1.00%	1.60	6.99%	6.99
2.00%	1.60	9.32%	4.66
5.00%	1.60	16.31%	3.26
Soft Market Cycle Levels			
0.50%	0.75	3.35%	6.70
1.00%	0.75	4.51%	4.51
2.00%	0.75	6.84%	3.42
5.00%	0.75	13.83%	2.77

Component Perils

Since 1997 almost 40% by value of all ILS issued have involved more than one peril, that is, are multi-peril bonds. The rest are single peril. When single peril bonds are issued the risk analysis will be provided for that specific peril. When multi-peril bonds are issued, the prospectus typically also provides the fraction of overall expected loss that emanates from the component perils of the bond. The historical result of this by expected loss and by potential limit was shown in Figure 1.

The combination of this expected loss by peril data means that Model (1) can be amended to allow for peril exposure. Thus Model (1') is:

$$Premium\ Spread = a + \sum_p [b_p * (Expected\ Loss)_p] \quad (4)$$

Where parameter b_p is the coefficient associated with peril p and $(Expected\ Loss)_p$ is the expected loss from peril p . In the empirical analysis that follows, the set of perils considered is the same as the one displayed in Figure 1: US Wind, US Quake, Euro Wind, Euro Quake, Japan Wind, Japan Quake and "Other". Other includes catastrophes in other parts of the world rather than the six already listed: examples include CAT-Mex covering Mexico earthquake and GlobeCat LAQ covering Guatemala earthquake. Clearly, ILS issues to date have been dominated by the four major perils; less than 10% of all limits (and an even smaller share of expected loss) have been Euro Quake, Euro Wind or Other. Model (2) is also expanded to include component perils, and it denoted Model (2').

Tables 5 and 6 show the result of refining the expected loss data into its component perils using Model (1') and Model (2'), respectively, over the same ten and five year periods.¹³ The explanatory power of these models is high, with a Multiple R higher than 90% (See Tables 4 and 5). However, some parameters are not statistically significant. "Other Perils" is not statistically significant in Model (1') and Model (2').

¹³ It can be shown that the degree of explanation under Model (2'), with Multiple R = 0.9672, is higher than the degree of explanation under Model (3'), that is Model (3) with decomposition by peril, with Multiple R = 0.9278.

Table 5. Catastrophe Risk Pricing – Model (1) with Component Perils Since Inception (1997-Q1 2008)

No intercept

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
US Wind	3.295162	0.123135	26.76055	5.46E-74
US Quake	3.043272	0.276142	11.02068	4.01E-23
Euro Wind	1.944646	0.290497	6.694208	1.52E-10
Euro Quake	1.412992	1.030007	1.371827	0.171398
Japan Wind	3.970125	1.117267	3.553424	0.000458
Japan Quake	2.55942	0.453563	5.642922	4.7E-08
Other Perils	-0.51689	0.444247	-1.16352	0.245774

Multiple R = 0.9317

Recent Issues (2003-Q1 2008)

No intercept

US Wind	3.317867	0.129194	25.68123	4.36E-61
US Quake	2.832238	0.299626	9.452575	2.25E-17
Euro Wind	1.793549	0.323083	5.551354	1.04E-07
Euro Quake	1.774677	1.082363	1.639632	0.102888
Japan Wind	2.975919	1.281416	2.322368	0.021371
Japan Quake	2.523373	0.492235	5.12636	7.81E-07
Other Perils	1.421159	0.842704	1.686428	0.093506

Multiple R = 0.9420

Table 6. Catastrophe Risk Pricing – Model (2) with Component Perils Since Inception (1997-Q1 2008)

No intercept

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
US Wind	2.723048	0.095658	28.46636	9.71E-79
US Quake	1.835132	0.212636	8.630401	8.7E-16
Euro Wind	1.383443	0.210754	6.564255	3.22E-10
Euro Quake	0.119274	0.740626	0.161045	0.872194
Japan Wind	0.995709	0.821662	1.211824	0.226777
Japan Quake	2.034328	0.325811	6.2439	1.93E-09
Other Perils	-0.31434	0.317601	-0.98974	0.3233
Cycle Index	0.035704	0.002347	15.21153	5.3E-37

Multiple R = 0.9659

Recent Issues (2003-Q1 2008)

No intercept

US Wind	2.787485	0.108703	25.64306	7.9E-61
US Quake	1.752864	0.246679	7.105855	3E-11
Euro Wind	1.314467	0.249004	5.278894	3.86E-07
Euro Quake	0.282996	0.832554	0.339913	0.734334
Japan Wind	0.830354	0.991505	0.837468	0.403485
Japan Quake	2.090056	0.37582	5.561329	1E-07
Other Perils	0.342672	0.647094	0.529556	0.597099
Cycle Index	0.033549	0.002958	11.34069	1.21E-22

Multiple R = 0.9672

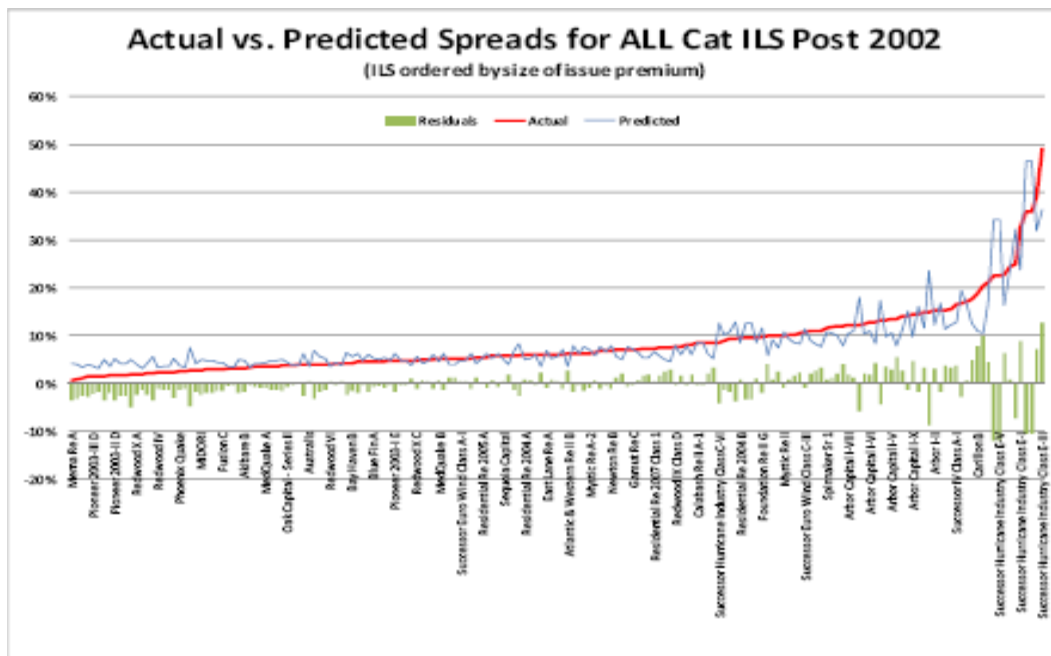
The parameters associated with Euro Quake, Japan Wind are all less than, or borderline statistically significant. Thus we cannot rely on the parameter estimates for those perils in the same way we can for US Wind, for example. The reason for the statistical caution may be due to the fact that there is not enough data on these issues to be confident about the statistical robustness of the estimates.

Nevertheless, qualitative if not exact quantitative observations can be made. Compare the most recent period before and after disaggregation into component perils. The multiple for expected loss as a whole is 2.332, with a cyclic load coefficient of 2.91% (see Table 2). However, when the expected loss is broken down to its component perils, the multiple for each peril is different. The US Wind peril demands the highest multiple of 2.787 whereas Euro Quake has a multiple of 1.314 (see Table 6). In other words, when pricing a bond with pure US Wind with an expected loss of, say, 2% the premium would be 5.575% plus the cyclic load component -- 3.355% in a neutral market (index =1) -- for a price of 8.930 (a multiple of 4.5). By the same reasoning using Model (2') for a Euro Wind risk with an expected loss of 2% would give a price of 5.984%. Table 5 also shows that Japan Wind has a multiple of less than one. A 2% Japan Wind risk, in the same neutral environment (index=1) would price at 5.05% - a multiple of 2.5. Therefore, a significant component of price is over and above expected loss, but it varies considerably by peril.

Coefficients of expected loss less than one could mean that credit is given for diversification. And in the very rare perils – Euro Quake and Other – the diversification credit is high. This is subject to the statistical significance of these estimates, which may not be always highly reliable. Furthermore, even when the coefficient is less than one (e.g., Japan Wind), it does not mean that the premium will be less than expected loss: there is an additional factor to be considered, the cyclic load, which is also part of the capital requirements.

The usefulness of these models for relative pricing of individual securities is not the subject of this paper; however, it can be illuminating to view these models from that perspective, as illustrated in Figure 4 below, where spreads are estimated using Model (2').

Figure 4. Actual vs Predicted Spreads for CAT ILS Post 2002.



Cross Section Analysis of Secondary Market Prices

After an ILS is issued in security form it is eligible to be traded in the secondary market. Investors who bought a particular transaction may later find that they see new opportunities and wish to readjust their portfolio by selling the ILS. Others may reassess the risk and want to sell. Still others may want to acquire more after failing to get a big enough initial allocation. All these motivations to trade are typical of security markets. Such after-issue trading is not, however, typical of the underlying reinsurance market. Reinsurers write a risk and hold it. Thus the ILS market brings to reinsurance an element not available in the traditional market, that is, secondary trading. With it comes the price of such secondary transactions and the information about current market conditions. Given a robust secondary market, transaction pricing is invaluable contemporary information to issuers and investors alike.

The ILS secondary market cannot be considered as robust, but it does exist and is growing. Dealers regularly provide prices at which they might transact, if asked to do so, or that they have recently seen transacted. This is the information contained in price indication sheets. And, while far from perfect, it is nevertheless valuable. The average of such price indication sheets from several dealers is available for two periods: end Q2 2006 and end Q1 2008. These two points respectively represent the hardest point in the recent post-Katrina hard market and the most recent quarter. Some 41 separately priced ILS tranches were outstanding in Q2 2006 and 83 were outstanding¹⁴ in Q1 2008, which illustrates how much the market has grown in the last two years.

An ILS issued with a premium spread of 8% over LIBOR might trade in the secondary market at 6% one year later. This is said to be its current yield. It is the premium spread at which an identical new bond might be issued. A fall of premium spread from 8% to 6% would indicate a considerable softening in the market. If expected losses were the same then the load above expected loss would have dropped significantly. This cyclical impact on pricing was identified and quantified in the previous section. The cross section analysis provides another light on the effect of market cycles by comparing the ILS priced on the secondary market at two different periods.

Before proceeding, however, a note of caution is needed. After initial issue, prices may change in the secondary market, but so may estimates of expected loss. While investors are given an initial estimate of expected loss, no one provides them with updates as time and new information becomes available to change an expected loss estimate. It is the responsibility of each ILS holder to make those estimates or to engage someone to do it for them. The upshot is that there is no information about secondary estimates of expected loss. In what follows, secondary market yields are compared with original issue estimates of expected loss.

Model (1) is adapted to capture the current yield and no intercept is allowed. Model (1b) is:

$$\text{Current Yield} = b * (\text{Original Expected Loss}) \quad (5)$$

The basic results of the cross-section analysis are displayed Table 7. The two periods are displayed alongside each other for comparison purposes. The comparison is quite dramatic: the multiple falls from 3.3374 to 1.6814 – a fifty percent reduction. The cross section analysis confirms what the cycle adjusted

¹⁴ The deals considered that the cross-section analyses contained certain Wind transactions that were eliminated from the statistical analysis because they were too close to maturity (3 months or less). At 3 months to maturity most seasonal risk has disappeared and pricing tends to be erratic.

time series showed, namely if the amount over expected loss is considered to be load then it varies considerably over time.

Table 7. Cross Section analysis – Model (1b)

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Second Quarter 2006 (41 observations)				
Original				
Expected Loss	3.337374	0.191179	17.45681	2.6E-20
Multiple R = 0.9402				
First Quarter 2008 (83 observations)				
Original				
Expected Loss	1.681447	0.142756	11.77848	2.57E-19
Multiple R = 0.7928				

Component Perils

The expected loss can also be broken into component perils, as it was done in the previous section. Model (1'), where the intercept is set at zero, becomes Model (1b')

$$Current\ Yield = \sum_p [b_p * (Original\ Expected\ Loss)_p] \quad (6)$$

However, note that in the case of Q2 2006 only 41 deals are available, so that the degrees of information for separating out the perils are even less than in the time series analysis. The results are shown in Table 8. The presence of so many “not significant” variables commends caution¹⁵.

Table 8. Cross Section Analysis – Model (1b) with Component Perils

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Second Quarter 2006 (41 observations)				
US Wind	3.44806	0.323957	10.64359	1.62E-12
US Quake	4.465869	1.40003	3.189838	0.002999
Euro Wind	3.365641	0.760508	4.425514	8.97E-05
Euro Quake	0	0	N/A	N/A
Japan Wind	6.921184	6.968441	0.993218	0.327421
Japan Quake	1.984895	0.900147	2.20508	0.034112
Other Perils	1.777366	1.848708	0.96141	0.342942
Multiple R = 0.9459				
First Quarter 2008 (83 observations)				
US Wind	2.396642	0.252253	9.500951	1.47E-14
US Quake	2.187185	0.39567	5.527794	4.38E-07
Euro Wind	-1.0669	0.618171	-1.72591	0.088428
Euro Quake	1.702749	1.169533	1.455922	0.149533
Japan Wind	1.726069	1.521742	1.134272	0.260246
Japan Quake	1.114602	0.785889	1.418269	0.1602
Other Perils	2.199775	1.031283	2.133046	0.036149
Multiple R = 0.8355				

¹⁵ Regressions have been re-run eliminating the “not significant” variables. The coefficients of the remaining variables change – but not markedly – so we have deliberately left the poorer fit to show the larger picture.

The component perils coefficients display the same drop in value as in the Model (1'). US Wind drops from 3.4481 to 2.3966. US Quake drops from 4.4659 to 2.1872. The Quake coverage was in much demand in Q2 2006, and there is anecdotal evidence for this. After Hurricane Katrina in August 2005, it became apparent that while physical property damage generate the main demand for catastrophe coverage, catastrophes can also affect workers compensation claims. Accordingly, reinsurers began to increase their coverage in US quake zones to allow for additional non physical claims from an earthquake.

Cross Section Analysis including ILWs and Re-Modeled Risk Statistics

Beyond the ILS market there is a related market for insurance risk that trespasses outside the traditional realm; it is the Industry Loss Warranty Market, or ILW market. Since much of the ILW trading takes place in the capital market. However, whereas the ILS quantities are a well known, the ILW sizes are not. Neither is there any declared expected loss associated with a particular ILW transaction. They are very much private and confidential transactions. There are, however, indicated prices and modeling agencies that will provide third party expected loss estimates.

We use in this analysis remodeled ILW and ILS estimates provided by Air Worldwide, which is one of the three largest catastrophe risk modeling firms with RMS and EQECAT. As one of three independent modelers they provide roughly one third of the original expected loss estimates and risk analyses for all issued ILS. An important question is whether all modeling firms model on the same basis. Is a 2% expected loss from AIR directly comparable to a 2% expected loss from RMS? Generally the answer is positive but there are differences that market participants are well aware of. One company tends to be conservative with a particular peril than another while another is more liberal elsewhere. Over time investors have been sensitized to such nuances and have come to demand for themselves analyses of risk on the same, or at least on a consistent, basis. The model companies have responded by re-modeling nearly all deals that are outstanding whether originally analyzed by themselves or their competitors.

One consequence of having a remodeled set of estimates is that they may differ from those originally provided with an ILS. We thus distinguish the original expected loss, oEL, and the remodeled expected loss, rEL. By way of double checking the series, Model (1b) is re-run with both series for the Q1 2008 ILS set with secondary yields. The results are shown in Table 9. The parameter estimates look very similar. Apparently, the market looks mostly at the original data, and not yet at remodeled statistics.

Table 9. Comparing the Use of Original vs Re-Modeled Expected Loss

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Observed data: First Quarter 2008 (83 observations)				
Intercept	0.041456	0.00435	9.529676	7.14E-15
Expected Loss	0.883247	0.12939	6.826238	1.46E-09
Multiple R = 0.6043				
Remodeled data: First Quarter 2008 (83 observations)				
Intercept	0.040351	0.005137	7.854396	1.45E-11
Expected loss	0.872826	0.159259	5.48055	4.67E-07
Multiple R = 0.5201				

Another virtue of having data modeled on the same basis is that we can make estimates of other statistics beyond expected loss. Theoretically this is possible with original risk analysis as well but prospectus

information is often given in such a way as to make the exercise difficult, or at least cumbersome. Two risk measures: Standard Deviation and Tail Value at Risk.

Standard Deviation is a well known measure of risk; it is essential to much capital market pricing where returns tend to be symmetric. Even in the reinsurance world there are those who believe prices should be set using standard deviation. Kreps (2005) has proposed the following pricing model:

$$\text{Premium Spread} = \text{Expected Loss} + \alpha * [\text{Standard Deviation}] \quad (7)$$

where alpha represents a fraction of the standard deviation that should be loaded on to the expected loss. It can be rewritten as:

$$\{\text{Premium Spread} - \text{Expected Loss}\} = \alpha * [\text{Standard Deviation}] \quad (7')$$

Or, equivalently, since $\{\text{Premium Spread} - \text{Expected Loss}\}$ can be viewed as the Expected Excess Return (EER)

$$\text{EER} = \alpha * [\text{Standard Deviation}] \quad (7'')$$

Then $\alpha * [\text{Standard Deviation}]$ can also be viewed as the expected return on capital required by the market on ILS of that riskiness.

An alternative risk measure is Tail Value at Risk. In this, a probability level is set, such as x%, and the tail value at risk is defined as the expected level of loss experienced in the next worst (1-x)% of outcomes. This is denoted as TVaR(1-x). For example, TVaR99 is the expected level of loss to be experienced in the worst 1% of outcomes. The level of loss experienced exactly at the chosen probability level is itself a risk measure known as Value at Risk or VaR. It has been popular for some time, but it has some less than satisfactory properties and generally the market is moving toward the use of TVaR.. Adapting the simple model for TVaR gives a linear function:

$$\text{Premium Spread} = \text{Expected Loss} + \alpha * \text{TVaR} \quad (8)$$

Reasoning similar to the standard deviation case shows that the term $\alpha * \text{TVaR}$ can be interpreted as the expected return on capital for an ILS with that level of TVaR riskiness. In what follows the regressions are conducted at several levels of TVaR.

Table 10 shows the result of the cross section analysis for Q2 2006 and Q1 2008 using secondary ILS prices, ILW price indications and expected losses. All measures are on a re-modeled consistent basis and in each case the regressions are conducted with

$$r\text{EER} = \text{Current Yield} - r\text{EL} = \alpha * [\text{risk measure}],$$

where rEER is the remodeled Expected Excess Return and the risk measure is either the standard deviation or TVaR.

Table 10. Secondary Market Remodeled Expected Excess Return vs Risk Measures

ILS and ILW 06Q2 (148 observation)				
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Std Dev	0.448869	0.025233	17.78891	1.76E-38
Multiple R = 0.8263				
TVaR90	0.17059	0.010945	15.58562	6.03E-33
Multiple R = 0.7893				
TVaR99	0.099455	0.005626	17.67798	3.3E-38
Multiple R = 0.8247				
ILS and ILW 08Q1 (213 observation)				
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Std Dev	0.247743	0.025202	9.830178	4.98E-19
Multiple R = 0.5596				
TVaR90	0.091548	0.010599	8.637379	1.39E-15
Multiple R = 0.5102				
TVaR99	0.054198	0.005134	10.55756	3.28E-21
Multiple R = 0.5870				

First, consider the standard deviation model. In Q2 2006 individual ILS, or for that matter individual ILWs, appear to have been priced on the basis of expected loss plus 44.88% of the standard deviation embedded in that particular ILS (see Table 10). For example, an ILS with an expected loss of 2% and a standard deviation of 5% would be priced at 4.2445%. That is with a load of 2.2445%, or equivalently an expected profit of 2.2445%. By Q1 2008, however, the estimated load for the standard deviation had dropped to 24.77% of the standard deviation. So the same ILS re-priced in 2008 would have a premium of 3.2385%. The near fifty percent drop of the estimated load has led in this example to a premium drop of some 25%.

Similar stories can be derived from the TVaR regressions. The TVaR90 coefficient drops from 0.1706 to 0.0915. Similarly, the TVaR99 coefficient drops from 0.0995 to 0.0542. In both cases they are cut in half. To give some intuition to the TVaR measures consider an ILS with an expected loss of 2% and a TVaR90 of 40%. In other words, in the worst 10% of cases the investor would expect to lose 40% of principal. The models would suggest premium shift for each of the periods from 8.824% to 5.366%, a drop of some 40%. The TVaR and the standard deviation models demonstrate dramatically the effect of a shift from a hard market to a soft market.

Concluding Remarks

Theory suggests that an important driver of reinsurance pricing is the amount of capital that is necessary to protect against extreme risk (see, for example, Froot (2007)). In practice, catastrophic risk pricing fluctuates quite dramatically following extreme events. However, identifying or quantifying an exact relationship is more elusive. Some of the principal practitioners of reinsurance are private companies located offshore where disclosure requirements are not as stringent as for onshore primary insurers. Even if required to report to regulators, however, it is unlikely they would reveal either their pricing practice or their capital allocation rules. Moreover, many of the companies that write catastrophe reinsurance do it as part of a multi-line business. There are only a few catastrophe reinsurance specialists. The industry has traditionally been quite opaque.

A window has now been opened to reinsurance pricing by the introduction during the last decade of Insurance Linked Securities (ILS), such as catastrophe bonds. These bonds, distributed broadly to investors in the capital markets, contain catastrophe risk and have explicit prices associated with them. They also associate premiums with a risk analysis. To the extent the embedded risks are typical of the traditional market the components of price can now be forensically examined. Since inception in 1996, in excess of 250 separately priced ILS tranches have been issued and priced. These bonds contain a wide variety of catastrophe perils. The principal peril classifications are windstorm and earthquake risk in the US, Europe and Japan. These have been complimented by others including earthquake risk in Mexico and Guatemala, and windstorm in the Caribbean. This set of perils is quite representative of the traditional reinsurance market as a whole.

This paper has found that the long term average over the whole set of bonds (1997 – Q1 200) is 2.69 times the expected loss. The most recent issues of the last five years show an average of 2.87. Whether this is too high is not a subject of discussion here; clearly these are freely derived market prices, so they are correct if we assume that these markets are competitive.

There is suggestion that the multiple will decline over time as the ILS market becomes more and more competitive. However, this is not a conclusion that can be easily drawn from the data. Simple multiples differ according to the point in the cycle; the level of risk attachment; the peril and zone; the risk profile of the coverage; and how it diversifies or concentrates the existing portfolio. Stripping out the volatile elements of the hard/soft market reinsurance cycles indicates that the cycle-adjusted multiple is estimated at 2.33.

Differentiating by peril shows that the multiple for US Wind peril is a relatively high 2.78 plus a cycle adjustment. On the other hand, Japanese wind shows a multiple of 0.88 plus the cycle adjustment. This is strong evidence that diversification pays. Japanese Wind deals appear to get a diversification credit from the market, i.e., they are priced at less than expected loss (when cyclic components are excluded). Similar remarks appear to apply to other non-peak perils.

Further detailed examination of these phenomena is conducted at two points in time using secondary market prices and Industry Loss Warranty (ILW) prices instead of the time series. This view points to the volatile nature of the markets. In the last hard market, June 2006, the capital load as represented by the long-term average multiple was 3.34. The most recent, first quarter 2008, was 1.68. And, correspondingly, the peril-specific multiples change dramatically between periods. US Wind peril falls from 3.45 to 2.4. US Quake falls from 4.47 to 2.19. Similar effects are perceived in the more diversifying perils. In other words, there is even a diversifying credit that is not immune to the larger capital market cycle.

The capital cost in a transaction is also affected by the profile of the embedded risk. Thus a deal with a long tail in its risk distribution may require more capital than one with more concentrated outcomes. Risk profiles are best summarized by risk measures such as Standard Deviation and Tail Value at Risk. This paper shows how higher capital loads are associated with higher risk measures. This, too, varies by cycle.

Catastrophe reinsurance prices, as represented by the ILS market, are determined freely in the capital markets. Dissecting the prices reveals that any ILS price is a function of a) expected losses, b) the wider capital market cycle, c) the risk profile of the transaction, d) the perils contained.

This paper has not proposed, nor discovered, any new models of catastrophe risk pricing behavior. Instead it has pulled together data from a number of disparate, and sometimes proprietary, sources to examine

certain simple intuitive ideas about pricing. The list of data includes: original issue ILS data, secondary market ILS data, a cyclic price index, ILW price indication sheets and a set of remodeled scenarios allowing evaluation of individual ILS and ILW and portfolios of the same. The quality of the data is uneven but to our knowledge has not been previously brought together in this fashion. The exercise is comforting in that the various relationships tested herein make intuitive sense and, just as important, highlight areas of insufficient knowledge.

Catastrophe risk pricing has for too long been a subject of actuarial scrutiny of statistical costs (expected losses) with little attention paid to market driven prices. The analysis herein shows that market prices follow a cycle, and they vary considerably by peril. Even low premium, low correlation perils appear to follow the same cycle, even if the cycle is amplified for peak perils. Pricing also depends on the risk characteristics of individual transactions.

None of these are surprising in themselves, and yet the analysis shows that each component is significant. It remains, of course, to bring all the elements together in a single model and quantify that relationship. That is for the future. The present study we believe takes the first step in that direction.

Annex 1. Data, Sources and Units

The data used in the regression exercises conducted in this paper comes from three proprietary sources; the ILS Data Base of Lane Financial LLC, Price Sheets of Access Re and the remodeled scenario set of AIR Worldwide. The secondary market ILS data is a compilation of Price Indication Sheets produced by various brokers at regular intervals. The indications are averaged at mid-market quotes. Similarly, ILW prices from Access Re are “indication only” prices. They represent neither actionable nor transacted prices. As such, statistical exercise results should be viewed with appropriate caution. Further, while every attempt has been made to be accurate, any errors that may have occurred are the responsibility of the author, not the source.

Where possible in the following, data on spreads and expected losses, etc., are quoted in decimal form. Thus a spread of 0.095 is the equivalent of 9.5% or 950 basis points. Unfortunately, not all sources use the same convention so attention should be paid to context. Also, while data is in that form, text may use the conventions interchangeably.

Annex 2. Lane Financial Long Term Index of Catastrophe Reinsurance prices
**Lane Financial Long Term Index of Catastrophe Reinsurance
Prices**

	Ten Year Index		Ten Year Index
12/31/1996	1.00	3/31/2003	1.05
3/31/1997	0.97	6/30/2003	1.07
6/30/1997	1.01	9/30/2003	1.03
9/30/1997	0.93	<u>12/31/2003</u>	0.99
<u>12/31/1997</u>	0.87	3/31/2004	0.92
3/31/1998	0.85	6/30/2004	0.96
6/30/1998	0.89	9/30/2004	0.92
9/30/1998	0.84	<u>12/31/2004</u>	0.91
<u>12/31/1998</u>	0.79	3/31/2005	0.87
3/31/1999	0.80	6/30/2005	0.90
6/30/1999	0.80	9/30/2005	0.86
9/30/1999	0.79	<u>12/31/2005</u>	1.02
<u>12/31/1999</u>	0.77	3/31/2006	1.08
3/31/2000	0.76	6/30/2006	1.65
6/30/2000	0.77	9/30/2006	1.57
9/30/2000	0.76	<u>12/31/2006</u>	1.46
<u>12/31/2000</u>	0.76	3/31/2007	1.30
3/31/2001	0.76	6/30/2007	1.35
6/30/2001	0.79	9/30/2007	1.19
9/30/2001	0.79	<u>12/31/2007</u>	1.06
<u>12/31/2001</u>	0.83	3/31/2008	1.06
3/31/2002	0.90		
6/30/2002	1.01		
9/30/2002	1.05		
<u>12/31/2002</u>	1.07		

Source: Lane Financials LLC (2008).

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