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Middle-Income Growth Traps

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Abstract

This paper studies the existence of middle-income growth traps in a two-period overlapping generations model of economic growth with two types of labor and endogenous occupational choices. It also distinguishes between "basic" and "advanced" infrastructure, with the latter promoting design activities, and accounts for a knowledge network externality associated with product diversification. Multiple steady-state equilibria may emerge, one of them taking the form of a low-growth trap characterized by low productivity growth and a misallocation of talent—defined as a relatively low share of high-ability workers in design activities. Improved access to advanced infrastructure may help escape from that trap. The implications of other public policies, including the protection of property rights and labor market reforms, are also discussed.

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MIDDLE-INCOME GROWTH TRAPS

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1 Introduction

Since the 1950s, rapid growth has allowed a significant number of countries to achieve middle-income status. However, very few have made the additional leap needed to become high-income economies. Rather, many have gotten stuck in what has been called a middle-income trap—or more appropriately perhaps, an imitation trap, as discussed later—characterized by a sharp deceleration in growth.[1] Most countries in Latin America and the Middle East for instance reached middle-income status during the 1960s and 1970s, and have remained there ever since. According to the World Bank (2012), of 101 middle-income economies in 1960, only 13 had become high income by 2008: Equatorial Guinea; Greece; Hong Kong; China; Ireland; Israel; Japan; Mauritius; Portugal; Puerto Rico; South Korea; Singapore; Spain; and Taiwan, China.[2]

In Asia, Malaysia and Thailand provide good examples of the growth slowdown that characterizes a middle-income trap. Despite the financial crisis of 1997-98, they ended the century with productivity levels that stood significantly closer to those recorded in advanced countries. However, the pattern of labor-intensive production and exports in these countries has remained broadly unchanged for the past two decades. At the same time, they have faced growing competition from low-cost producers, first China and India, and more recently Vietnam and Cambodia. Growth has slowed significantly as a result. Moving up the value chain and resuming rapid growth by breaking into fast-growing markets for knowledge and innovation-based products and services has remained elusive—not only for Malaysia and Thailand but also for a number of other middle-income countries (UNIDO (2009)).

In a more formal analysis, Eichengreen et al. (2011) found that growth slowdowns typically occur at per capita incomes of about $16,700 in 2005 constant international prices.[3] At that point, the growth rate of GDP per capita slows from 5.6 to 2.1 percent, or by an average of 3.5 percentage points.[4] They also found, using regression and standard growth accounting techniques, that growth slowdowns are essentially *productivity* growth slowdowns—with a drop in TFP growth accounting for about

[1] The term "middle-income trap" was apparently first used by Gill, Kharas, et al. (2007); see also the Growth Report of the Commission on Growth and Development (2008). "Middle income economies" are defined in accordance with the World Bank's classifications by income group, as given in http://data.worldbank.org/about/country-classifications.

[2] The ongoing crisis in many of these countries has of course brought into question the sustainability of these achievements.

[3] The authors define a growth slowdown based on three conditions: The first requires that prior to the slowdown the seven-year average growth rate is 3.5 percent per annum or greater. The second identifies a growth slowdown with a decline in the seven-year average growth rate by at least 2 percentage points. The third condition limits slowdowns to cases in which per capita GDP is greater than $10,000 in 2005 constant prices—thereby ruling out episodes related to countries that have not yet successfully developed. The authors also claim that weakening some of these conditions does not affect the results significantly.

[4] However, there is considerable dispersion in the income levels at which growth slowdowns occur: the standard deviation is $6,000. Although the mean per capita income is $16,700, the minimum is $10,000 and the maximum $40,000.

85 percent, or 3 percentage points, of the absolute reduction in the growth rate of GDP per capita—and that the peak probability of a growth slowdown occurs when manufacturing accounts for about 23 percent of total employment in the economy.

A common explanation of growth slowdowns is based on a Lewis-type development process. In that perspective, factors and advantages that generate high growth during an initial phase of rapid development—low-cost labor and imitation of foreign technology—disappear when middle- and upper-middle-income levels are reached, thereby requiring new sources of growth to maintain sustained increases in per capita income. Indeed, during a first phase, low-income countries can compete in international markets by producing labor-intensive, low-cost products using technologies imported from abroad. These countries can achieve large productivity gains initially through a reallocation of labor from low-productivity agriculture to high-productivity manufacturing. However, once these countries reach middle-income levels, the pool of underemployed rural workers drains and wages begin to rise, thereby eroding competitiveness. Productivity growth from sectoral reallocation and technology catch-up are eventually exhausted, while rising wages make labor-intensive exports less competitive on world markets—precisely at the time when other low-income countries get themselves engaged in a phase of rapid growth. Put differently, growth slowdowns coincide with the point in the growth process where it is no longer possible to boost productivity by shifting additional workers from agriculture to industry and where the gains from importing foreign technology diminish.[5] This process is well supported by the evidence on productivity slowdowns provided by Eichengreen et al. (2011), as indicated earlier. It is also consistent with the results in Perez-Sebastian (2007), where imitation is the main source of productivity growth at early stages of development, whereas broad-based innovation—defined as the application of new ideas, technologies, or processes to productive activities—becomes the main engine of growth as the economy approaches the technology frontier.

The implication then is that to avoid falling into a middle-income trap, countries must address its root structural cause early on and find new ways to boost productivity. The main sources of higher productivity are a shift to high-value services and the promotion of home-grown innovation, rather than continuing to rely on imitation of foreign technology. Put differently, the key issue that needs to be addressed is how to switch from "imitation" activities to a broad-based innovation strategy.

This paper offers an alternative view on the cause of a middle-income trap. Although it fundamentally agrees on productivity slowdowns as being a source of these traps, it differs from the existing literature in terms of the reasons why productivity growth may be constrained, and what type of public policies can be implemented to promote a broad-based innovation strategy and escape a middle-income trap. We emphasize interactions between three determinants of productivity growth: individual

[5] A related consideration is that by specializing initially in low-skilled intensive activities, low-income countries may have diminished incentives to invest in education—hence reducing the rate of accumulation of human capital needed to promote broad-based innovation.

decisions to acquire skills, access to different types of public infrastructure, and *knowledge network externalities*—which we define as the possibility that a higher share of workers with advanced levels of education has a positive impact on their performance, that is, their ability to take advantage of existing knowledge.[6] As far as we know, this paper is the first to bring these interactions to the fore and analyze their implications for the existence of multiple equilibria in a growth framework.

We conduct our analysis in the context of an overlapping generations (OLG) model in which we distinguish between two types of labor skills, basic and advanced. In contrast to most of the literature, we focus on embodied human capital; advanced skills are defined as specialized knowledge that can be acquired by devoting a given amount of time to higher education in early adulthood. Thus, in contrast to models with disembodied knowledge and endogenous schooling time allocation in the Lucas-Uzawa tradition, human capital cannot be accumulated indefinitely. This reduces the dimensionality of the model and allows a full analytical treatment.[7] Individuals with either basic or advanced skills can both work in the production of final goods (or manufacturing), whereas only those with advanced skills can work in the innovation sector (or, more precisely here, design activities). Because labor is relatively more productive in the design sector, an increase in the supply of workers with advanced skills is growth-enhancing. We also assume that occupational choices are endogenous; individuals choose to invest in education only if wages in the design sector are high enough, compared to manufacturing. And unlike existing models, where the marginal productivity gain associated with the stock of ideas is either constant (as in Romer (1990)) or decreasing (as in semi-endogenous growth models in the tradition of Jones (1995)), we focus on the case where, due to the combination of a standard knowledge effect and a learning by doing effect, this gain is increasing—at least over a certain range. We argue that the learning by doing effect may be particularly relevant for developing countries. This is also corroborated by empirical evidence.

In addition, we also consider two types of infrastructure: *basic* infrastructure (which consists of roads, electricity, and basic telecommunications) and *advanced* infrastructure, which consists of advanced information and communication technologies (ICTs) in general, and high-speed communication networks in particular. It is now well established that access to broadband facilitates the buildup of domestic and international knowledge networks, thereby promoting dissemination and research (see Canuto et al. (2010)). Broadband networks also provide a platform that other sectors can leverage to develop other platforms (such as distance education and telemedicine) and enable the development of digital content—all of which can help to promote innovation. In our framework, basic infrastructure helps to promote manufacturing, whereas advanced

[6] Indirect evidence of the importance of (local) knowledge networks is provided by Weinberg (2011), who found that per capita GDP in developing countries is positively related to the number of important scientists born in and staying in a country.

[7] Dinopoulos and Segerstrom (1999) also treat the acquisition of skills endogenously in a model of innovation, trade, and growth. However, they do so in a setting with representative households and are unable to study transitional dynamics.

infrastructure serves to promote design activities. Because labor supply decisions are endogenously related to relative wages, there is a two-way interaction between these activities and the proportion of the population acquiring advanced skills.

The key result that we are able to establish is that if the marginal benefits associated with nonrival (disembodied) knowledge depends in a nonlinear fashion on the share of the population involved in design activities (being high for a range of values for that share), as a result of a knowledge network effect, then multiple equilibria may emerge, one of them (the lower-growth equilibrium) being synonymous with a middle-income trap. Productivity growth is low as well, in line with the evidence discussed earlier. We also show that escaping from that trap may be achieved by a sufficiently large increase in investment in advanced infrastructure. Intuitively, to benefit from existing ideas, there must be enough high-ability individuals involved in the design sector; but if productivity in that sector is low, because access to advanced infrastructure is limited, wages will be low—implying that few high-ability individuals will choose to invest in the advanced skills needed to operate in that sector. Thus, the lower-growth equilibrium is also characterized by a *misallocation of talent.* Improving access to advanced infrastructure boosts productivity and wages in the design sector, which draws more labor there and triggers the shift in labor supply that magnifies (at least temporarily) the benefit associated with exploiting the existing stock of ideas.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 defines the equilibrium whereas Section 4 analyzes its properties. Section 5 introduces knowledge network externalities and considers their implication for multiple equilibria and the existence of middle-income traps. Section 6 considers various extensions of the model, namely, the enforcement of property rights, the link between advanced infrastructure and the acquisition of skills, and the role of labor market rigidities. Section 7 draws together the policy implications of the analysis, and the final section offers some concluding remarks.

2 The Model

The economy that we consider is populated by individuals with different innate abilities, who live for two periods, adulthood and old age. Total population is constant. Each individual is endowed with one unit of time in the first period of life, and zero unit in old age. Wages are the only source of income. Agents have no other endowments, except for an initial stock of physical capital at $t = 0$, which is the endowment of an initial old generation. Savings can be held only in the form of physical capital.

In addition to individuals, the economy is populated by firms and a government. There are three production sectors in the economy: the first produces a final good (manufacturing, for simplicity), the second intermediate inputs (which depreciate fully after use), and the third designs, which consist of blueprints used for the production of intermediate inputs.[8] Firms in the design sector use labor and public capital, whereas

[8] In the broad sense used here, innovation encompasses a wide range of activities in addition to

firms producing the final good use labor, physical capital, and intermediate goods. The final good can be either consumed in the period it is produced, or stored to yield physical capital at the beginning of the following period. Blueprints produced by the design sector are patented and thus *excludable*, but *nonrival* (anybody willing to pay the patent price can acquire the right to use a new design); they are indispensable for producing intermediate inputs. As in Romer (1990), each new design involves the production of a new intermediate good, using a technology similar to the one used to produce the final good.

The government invests in two types of infrastructure: " basic" (roads, electricity, basic telecommunications) and "advanced" (such as advanced information networks). It spends on some other items as well, and finances its expenditure by taxing wages. It cannot borrow and therefore must run a balanced budget in each period. Finally, all markets clear and there are no debts or bequests between generations.

2.1 Individuals

Individuals have identical preferences but they are born with different abilities, or cognitive skills. To simplify matters, suppose that a fraction $\delta \in (0,1)$ of individuals are born with average cognitive skills (L-type for short) and a fraction $1-\delta$ with high cognitive skills (or H-type for short). Ability is instantly observable by all, implying that, in particular, a low-ability individual cannot masquerade as a high-ability individual. Whatever his or her type, an individual can work in the final good sector (identified with superscript Y). However, only individuals with high cognitive skills can work in the design sector (identified with superscript R). For that, he/she must also engage in schooling activity and acquire additional, specialized skills, which in turn requires spending a fraction $\varepsilon \in (0,1)$ of his or her time endowment at the beginning of adulthood.[9] Schooling is publicly-provided and free of charge, and completion rate is unity. There are no barriers to entry in the design sector, so any individual with the appropriate skills can work in that sector if he or she is willing to do so.

Let $c_{t+j}^{h,t}$ denote consumption at period $t+j$ of an individual working in sector h, where $h = Y, R$, born at the beginning of period t, with $j = 0,1$. The individual's discounted utility function is given by

$$U_t^h = \ln c_t^{h,t} + \frac{\ln c_{t+1}^{h,t}}{1+\rho}, \tag{1}$$

where $\rho > 0$ is the discount rate.

R&D, such as organizational changes, training, testing, marketing and design. We therefore choose the term "design sector", rather than "R&D sector" as is often the case in the literature, to reflect this broader view.

[9]For a conceptually similar setup, with uniformly distributed abilities, see for instance Galor and Moav (2000). For simplicity, and in the absence of a theory of (innate) cognitive skills, we keep δ exogenous, although it could be assumed that parents who choose to acquire specialized skills are more likely to produce children with high ability.

Because taxes are levied only on labor, and the price of the final good is normalized to unity, the period-specific budget constraints, which depend on the sector of employment in adulthood, are given by

$$c_t^{Y,t} + s_t^Y = (1-\tau)w_t^Y, \tag{2}$$

$$c_t^{R,t} + s_t^R = (1-\tau)(1-\varepsilon)w_t^R, \tag{3}$$

$$c_{t+1}^{h,t} = (1+r_{t+1})s_t^h, \tag{4}$$

where w_t^h is the wage rate in sector h, $\tau \in (0,1)$ a constant tax rate, s_t^h the savings rate, and $1+r_{t+1}$ the rate of return on holding (physical) assets between periods t and $t+1$. Note that here, an H-type individual may choose to work in manufacturing, rather than the design sector; as a result, there is no one-to-one correspondence between the distribution of abilities and the sector of employment.

2.2 Production of the Final Good

The final good is produced by a continuum of unit mass competitive firms indexed by $i \in (0,1)$. Production requires the use of labor, $N_{i,t}^Y$, private capital, $K_t^{P,i}$, basic public infrastructure, K_t^B, and a Dixit-Stiglitz combination of a continuum of intermediate inputs, $x_{s,t}^i$, where $s \in (0, M_t)$.

The production function of firm i takes the form

$$Y_t^i = [\frac{K_t^B}{(K_t^P)^{\zeta_K}(N_t^Y)^{\zeta_N}}]^{\omega}(K_t^{P,i})^{\alpha}(N_{i,t}^Y)^{\beta}[\int_0^{M_t}(x_{s,t}^i)^{\eta}ds]^{\gamma/\eta}, \tag{5}$$

where $\alpha, \beta, \gamma \in (0,1)$, $\eta \in (0,1)$, $\omega > 0$, $\zeta_K, \zeta_N > 0$, $K_t^P = \int_0^1 K_t^{P,i}$ the aggregate private capital stock, $N_t^Y = \int_0^1 N_{i,t}^Y$ total employment in the final good sector, and $1/(1-\eta) > 1$ is (the absolute value of) the price elasticity of demand for each intermediate good.[10] In addition, the production function distinguishes between the returns to specialization, as measured by γ, and the parameter that determines the elasticity of demand for each input, η. For reasons that will be made clear later, we also impose $\eta > \gamma$. This implies that intermediate goods are gross substitutes ($\partial Y_t^i/\partial x_{j,t}^i \partial x_{k,t}^i < 0$) in the production of final goods.

We assume constant returns in all private inputs, so that

Assumption 1: $\alpha + \beta + \gamma = 1$.

Specification (5) also implies that basic public capital, although nonexcludable, is partially rival and subject to congestion, as measured by the aggregate private capital

[10] The assumption on η implies that the various varieties of intermediate goods are imperfect substitutes—which in turn implies that newly-introduced intermediate goods do not make any existing good obsolete. As discussed later, it also effectively constrains the ability of the monopolist suppliers to set prices above marginal costs.

stock and the total number of workers in manufacturing.[11] The strength of congestion effects is measured by the parameters ζ_K and ζ_N.

With the price of the final good normalized to unity, and assuming for simplicity full depreciation of private capital, profits of manufacturing firm i, $\Pi_t^{Y,i}$, are given by

$$\Pi_t^{Y,i} = Y_t^i - \int_0^{M_t} p_t^s x_{s,t}^i ds - w_t^Y N_{i,t}^Y - r_t K_t^{P,i},$$

where p_t^s is the price of intermediate good s, w_t the wage rate in manufacturing, and r_t the net rental rate of private capital.

Each producer maximizes profits subject to (5) with respect to private inputs, labor, capital, and quantities of all intermediate goods $x_{s,t}^i$, $\forall s$, taking factor prices and M_t as given. This yields

$$r_t = \alpha \frac{Y_t^i}{K_t^{P,i}}, \quad w_t^Y = \beta \frac{Y_t^i}{N_{i,t}^Y}, \tag{6}$$

$$x_{s,t}^i = (\frac{\gamma Z_t^i}{p_t^s})^{1/(1-\eta)}, \quad s = 1, ...M_t, \tag{7}$$

where

$$Z_t^i = Y_t^i / \int_0^{M_t} (x_{s,t}^i)^\eta ds. \tag{8}$$

Each firm demands the same amount of each intermediate input. Equation (7) therefore implies that the aggregate demand for intermediate input s is

$$x_{s,t} = \int_0^1 x_{s,t}^i = \int_0^1 (\frac{\gamma Z_t^i}{p_t^s})^{1/(1-\eta)}. \tag{9}$$

Because all firms producing the final good are identical and their number is normalized to unity, $K_t^P = K_t^{P,i}$, $Z_t = Z_t^i$, $\forall i$, and the total demand for intermediate inputs is the same across firms, $x_t^i = x_t$, $\forall i$. Moreover, in a symmetric equilibrium, $x_{s,t}^i = x_t^i$, $\forall s$. Thus, $\int_0^1 [\int_0^{M_t} (x_{s,t}^i)^\eta ds]^{1/\eta} di = M_t^{1/\eta} x_t$. Using these results, equation (5) and the constant-returns-to-scale assumption imply that aggregate output of the final good is

$$Y_t = \int_0^1 Y_t^i = (N_t^Y)^{\beta - \omega\zeta_N} (\frac{K_t^B}{K_t^P})^\omega [(\frac{M_t}{K_t^P})^{\gamma/\eta} x_t^\gamma](K_t^P)^{\alpha+\gamma/\eta+\omega(1-\zeta_K)}, \tag{10}$$

To ensure steady-state growth and eliminate the (weak) scale effect associated with population, the following restrictions on the congestion parameters ζ_K and ζ_N are imposed:

Assumptions 2: $\alpha + \gamma/\eta + \omega(1 - \zeta_K) = 1$, $\beta - \omega\zeta_N = 0$.

[11] Rather than the number of workers in the manufacturing sector, the total labor force could be used instead. This would complicate the analysis without adding much insight; the key reason, as discussed further later, is that the design sector is more labor intensive than the manufacturing sector. In addition, the proportion of research workers in the workforce is typically very small in practice.

By implication,

$$Y_t = (k_t^B)^\omega m_t^{\gamma/\eta} x_t^\gamma K_t^P, \tag{11}$$

where $k_t^B = K_t^B/K_t^P$ is the ratio of basic public capital to private capital, and $m_t = M_t/K_t^P$ is what we will refer to from now on as the (disembodied) knowledge-private capital ratio, or simply the knowledge-capital ratio.

2.3 Production of Intermediate Goods

Firms in the intermediate sector are monopolistically competitive. There is only one producer of each input s, and each of them must pay a fee (a patent) to use the blueprint for that input created by design producers. Production of each unit of an intermediate good s requires a single unit of the final good; the marginal cost of producing s is thus one.

Once the fee involved in purchasing a patent has been paid, each intermediate good producer sets its price to maximize profits, $\Pi_{s,t}^I$, given the perceived demand function for its good (which determines marginal revenue), $x_{s,t}$:

$$\Pi_{s,t}^I = (p_t^s - 1)x_{s,t}. \tag{12}$$

Substituting (9) in this expression and imposing $Z_t^i = Z_t$, $\forall i$, yields

$$\Pi_{s,t}^I = (p_t^s - 1)(\frac{\gamma Z_t}{p_t^s})^{1/(1-\eta)}.$$

Maximizing this expression with respect to p_t^s, taking Z_t as given, yields the optimal price as

$$p_t^s = p_t = \eta^{-1}, \quad \forall s \tag{13}$$

which implies, using (9), that the optimal quantity of each intermediate input demanded by producers of the final good is

$$x_{s,t} = x_t = (\gamma\eta Z_t)^{1/(1-\eta)}. \quad \forall s \tag{14}$$

From the definition of Z_t^i in (8), in a symmetric equilibrium $Z_t = Y_t/M_t x_t^\eta$. Substituting this expression in (14) yields

$$x_t = \gamma\eta(\frac{Y_t}{M_t}). \tag{15}$$

Substituting (13) and (15) in (12) yields the maximum profit for an intermediate-input producer:

$$\Pi_t^I = (1-\eta)\gamma(\frac{Y_t}{M_t}), \tag{16}$$

which is constant in equilibrium if Y_t/M_t is constant.

A potential producer of an intermediate input decides to enter the market by comparing the discounted stream of profits generated by producing that input, and the price that must be paid for the patent or new design, p_t^R. If the market for new designs is competitive, standard arbitrage implies that the price of a patent must be equal to the present discounted stream of profits that the potential producer could make by producing the intermediate input s. For simplicity, we assume that intermediate-input producing firms last only one period, and that patents are auctioned off randomly to a new group of firms in each period. Thus, each producer of a new intermediate good holds a patent only for the period during which it is bought, implying monopoly profits during that period only; yet patents last forever.[12] The arbitrage condition requires therefore that, in a symmetric equilibrium,

$$p_t^R = \Pi_t^I. \tag{17}$$

2.4 Design Sector

Firms engaged in the design sector generate blueprints for new intermediate inputs, using the same technology. There is no aggregate uncertainty in design activities. In line with Romer (1990), we assume that the growth rate of the production of new designs depends on the number of H-type individuals with higher education in the population, adjusted for the time spent acquiring advanced skills, $(1-\varepsilon)N_t^R$, and a productivity factor, A_t. In addition, as in Dinopoulos and Thompson (2000), we also account for a *dilution effect*, which captures the fact that creating new designs becomes more difficult as the size of the market, measured by the size of the population, N_t, grows—possibly as a result of organizational costs related to product distribution. Thus, the aggregate stock of designs evolves according to

$$M_{t+1} - M_t = A_t \frac{(1-\varepsilon)N_t^R}{N_t}. \tag{18}$$

Productivity itself depends on three factors: access to basic public capital (which again is subject to congestion), access to advanced public capital, and the existing stock of designs:

$$A_t = [\frac{K_t^B}{(K_t^P)^{\zeta_M}}]^{\kappa_1}(K_t^A)^{\kappa_2} M_t^{1+\phi}, \tag{19}$$

where $\kappa_1, \zeta_M > 0$ and $\phi, \kappa_2 \geq 0$.

[12] If firms are assumed to last forever, and patents are of infinite duration, then we would need to discount profits by using the gross rental rate of capital, which is endogenous here—in contrast to simple, Romer-type representative-agent models. Alternatively, if firms last forever and patents last for one period only (implying that intermediate good producers make zero profits thereafter), we would need to distinguish between the price charged by monopolists for new designs produced at t, that is, $M_t - M_{t-1}$, which is given by (13), and the price charged by competitive firms for goods produced before t, that is, in the interval $(0, M_{t-1})$. That price would be equal to marginal cost, which is unity here (the price of the final good). However, either assumption would complicate the solution of the model, without shedding much additional light on the issue at stake.

The usual reason for introducing a positive impact of the stock of designs (or ideas) on productivity is the existence of a *standing-on-shoulders* effect, that is, the assumption that each design creates a positive externality for future design activities or the development of new ideas.[13] In Romer (1990), the marginal effect of M_t on A_t is taken to be linear, implying therefore that $\phi = 0$. Here, by contrast, we consider the case where the marginal effect is increasing, so that $\phi > 0$. Our view is that the effect of the stock of designs on productivity captures two effects: a standing-on-shoulders effect, as in the previous contributions, but also a *learning-by-doing* effect, which results from the fact that the production of designs helps to improve efficiency. This mechanism may be particularly important for developing countries, as suggested in some studies. Addison (2003) for instance, in a cross-country study of 29 countries, found that the largest source of productivity growth in developing countries is product variety imitation, which represents a key source of (nonrival) knowledge. Amiti and Konings (2007) also found a link between the import variety of intermediate inputs and productivity gains at the firm level for Indonesia. As it turns out, the assumption that $\phi > 0$ (if only temporarily, as discussed later) turns out to be critical for generating multiple equilibria in our framework.[14]

Note also that although in general both types of capital are essential for productivity, their marginal benefit depends on the value of parameters κ_1 and κ_2. When $\kappa_2 = 0$ access to advanced capital has no external effect on the productivity of workers in the design sector. Consequently, rather than distance to the world technological frontier and the quality of labor (skilled or unskilled) used in producing designs, as in Vandenbussche et al. (2006) for instance, the distinction between imitation and (broad-based) innovation can be captured by considering alternative assumptions about κ_1 and whether or not advanced public capital is an essential input in productivity. A *pure imitation regime*, in particular, to the extent that it involves (mostly) copying of imported blueprints, corresponds to $\kappa_1 > 0$ and $\kappa_2 = 0$. By contrast, an *innovation regime* can be captured by assuming that κ_1 is small and $\kappa_2 > 0$. This interpretation is quite useful and allows us to consider only a single design sector when thinking about the switch from imitation to innovation. We will consider later the case where the marginal effect of advanced public capital, as measured by κ_2, is subject to a threshold externality, related to the stock of advanced infrastructure itself.[15]

To ensure that the *growth rate* of designs—or, equivalently, the ratio A_t/M_t—is

[13] See Jones (2005) and Gancia and Zilibotti (2005). UNIDO (2009, Chapter 2) presents evidence that over a wide range of income the diversification of manufacturing raises productivity. See also World Bank (2008).

[14] In Addison (2003), an interactive term between distance from variety frontier (which can be viewed as a proxy for the domestic stock of designs) and educational attainment has a large effect on TFP growth in developing countries. Although this is not a strict test of the nonlinearity that we discuss later, it is consistent with it.

[15] Note also that in (19), for simplicity, congestion is measured solely in terms of the private capital stock. If we were to account also for the number of workers in manufacturing as a congestion factor (as in (5) for instance), or total population, the dilution effect in (18) could be assumed to be decreasing in N_t. However, a linearity assumption would still be required to eliminate the scale effect.

constant in the steady state, the following assumption on the degree of congestion is imposed:

Assumption 3: $\zeta_M = 1 + (\kappa_2 + \phi)/\kappa_1$.

Firms in the design sector choose labor so as to maximize profits, Π_t^R, subject to (18), and taking the wage rate, the patent price, and productivity as given:

$$\max_{N_t^R} \Pi_t^R = p_t^R(M_{t+1} - M_t) - w_t^R(1-\varepsilon)N_t^R. \tag{20}$$

The first-order condition is given by

$$w_t^R \geq \frac{p_t^R A_t}{N_t}, \tag{21}$$

with equality if $N_t^R > 0$. In that case, and given the linearity of the production technology with respect to labor, (21) is also the zero-profit condition implied by free entry.

2.5 Government

As noted earlier, the government taxes only adult wages. It spends a total of G_t^B and G_t^A on basic and advanced infrastructure investment, and G_t^U on other items. All its services are provided free of charge. It cannot issue bonds and must therefore run a balanced budget:

$$G_t = \sum G_t^h = \tau[w_t^Y N_t^Y + w_t^R(1-\varepsilon)N_t^R]. \quad h = A, B, U \tag{22}$$

Shares of public spending are all assumed to be constant fractions of government revenues:

$$G_t^h = \upsilon_h \tau[w_t^Y N_t^Y + w_t^R(1-\varepsilon)N_t^R], \quad h = A, B, U \tag{23}$$

Combining (22) and (23) therefore yields

$$\sum_h \upsilon_h = 1. \tag{24}$$

Assuming full depreciation, public capital in infrastructure evolves according to

$$K_{t+1}^j = G_t^j, \quad j = A, B \tag{25}$$

2.6 Market-Clearing Condition and Labor Supply

The asset market clearing condition requires private capital stock in $t+1$ to be equal to savings in period t by individuals born in $t-1$:

$$K_{t+1}^P = s_t^Y N_t^Y + s_t^R N_t^R. \tag{26}$$

As noted earlier, all H-type individuals who choose not to invest in specialized skills work in the final good sector. Let θ_t denote the number of high-ability individuals who decide to acquire more education; the numbers of workers in the design and final good sectors are given by, respectively,[16]

$$N_t^R = (1-\delta)\theta_t N_t, \tag{27}$$

$$N_t^Y = [\delta + (1-\delta)(1-\theta_t)]N_t. \tag{28}$$

As noted earlier, total population is assumed constant. But because θ_t is endogenous, the distribution of the workforce across sectors will change over time. The "maximum" size of the labor force in the design sector is $1-\delta$, which obtains when $\theta_t = 1$. All H-type individuals are then employed in design activities.

3 Equilibrium

In this economy an *equilibrium with imperfect competition* is a sequence of consumption and saving allocations $\{c_t^{h,t}, c_{t+1}^{h,t}, s_t^h\}_{t=0}^{\infty}$, for $h = Y, R$, private capital stock $\{K_t^P\}_{t=0}^{\infty}$, public capital stocks $\{K_t^A, K_t^B\}_{t=0}^{\infty}$, prices of production inputs $\{w_t^Y, w_t^R, r_{t+1}\}_{t=0}^{\infty}$, prices and quantities of intermediate inputs $\{p_t^s, x_{s,t}\}_{t=0}^{\infty}$, $\forall s \in (0, M_t)$, existing varieties (or disembodied knowledge), $\{M_t\}_{t=0}^{\infty}$, such that, given initial stocks $K_0^P > 0$, $K_0^A, K_0^B > 0$, and $M_0 > 0$,

a) individuals working in both manufacturing and the design sector maximize utility by choosing consumption subject to their intertemporal budget constraint, taking factor prices and the tax rate as given;

b) firms in the final good sector maximize profits by choosing labor, private capital, and intermediate inputs, taking input prices as given;

c) intermediate input producers set prices so as to maximize profits, while internalizing the effect of their decisions on the perceived aggregate demand curve for their product;

d) producers of blueprints in the design sector maximize profits by choosing labor, taking wages, patent prices, productivity, and population, as given;

e) the equilibrium (patent) price of each blueprint extracts all profits made by the corresponding intermediate input producer;

f) the government budget is balanced; and

g) all markets clear.

A *balanced growth equilibrium* is an equilibrium with imperfect competition in which

a) $\{c_t^{h,t}, c_{t+1}^{h,t}, s_t^h\}_{t=0}^{\infty}$, for $h = Y, R$, and K_t^P, K_t^A, K_t^B, Y_t, M_t, w_t^Y, w_t^R, grow at the constant, endogenous rate $1+g$, implying that the knowledge-private capital ratio, as well as the public-private capital ratios, are also constant;

b) the net rate of return on private capital r_{t+1} is constant;

[16] It could be assumed that when used in manufacturing one unit of H-type labor is worth, say, a multiple $\mu > 1$ units of L-type labor. As long as μ is fixed, this would alter our results.

c) the price of intermediate goods p_t and the patent price p_t^R are constant; and

d) the fraction of high-ability individuals who choose to acquire higher education, θ_t, and thus working in the design sector, is constant. By implication, the fraction of the labor force employed in the final good sector is also constant.

Consider now the properties of a balanced growth equilibrium. The first property is that, as shown in the Appendix, the public-private capital ratios are given by

$$k_{t+1}^h = J_h = \frac{\upsilon_j \tau}{\sigma(1-\tau)}, \quad h = A, B \tag{29}$$

which implies that they are both constant over time.

The second property relates to the condition under which an H-type individual will choose to acquire specialized skills. In general, he (or she) will do so if discounted utility is at least equal to the alternative of working in the manufacturing sector. As shown in the Appendix, the wage ratio must satisfy the condition

$$\frac{w_t^R}{w_t^Y} \geq \frac{1}{1-\varepsilon}, \tag{30}$$

which indicates that the wage premium in the design sector compensates for the opportunity cost of studying (the manufacturing wage). Equivalently, for any value of w_t^R that is less than $(1-\varepsilon)^{-1}w_t^Y$, no H-type individual will choose to acquire an education ($N_t^R = 0$) and the model generates a corner solution with zero growth.

From the supply side, and using (29), the wage ratio is derived in the Appendix as

$$\frac{w_t^R}{w_t^Y} = (1-\eta)\gamma \frac{J_B^{\kappa_1} J_A^{\kappa_2} m_t^{\phi}}{\beta}[\delta + (1-\delta)(1-\theta_t)], \tag{31}$$

which shows, in particular, a negative partial relationship (as one would expect) between the relative wage in manufacturing and the proportion of the labor force employed there.

Equating (30), holding with equality, and (31) yields therefore a solution for the proportion of H-type individuals employed in the design sector:

$$\theta_t = 1 - \frac{1}{1-\delta}\left\{[(1-\varepsilon)(1-\eta)\gamma \frac{J_B^{\kappa_1} J_A^{\kappa_2} m_t^{\phi}}{\beta}]^{-1} - \delta\right\}, \tag{32}$$

which depicts a positive relationship between m_t and θ_t: a higher knowledge-capital ratio makes education more desirable because it raises productivity and wages in the design sector.

Inspection of (31), and (32) shows that the relative wage in the design sector, as well as the number of H-type individuals employed in that sector, are always lower when advanced capital is not an essential input ($\kappa_2 = 0$). This is of course due to its positive effect on productivity. Note also that if m_t is too low, namely, below $m^S = \{\beta/[(1-\varepsilon)(1-\eta)\gamma J_B^{\kappa_1} J_A^{\kappa_2}]\}^{1/\phi}$, then the inequality (30) is reversed, θ_t (which cannot be negative) is equal to zero, and a corner solution with no activity in the design sector obtains; the economy stagnates. We will therefore assume in what follows that initially $m_0 > m^S$ to ensure that $\theta_0 > 0$.[17]

Now, let m^C be the value of m_t for which $\theta_t = 1$, that is, all H-type individuals born at t choose to engage in higher education and work in the design sector. From (32) this value is

$$m^C = [\frac{\beta\delta^{-1}}{(1-\varepsilon)(1-\eta)\gamma J_B^{\kappa_1} J_A^{\kappa_2}}]^{1/\phi}, \tag{33}$$

so that $m^C > m^S$, given that $\delta > 0$.

This result implies that the threshold value of the knowledge-private capital ratio above which all available H-type individuals choose to invest in skills is decreasing in the advanced public capital-private capital ratio if advanced capital is an essential input in the design sector ($\kappa_2 > 0$). Intuitively, a higher J^A raises wages in the design sector, which thereby induces (all else equal) more H-type individuals to choose to acquire higher skills.

If all H-type agents become skilled for $m_t > m^C$, θ_t is a nonmonotonic function:

$$\theta_t = \begin{cases} f(m_t) & \text{if } m_t < m^C \\ 1 & \text{if } m_t \geq m^C \end{cases}, \tag{34}$$

where $f()$ is defined in (32).

The Appendix also shows that the dynamics of the knowledge-capital ratio are determined by

$$m_{t+1} = \Phi(m_t, \theta_t), \tag{35}$$

where

$$\Phi() = \frac{(1+\chi_2 J_B^{\kappa_1} J_A^{\kappa_2} m_t^{\phi}\theta_t) J_B^{-\omega/(1-\gamma)}}{\Lambda_1 \sigma(1-\tau)(\beta + \chi_1 J_B^{\kappa_1} J_A^{\kappa_2} m_t^{\phi}\theta_t)} m_t^{(1-\gamma\eta^{-1})/(1-\gamma)},$$

with $\chi_1 = (1-\varepsilon)(1-\delta)(1-\eta)\gamma$, $\chi_2 = (1-\varepsilon)(1-\delta) > \chi_1$, and $\Lambda_1 = (\gamma\eta^2)^{\eta\gamma/(1-\eta\gamma)}$. The steady-state growth rate is given by

$$1 + g = \Lambda_2 J_B^{\omega/(1-\gamma)} \tilde{m}^{\gamma(\eta^{-1}-1)/(1-\gamma)}(\beta + \chi_1 J_B^{\kappa_1} J_A^{\kappa_2} \tilde{m}^{\phi}\tilde{\theta}), \tag{36}$$

where $\Lambda_2 = \Lambda_1\sigma(1-\tau)$, and $\tilde{\theta}$ and $\tilde{m}$ are the steady-state solutions obtained by solving (32) and (35) simultaneously. This equation shows that the growth rate is increasing in the two public-private capital ratios and depends both directly and indirectly on the proportion of H-type individuals in the economy and the knowledge-capital ratio.[18]

[17] The threshold value m^S is decreasing in both public-private capital ratios, implying, in particular, that a low-income economy without a sufficiently developed infrastructure to begin with, is condemned to stagnation. In particular, initiating an imitation regime ($\kappa_1 > 0$, $\kappa_2 = 0$) is easier if the stock of basic infrastructure J_B is high. This creates the case for a Big Push in public investment. This argument is quite distinct from those commonly proposed in the literature.

[18] If the population variable used to account for congestion in (5) had been measured in terms

4 Properties of Equilibrium

The fundamental dynamic equation of the model is (35), which given (34) can be rewritten as

$$m_{t+1} = \Phi(m_t, \theta_t) = \begin{cases} \Phi^1(m_t, \theta_t) & \text{if } m_t < m^C \\ \Phi^2(m_t) & \text{if } m_t \geq m^C \end{cases}, \tag{37}$$

where $\Phi^1()$ corresponds to the right-hand side of (35) and $\Phi^2()$ is given by

$$\Phi^2(m_t) = \frac{\Lambda_1^{-1}(1 + \chi_2 J_B^{\kappa_1} J_A^{\kappa_2} m_t^{\phi}) J_B^{-\omega/(1-\gamma)}}{\sigma(1-\tau)(\beta + \chi_1 J_B^{\kappa_1} J_A^{\kappa_2} m_t^{\phi})} m_t^{(1-\gamma\eta^{-1})/(1-\gamma)}. \tag{38}$$

The transition function $\Phi(\cdot)$ is highly nonlinear and an explicit steady-state value solution $\tilde{m}$ cannot be derived, regardless of whether $\tilde{m} \lesseqgtr m^C$. However, the steady-state equilibrium can be characterized directly in terms of $\tilde{\theta}$ and $\tilde{m}$, given that these variables are interdependent, as implied by (32) and (35). In Figures 1 and 2, curve FF describes (32), which holds at all times, whereas curve MM corresponds to the steady-state solution of the implicit equation (37). The intersection of FF and MM (if any) determines the equilibrium values of the share of the H-type population that chooses to engage in higher education and the knowledge-private capital ratio.

Curve FF is upward sloping and concave and reaches a maximum for $m_t \geq m^C$ at which point it becomes horizontal. Curve MM is upward-sloping, but it can be either convex, concave, or concave-convex, depending on the value of the underlying structural parameters. Intuitively, although a higher m raises the proportion of high-ability individuals in the design sector (which therefore also tends to increase the stock of designs in the long run) it also increases savings and the private capital stock; although the former effect dominates (for plausible parameter values) the rate at which the knowledge-capital ratio increases may be either positive or negative. The critical parameters to distinguish between the two cases are the values of γ (the elasticity of output of final goods with respect to intermediate inputs, which has a direct positive effect on the patent price, and thus wages in the design sector), η (which determines the elasticity of demand for each intermediate good, $1/(1-\eta)$, and negatively affects the patent price), and ϕ (which measures the knowledge externality in the design sector).[19]

In Figure 1, MM is shown as a concave curve. This shape is obtained when the difference between γ and η is relatively significant (to draw the curve we used $\eta = 0.25$,

of the *total* number of workers, instead of only the number of workers in manufacturing, then a direct additional term in θ, to the power β, would have appeared in (36), with a negative effect on growth. This would simply reflect the fact that, as the proportion of H-type workers in the design sector increases, the proportion of workers in the manufacturing sector naturally decreases—thereby mitigating the growth in output. However, as can be inferred from these equations, this term would be dominated by the direct positive effect of θ, for any admissible values of η and γ. This essentially reflects the assumption that the design sector is more labor intensive than the manufacturing sector.

[19] In drawing curves FF and MM we used the following values for the other variables: $\delta = 0.1$, $\varepsilon = 0.2$, $\sigma = 0.5$ (corresponding to an annual discount rate of 0.04), $\tau = 0.3$, and $\beta = 0.6$. These values are well in the range of those used in the literature.

implying an elasticity of demand for intermediate goods equal to 1.3, $\gamma = 0.15$, and $\phi = 0.1$). A unique equilibrium can be obtained either at point E or at point E' (located beyond m^C) but there may also be no equilibrium, as shown by curve MM'' in the figure; numerically, this occurs for higher values of ϕ. Experiments with other values of η, γ, and ϕ in a plausible range indicated that multiple equilibria (although possible) do not occur, as long as the difference between η and γ is relatively large.

In Figure 2, MM is shown as a concave-convex curve, drawn for $\eta = 0.25$, $\gamma = 0.24$, and $\phi = 0.5$. Thus, this shape is obtained when the difference between γ and η is relatively small and ϕ is relatively large.[20] Multiple equilibria are now more likely, depending again on the values of the parameters. As before, curve MM may or may not intersect curve FF, and if it does, it can be either before or after $\tilde{m} = m^C$. In the first case, MM is quite steep and there is no equilibrium (see MM'''). In the third, there is only one equilibrium, located at point B on MM''. In the second case, curves MM and FF can intersect twice, either at points E and E', or at points A and A', with the latter located beyond point m^C. The "low" equilibrium is characterized by a low fraction of H-type individuals who choose to acquire an improved education and a low knowledge-private capital ratio, and vice-versa for the "high" equilibrium. From equation (36), the low (high) equilibrium is thus also characterized by low (high) growth.

The next question is to determine whether the equilibrium (unique or not) is locally stable. In general, as shown in the Appendix, it cannot be established a priori whether $|dm_{t+1}/dm_t| < 1$ in the vicinity of the steady state, to ensure local stability.[21] First, it can be noted that in the "Romer case" where $\phi = 0$, equation (32) implies that θ_t is constant $\forall t$; and from (35), stability requires only that $|(1 - \gamma\eta^{-1})/(1 - \gamma)| < 1$, or equivalently $1 - \gamma\eta^{-1} < 1 - \gamma$. This condition boils down to $\eta < 1$, which is always satisfied, given the assumption of (gross) complementarity between intermediate inputs. If so, *multiple equilibria cannot arise.* Indeed, for $\phi = 0$, FF in Figure 1 is a flat curve and a unique equilibrium, shown at points B or B', always exists as long as the expression on the right-hand side of (32) is such that $\theta \in (0, 1)$. A similar result would hold in Figure 2.

By continuity, it can be assumed that for ϕ positive but small, the economy remains stable. Indeed, numerical experiments in the range of parameter values mentioned earlier indicate that the adjustment process remains stable for values of ϕ that are positive but close enough to 0.

[20]When η and γ are close to each other, the last term in m_t in (38) has an exponent that is close to zero, so the dynamics become driven by the term m_t^{ϕ}, which appears both in the numerator and the denominator of that equation.

[21]As noted in the Appendix, neither the sign nor the magnitude of dm_{t+1}/dm_t can be established analytically. Consequently, a variety of outcomes are possible (see Azariadis (1993): *a*) $dm_{t+1}/dm_t = 1$, a condition for the economy to exhibit a continuum of unstable steady-state equilibria; *b*) $dm_{t+1}/dm_t < 0$, a necessary (but not sufficient) condition for periodic equilibria (including two-period cycles; and *c*) dm_{t+1}/dm_t may change sign, a necessary (although not sufficient) condition for complex dynamics (cycles of higher order, and even aperiodic equilibria). Given the issue at hand, however, we abstract from a detailed discussion of these cases.

The adjustment process of the knowledge-capital ratio, as characterized in equation (37) is displayed in a standard 45-degree diagram in Figure 3 with $\phi = 0$ and $\phi > 0$ but close to zero. The figure also assumes that $1 - \gamma\eta^{-1} > 0$ or equivalently $\eta > \gamma$, to eliminate oscillatory convergence. If the steady-state equilibrium point is located *before* the threshold level m^C, it will be reached in a smooth fashion (as shown at point E^1), regardless of whether ϕ is zero or close to zero. By contrast, if the steady-state equilibrium is located *after* m^C (as for instance at point E^2), and if ϕ is small but positive, the slope of the transition curve will generally change at $m_t = m^C$, as occurs at point B.[22] For higher values of ϕ, as discussed earlier, the transition curve may turn convex, implying either no equilibrium or two equilibria.[23] Moreover, it can easily be established that if there are two equilibria, only the lower one is stable.

Using Figures 1 and 3, a variety of comparative-statics exercises can be performed to illustrate the properties of the model; these include changes in the composition of government spending and the time that must be allocated to higher education, as well as changes in structural parameters. By adding a cost of acquiring education, subsidy policies could also be studied. However, given the purpose at hand, we leave this to the interested reader and instead turn our attention to the possibility that threshold externalities can generate *multiple* stable equilibria—one of which possibly representing a middle-income growth trap.

5 Middle-Income Trap and Public Policy

Suppose that when the share of H-type workers employed in the design sector is low ($\theta_t < \theta_L$), the productivity effect of knowledge is limited and $\phi = \phi_L$. We assume that ϕ_L is either 0, as in Romer (1990), or close enough to 0, to ensure that the adjustment process of m_t is stable, as discussed earlier. Once a critical mass θ_L is achieved, that is, for $\theta_t > \theta_L$, the marginal gains increase significantly, and $\phi = \phi_H > \phi_L$. Beyond another point, $\theta_t > \theta_H$, the network effect is exhausted, and ϕ falls back to a lower value (say, back to ϕ_L, to fix ideas). There is therefore an intermediate stage where the critical mass of H-type workers employed in the design sector is large enough to generate strong marginal benefits associated with the existing stock of disembodied knowledge; there is a *knowledge network externality*, as defined earlier.[24]

Given that from (32) the knowledge-private capital ratio is positively and monotonically related to θ_t (until the maximum value of unity is reached), the critical values

[22] As shown in the Appendix, $dm_{t+1}/d\theta_t$ may be either positive or negative. Thus, when m_t reaches m_C, from which point θ_t remains constant at unity, the transition curve may either become steeper or flatter. In the case shown in Figure 3, the latter occurs.

[23] The reason why (nontrivial) multiple equilibria can be displayed in a figure similar to Figure 3 when the transition curve is convex is because, as noted earlier, the initial value m_0 must be higher than the minimum value m^S needed to ensure that $\theta_0 > 0$.

[24] Changes in the value of ϕ could also be related to the *fishing out effect* discussed by Jones (1995, 2005).

θ_L and θ_H are associated with two threshold values of m_t, given by m_L and m_H.[25] Put differently, the strength of the (marginal) effect of the stock of designs on productivity depends on how much talent is already engaged in design activities, or equivalently here the initial (relative) stock of disembodied knowledge itself.

In the presence of this externality, it is easy to see that there may be multiple, locally stable steady-state equilibria. The reason is that, if ϕ_H is high enough, the transition curve will generally be convex in the interval (m_L, m_H). If so, there are three possible outcomes: no equilibrium, one equilibrium, or three equilibria. Figure 4 displays the case where there are indeed three steady states: one with a low knowledge-private capital ratio, $\tilde{m}_1$, corresponding to Point A^1; one with a high knowledge-private capital ratio $\tilde{m}_3$, corresponding to point A^3; and an intermediate equilibrium, $\tilde{m}_2$, corresponding to point A^2. Points A^1 and A^3 are stable, whereas A^2 is unstable.[26] To the low (high) knowledge-private capital ratio corresponds a low (high) share of H-type workers in the design sector, and a low (high) growth rate. We may therefore define the equilibrium at A^1 as a *middle-income growth trap*. Initial conditions determine the economy's long-run steady-state equilibrium: economies with a relatively low initial knowledge-private capital ratio—or, equivalently here, a low initial share of high-ability workers engaged in the design sector—may converge to a lower-growth trap. For instance, if the initial knowledge-private capital ratio is such that $m^S \leq m_0 < \tilde{m}_2$, the economy will converge to $\tilde{m}_1$, whereas if $m_0 > \tilde{m}_2$, it will converge to $\tilde{m}_3$. Put differently, the level of the knowledge-private capital ratio that corresponds to the unstable equilibrium A^2 acts as a threshold level that a country must cross if it wants to evolve from "moderate" growth to high growth.[27] In the first scenario, the knowledge-capital ratio falls during the transition to the trap, and so does productivity. At equilibrium A^1, given that (as noted earlier) the ratio A_t/M_t is constant, the growth rate of productivity is lower as well, consistent with the "productivity slowdown" documented by Eichengreen et al. (2011).

Note also that at equilibrium point A^1, $\tilde{\theta}_1 < 1$; a proportion $1 - \tilde{\theta}_1$ of H-type workers has chosen not to engage in higher education and instead prefer to work in the manufacturing sector; there is, therefore, a *misallocation of talent*, due fundamen-

[25] Note that although θ_L and θ_H are exogenous, m_L and m_H are *endogenous*, as they depend also on the advanced public capital-private capital ratio, and thus υ_A; this matters when considering spending shifts, as discussed later.

[26] This occurs if $\Phi(m_L) < m_L$, $\Phi(m_H) > m_H$, $\Phi(m_t) < m_t$, $\forall m_t \in (\tilde{m}_1, \tilde{m}_2)$, and $\Phi(m_t) > m_t$, $\forall m_t \in (\tilde{m}_2, \tilde{m}_3)$. Note that in both Figures 3 and 4, it is assumed that the high-growth equilibrium is located to the left of the threshold point m^C, implying that at that point the share of H-type workers who have chosen to acquire higher education is less than $1 - \delta$. This is simply to avoid cluttering the graph; m^C and $\tilde{m}_3$ could be made arbitrarily close to each other.

[27] Note that the high-growth steady state remains inefficient for standard reasons associated with the decentralized equilibrium of Romer-type models—monopoly power (which allows intermediate goods firms to charge a price in excess of marginal cost) and the fact that firms in the design sector do not internalize the intertemporal spillover associated with the creation of ideas. In addition, because entry of new firms in the design sector does not dissipate rents, growth may be suboptimally low. See Gancia and Zilibotti (2005) for a discussion.

tally to the fact that the wage in the design sector (which would need to be adjusted downward in the presence of a non-zero cost to acquiring higher education) is too low relative to the going wage in manufacturing.[28]

The role of public policy in the presence of this type of threshold externalities is illustrated in Figure 5. Suppose that the economy is initially at a knowledge-private capital ratio m_0, located to the *left* of the unstable equilibrium, $\tilde{m}_1$, and consider a budget-neutral increase in the share of government spending on investment in advanced infrastructure, υ_A, financed by a cut in unproductive spending ($d\upsilon_A + d\upsilon_U = 0$). As can be inferred from the foregoing discussion, such a policy has two opposite effects on the knowledge-private capital ratio. On the one hand, it tends to increase that ratio, because greater access to advanced infrastructure raises the productivity of workers in the design sector, which in turn raises wages and the supply of labor in that sector. On the other, higher wages translate into higher saving per H-type worker, which tends to increase investment and the private capital stock. But the shift in the workforce toward the design sector also means less workers, and thus less savings, from those employed in the manufacturing sector. Even though the reduction of employment in the manufacturing sector puts upward pressure on wages there, wages in the design sector increase by more, for inequality (30) to continue to hold and for the transfer of labor across sectors to actually occur. Thus, the positive effect on the degree of product diversification dominates, and the knowledge-capital ratio increases following a rise in the share of spending on advanced infrastructure. Graphically, the transition curve $\Phi(m_t)$ shifts upward, whereas the threshold values m_L and m_H shift to the left.[29] If this shift is large enough (that is, if the increase in υ_A is of a sufficient magnitude) it may be such that the initial knowledge-capital ratio m_0 may now be positioned to the *right* of the new unstable equilibrium (located at point B^2), thereby ensuring convergence to the new higher-growth equilibrium, located at B^3.[30]

In the foregoing discussion it was assumed that $\kappa_2 > 0$; of course, if advanced infrastructure is not an essential input in productivity ($\kappa_2 = 0$) the policy shift considered earlier does not work. More generally, suppose that the parameter κ_2 is zero if advanced public capital is below a certain threshold k_C^A, and positive above that value, that is $\kappa_2 = 0$ if $k_t^A < k_C^A$, and $\kappa_2 > 0$ if $k_t^A \geq k_C^A$. This specification helps to capture in a simple manner a "critical mass" or "network" effect associated this time with the

[28] At the higher-growth equilibrium A^3 there may of course be also some misallocation of talent; the point, however, is that it is less significant, and less costly to the economy, than what occurs at the lower-growth equilibrium.

[29] As noted earlier, equation (32) implies that the threshold levels m_L and m_H depend negatively on υ_A. To avoid cluttering Figure 5, the shift in m_L and m_H is not shown. Note that if the shift in m_L is large enough, the lower-growth equilibrium point B^1 may actually disappear, leaving only the higher-growth equilibrium.

[30] Of course, the upward shift in the transition curve could be so large that it eliminates entirely the possibility of multiple equilibria, leaving only a single, stable equilibrium. This is the same outcome that would obtain if the leftward shift in m_L is large enough, as noted in the previous footnote. Note also that from (33), the increase in υ_A lowers the threshold m^C above which all H-type individuals choose to engage in higher education. Again, this shift is not shown to avoid cluttering the figure.

stock of infrastructure itself (see Agénor (2010) and Agénor and Neanidis (2010)). It is also consistent with the evidence in Czernich et al. (2011) for instance, who found that broadband matters for growth only above a threshold penetration rate (measured as the number of broadband subscribers per 100 inhabitants) of 10 percent. What this implies essentially is that the increase in υ_A must now be large enough to satisfy *two* constraints in order to help an economy escape from a middle-income growth trap: not only does it need to be large enough to induce enough H-type individuals to engage in higher education (as before), it must also be large enough to ensure that sufficient capacity in advanced infrastructure is created.

Finally, could this outcome have been achieved by increasing the share of spending on *basic* infrastructure? No, if κ_1 is too small. A budget-neutral increase in υ_B, financed by a cut in spending on unproductive items ($d\upsilon_B + d\upsilon_U = 0$), would raise the marginal product of labor in the manufacturing sector and increase wages there—thereby reducing incentives to acquire advanced skills. As a result, the share of H-type workers engaged in the design sector would fall, reducing the knowledge-capital ratio and thus productivity and wages there. The implication is that this policy shift may lead to a *downward* movement in the transition curve, making it in fact more, rather than less, likely that the economy would converge to the lower-growth trap.

6 Extensions

The foregoing analysis can be extended in several directions. In line with recent debates about the role of public policy in helping middle-income countries avoid, or escape from, an imitation trap, we consider three extensions of our basic framework: the degree of enforcement (or lack thereof) of property rights, a direct impact of advanced infrastructure on time allocated to education, and the role of labor market rigidities.

6.1 Enforcement of Property Rights

In the foregoing analysis, it was assumed that there is a single producer for each intermediate good. Producers operate under monopolistic competition and each of them purchases the exclusive rights to a blueprint, making their technology fully protected. However, in practice, the enforcement of patents may be lacking, as is often the case in developing countries.

A poorly functioning system to administrate patents and enforce property rights may create a deadweight loss for the economy, in the sense that it may translate into a lower ability of firms in the design sector to appropriate the rent created by their activity—that is, the profits secured by intermediate good producers.[31] In the present setting, this can be captured by assuming that even though the price of the patent paid by each intermediate good producer is p_t^R, inefficiencies in enforcing property rights

[31] See Lorenczik and Nowiak (2011) and Newiak (2011) for a more detailed discussion of the implications of protection of property rights on innovation.

imply that design producers receive only a fraction $\chi^R < 1$ of that price. Formally, this boils down to replacing in (20) p_t^R by $\chi^R p_t^R$. From condition (21), this means that the wage in the design sector is lower, and thus that incentives to acquire advanced education are reduced. Graphically, this means that the transition curve in Figure 4 shifts down, making it more likely that the economy would converge to the lower-growth trap. Conversely, improved enforcement of property rights translates into higher wages in the design sector, which would draw more high-ability workers into that sector—making it more likely that the knowledge network externality would kick in and set the economy on a path to higher steady-state growth.

6.2 Infrastructure and Time Allocation

In the foregoing analysis, it was assumed that the amount of time that must be devoted to the acquisition of advanced skills, ε, is given. Suppose instead that such time consists of two components: a compulsory "classroom" time ε_m, which is again exogenous, and "homework" time, which is endogenous and inversely related to access to advanced infrastructure. The idea here is that access to high-speed internet, in particular, makes it easier to access information needed to complete course assignments, and therefore reduces the total amount of time needed to complete higher education.[32] Thus, we now have,

$$\varepsilon = \varepsilon_m + e(J_B),$$

with $e' < 0$. Because, under our assumptions, the public-private capital ratio is constant for both types of infrastructure, this does not qualitatively affect the nature of the initial equilibrium (or equilibria). However, a policy shift toward advanced infrastructure affects now the economy through an additional channel, because it makes studying time more efficient and therefore allows H-type individuals to devote more time to work in the design sector—which then, through higher wages, serves to attract even more workers in that sector. At the same time, a reduction in ε means that the required premium that H-type individuals demand to invest in higher skills falls—thereby inducing more of them to do so. Thus, in the presence of a time allocation effect, investment in advanced infrastructure is now more likely to succeed in putting the economy on a convergent path to the higher-growth equilibrium.

6.3 Labor Market Rigidities

It is well recognized that labor market rigidities may discourage hiring. Does that affect the likelihood of being caught in a middle-income trap? It depends. Suppose first that the impact of labor market distortions (firing costs, etc.) on labor costs in the manufacturing and design sectors can be captured by a proportional effect on the gross

[32] A related idea is to assume the cost of acquiring an education (which we assumed to be zero) is inversely related to access to advanced infrastructure. The results would be qualitatively similar to those described here, as long as this cost is proportional to the wage in the design sector.

wage in each sector, so that the "effective" wage is now $(1 + \Lambda^h)w_t^h$, where $\Lambda^h > 0$ and $h = Y, R$. It is immediately clear that if the distortion affects both sectors in the same way, then the foregoing analysis remains essentially the same, because what matters for the dynamics (the allocation of H-type labor across sectors) is the *relative* wage.

However, it could be argued that some types of labor market restrictions, especially those on firing costs, may be particularly detrimental to design or innovation activities. The reason is that in such activities, it is often more difficult to observe the productivity of a worker before hiring—in contrast to "routine" tasks in manufacturing, where observability, both *ex ante* and *ex post*, is less costly. Thus, the risk of hiring a worker who turns out to be a "lemon" (a poor performer) is higher in activities where a college degree does not necessarily provide a reliable signal about future performance. If so then, the proportional cost may be such that $\Lambda^R > \Lambda^Y$. In such conditions, and given that the equilibrium wage is inversely proportional to the gross cost (that is, for the design sector, $w_t^R = p_t^R A_t/(1+\Lambda^R)N_t$) the labor market distortion acts as a disincentive to seek higher education—with adverse consequences for innovation and growth. By exacerbating the misallocation of talent, labor market distortions may make it more likely that the economy will end up in a lower-growth trap.

7 Policy Implications

The foregoing analysis has important implications for the ongoing debate on how countries can avoid falling into a middle-income (or imitation) growth trap, or more generally for the transition from imitation to true innovation. This debate has recently focused on China's economic prospects for the next 20 years (see World Bank (2012)) but it is of equal relevance for many other countries in Asia and Latin America, such as Brazil and Malaysia.

The first point relates to changes in the composition of the labor force during the development process. The common view on this issue, as discussed in the introduction, is that imitating available techniques is an easier task than "true" innovation. Thus, in the early stages of development, when the main task is to copy and adapt available technologies, a relatively low level of skills, or specialization in basic technical skills, helps to promote growth.[33] At later stages, however, true innovation requires more advanced skills, in a wider range of areas. The shift from low-technology to more advanced manufacturing activities then becomes the main vehicle for productivity change in an economy. Technological learning spurs productivity growth and increases real wages, which in turn causes firms to exit low-technology, labor-intensive activities and en-

[33] Imitations activities may take the form of task-based production, that is, specialization in some stage of a value chain rather than in final products. However, task-based production may reinforce a country's specialization in low-technology, less sophisticated activities—which can be viewed as another form of the "imitation trap." UNIDO (2009) compared the sophistication of product- and task-based manufacturing and finds no evidence that task-based production is less technologically sophisticated than production of final products.

ter more capital-intensive, technologically sophisticated sectors. Because these sectors have stronger learning effects, and possibly more spillovers to the rest of the economy in terms of skills development and knowledge, they strengthen growth further.

Our analysis, while consistent with the view, helps to shed new light on it. In our approach, the fundamental distinction between "imitation" and "innovation" is the productivity of labor in the design sector: in an imitation trap, productivity and wages are relatively low in that sector, thereby mitigating incentives to invest in higher education. In turn, the lack of highly educated workers constrains production in design activities, and prevents the exploitation of externalities associated with knowledge networks. Thus, in our analysis there is a *two-way causality* between education and innovation. Countries may remain caught in a low or moderate growth equilibrium because they are unable to get enough high potential workers into innovation activities; and because wages are low as a result, a fewer number of individuals with high potential are willing to make the investment needed to acquire the skills needed to seek employment in the innovation sector. The composition of the labor force depends therefore on the interaction between supply and demand factors, and a middle-income growth trap can also be characterized by a misallocation of talent.

Second, the idea that market failures of some type, such as the partial excludability of knowledge, lead to under-investment in research has long been the principal rationale for government funding of research and development (see OECD (2010)). However, the presence of bottlenecks or other failures that impede innovation activities can constitute equally crucial (if not more important) obstacles to these activities. Our analysis argues that the lack of advanced infrastructure, which is particularly productive in the design sector (in part because it promotes knowledge networks), plays a critical role in helping a country escape from a lower-growth trap, not only because of its direct effect on productivity but also because of its effect on the supply of high-skilled labor. In turn, a growing skill base facilitates a shift in production from labor-intensive to skill-intensive activities and an increase in the pace of innovation. Somewhat paradoxically, a reallocation of (limited) government resources from direct subsidies to research and innovation activities toward the provision of advanced infrastructure may actually be more effective at promoting these activities and magnifying their impact on economic growth.

Third, our analysis suggests that, in addition to advanced infrastructure, improving the enforcement of property rights (in particular the administration of patents), and removing (some types of) labor market rigidities may help to accelerate the pace of innovation and be quite effective at helping a country avoid a middle-income growth trap. Labor market rigidities may be especially detrimental to innovation activities (compared to sectors where more routine tasks are performed), because of the difficulty of inferring *ex ante* workers' potential to perform in the future. By making firing more difficult, these rigidities may deter hiring in the first place and constrain innovation. To the extent that they translate into relatively higher labor costs in knowledge-intensive sectors, they may also lead to lower wages there and reduce incentives to invest in advanced education.

8 Concluding Remarks

This paper developed a two-period, three-sector OLG model with labor heterogeneity and analyzed the channels through which access to public capital may help a country escape a middle-income growth trap. In the model, high- and low-ability individuals can both work in the production of final goods (or manufacturing), but only those with high ability and advanced skills can operate in the innovation sector (or, more precisely here, design activities). The sectoral distribution of the labor force, and the share of high-ability individuals who choose to engage in education, are endogenously determined as a function of relative wages. In addition, we also considered two types of infrastructure: basic infrastructure and advanced infrastructure (which affects positively productivity only in design activities). We also account for a two-way interaction between (disembodied) knowledge and the proportion of the high-ability population acquiring advanced skills. We showed that if the marginal benefits associated with disembodied knowledge depend in a nonlinear fashion on the share of the population involved in design activities (being particularly high during a range of values for that share), as a result of a knowledge network externality, then multiple equilibria may emerge, one of them corresponding to a middle-income growth trap. The trap is characterized both by low productivity growth (consistent with the evidence) and a misallocation of talent, in the sense of a relatively low share of high-ability workers in design activities. We also show that escaping from that trap may be achieved by a sufficiently large increase in investment in advanced infrastructure. We then discussed various extensions of the analysis (namely, the role of enforcement of property rights and labor market distortions) and drew together its policy implications.

Our analysis can be further extended in a number of directions. First, by assuming a constant population, we essentially abstracted from demographic issues. Yet, one reason to doubt that high growth in a relatively poor developing country will continue forever (in addition to those discussed in the introduction) relates to changes in the fertility rate with the level of development. In the early stages of economic growth, high fertility rates provide a dividend; they translate into a rising share of the population in the labor force, causing per capita output to grow more rapidly than otherwise. However, in the late stages of development, the fertility rate tends to fall and (with the mortality rate having fallen earlier) the age dependency ratio tends to increase. Although the increase in savings (associated with life-cycle effects) may promote investment in physical capital, the fall in the labor force puts upward pressure on wages and hampers economic growth. Accounting for this demographic transition would help the model to explain some of the evidence regarding fertility and growth, especially with regard to East Asia's experience since the 1960s.

Second, in the model there is no explicit distinction between "imitation" and "true innovation" activities; there is a single design sector, and the key difference between a lower-growth (imitation) trap and a higher-growth equilibrium is the productivity of labor in design activities. While this is sufficient to illustrate our main point—the role of the composition of public capital in escaping from a middle-income trap—

the model does not account for the coexistence, in a long-run equilibrium, of both types of activities, possibly with imitation requiring only workers with basic skills and basic capital, and true innovation requiring both advanced skills and advanced public capital.[34] While the resulting model may well prove intractable analytically, its properties can be studied numerically. An interesting question in that context is under what conditions would both types of activities coexist, and only one type would prevail. We suspect that some of the channels identified in the present paper will matter (especially the composition of public capital), but this remains to be examined more carefully.

Notwithstanding these extensions, it is our belief that as it stands this paper sheds some useful new light on the ongoing debate on middle-income growth traps and on policies to avoid them. Put in a broader perspective, our view is that for a low-income country the provision of basic infrastructure is essential to initiate the transfer of labor from agriculture to manufacturing, and facilitate the adoption, adaptation, and diffusion of pre-existing technologies imported from more advanced countries. There is also a learning-by-doing effect in the process, which promotes human capital accumulation and productivity in general. Indeed, knowledge may grow in part because of imitation of foreign products, even though this may occur with decreasing returns. However, beyond this initial stage, to sustain productivity growth and avoid a growth slowdown, it is critical to invest in the type of infrastructure that will generate strong marginal benefits in terms of knowledge diffusion and knowledge networks.

[34] Studies that focus on the transition from imitation to innovation include Walz (1996), Currie et al. (1999), Perez-Sebastian (2007), Mondal and Gupta (2009), and Iacopetta (2010). However, none of them discussed the role of public capital in that context.

Appendix

From (2) and (4), the household's consolidated budget constraint is, for L-ability individuals and for H-ability individuals who decide not to seek higher education and choose to work in final-good sector,

$$c_t^{Y,t} + \frac{c_{t+1}^{Y,t}}{1+r_{t+1}} = (1-\tau)w_t^Y, \tag{A1}$$

whereas from (3) and (4), for those H-type workers who decide to invest in specialized skills and are hired in the design sector,

$$c_t^{R,t} + \frac{c_{t+1}^{R,t}}{1+r_{t+1}} = (1-\tau)(1-\varepsilon)w_t^R. \tag{A2}$$

Maximizing (1) subject to either (A1) or (A2) yields the standard Euler equation

$$\frac{c_{t+1}^{h,t}}{c_t^{h,t}} = (\frac{1+r_{t+1}}{1+\rho}). \quad h = Y, R \tag{A3}$$

Substituting this result in (A1) and (A2) yields

$$c_t^{Y,t} = (\frac{1+\rho}{2+\rho})(1-\tau)w_t^Y, \quad c_t^{R,t} = (\frac{1+\rho}{2+\rho})(1-\varepsilon)(1-\tau)w_t^R, \tag{A4}$$

so that

$$s_t^Y = \sigma(1-\tau)w_t^Y, \quad s_t^R = \sigma(1-\tau)(1-\varepsilon)w_t^R, \tag{A5}$$

where $\sigma = 1/(2+\rho)$ is the marginal propensity to save, which is the same for both types of workers.

A H-type individual will choose to engage in higher education if the discounted present value of utility from doing so is at least equal to the alternative ($U_t^R \geq U_t^Y$). Substituting (A3) and (A4) in (1) yields, after simplification,

$$\ln[(1-\varepsilon)w_t^R] + \frac{\ln[(1-\varepsilon)w_t^R]}{1+\rho} - \ln w_t^Y - \frac{\ln w_t^Y}{1+\rho} \geq 0,$$

which yields

$$(\frac{2+\rho}{1+\rho})\ln[\frac{(1-\varepsilon)w_t^R}{w_t^Y}] \geq 0,$$

or equivalently

$$\frac{w_t^R}{w_t^Y} \geq \frac{1}{1-\varepsilon}. \tag{A6}$$

To study the dynamics, note first that substituting (15) in (11) yields

$$Y_t = (k_t^B)^\omega m_t^{\gamma/\eta}[\gamma\eta(\frac{Y_t}{K_t^P})m_t^{-1}]^\gamma K_t^P,$$

which can be rearranged to give

$$\frac{Y_t}{K_t^P} = \Lambda_1 (k_t^B)^{\omega/(1-\gamma)} m_t^{\gamma(\eta^{-1}-1)/(1-\gamma)}, \tag{A7}$$

where $\Lambda_1 = (\gamma\eta)^{\gamma/(1-\gamma)}$.

Under Assumption 3, equation (19) becomes

$$A_t = (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi} M_t. \tag{A8}$$

To determine the equilibrium wage ratio, note that from equations (6), (21), holding with equality, and (A8),

$$w_t^Y = \beta Y_t / N_t^Y, \tag{A9}$$

$$w_t^R = \frac{p_t^R A_t}{N_t} = \frac{p_t^R (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi} M_t}{N_t}. \tag{A10}$$

Thus

$$\frac{w_t^R}{w_t^Y} = \frac{p_t^R (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi} M_t}{\beta Y_t} (\frac{N_t^Y}{N_t}). \tag{A11}$$

From (28), $N_t^Y / N_t = \delta + (1-\delta)(1-\theta_t)$, and from (16) and (17),

$$p_t^R = (1-\eta)\gamma(\frac{Y_t}{M_t}). \tag{A12}$$

Substituting these two results in (A11) yields

$$\frac{w_t^R}{w_t^Y} = (1-\eta)\gamma \frac{(k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi}}{\beta} [\delta + (1-\delta)(1-\theta_t)]. \tag{A13}$$

Equating (A6), holding with equality, and (A13) yields

$$(1-\eta)\gamma \frac{(k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi}}{\beta} [\delta + (1-\delta)(1-\theta_t)] - \frac{1}{1-\varepsilon} = 0,$$

which implies that

$$\theta_t = 1 - \frac{1}{1-\delta} \left\{ [(1-\varepsilon)(1-\eta)\gamma \frac{(k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi}}{\beta}]^{-1} - \delta \right\}. \tag{A14}$$

Now, consider the dynamics of the private capital stock. From (26) and (A5),

$$K_{t+1}^P = \sigma(1-\tau)[w_t^Y N_t^Y + w_t^R (1-\varepsilon) N_t^R]. \tag{A15}$$

Using (A9) and (A10) yields

$$K_{t+1}^P = \sigma(1-\tau)[\beta Y_t + p_t^R (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^{\phi} M_t (1-\varepsilon)(\frac{N_t^R}{N_t})].$$

Using (A12) to eliminate p_t^R and noting from (27) that $\theta_t = N_t^R/(1-\delta)N_t$ yields

$$K_{t+1}^P = \sigma(1-\tau)[\beta + \chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t]Y_t, \tag{A16}$$

where $\chi_1 = (1-\varepsilon)(1-\delta)(1-\eta)\gamma$.

Equation (18) can be rewritten as, using (27) again and (A8),

$$\frac{M_{t+1}}{M_t} = 1 + \chi_2 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t,$$

where $\chi_2 = (1-\varepsilon)(1-\delta) > \chi_1$.

Dividing this expression by (A16) yields, noting that $M_t/Y_t = (Y_t/K_t^P)^{-1}m_t$,

$$m_{t+1} = \frac{1 + \chi_2 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t}{\sigma(1-\tau)[\beta + \chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t]}(\frac{Y_t}{K_t^P})^{-1}m_t,$$

that is, using (A7),

$$m_{t+1} = \frac{\Lambda_1^{-1}[1 + \chi_2 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t](k_t^B)^{-\omega/(1-\gamma)}}{\sigma(1-\tau)[\beta + \chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t]} m_t^{(1-\gamma\eta^{-1})/(1-\gamma)}, \tag{A17}$$

which corresponds to equation (35) in the text.

Consider first the case where $\phi = 0$, that is, the Romer case. The condition for stability is then $|(1-\gamma\eta^{-1})/(1-\gamma)| < 1$, or equivalently $1-\gamma\eta^{-1} < 1-\gamma$. This condition boils down to $\eta < 1$, which is always satisfied.

Consider now the case where $\phi > 0$. From (A17),

$$\frac{dm_{t+1}}{dm_t} = (\frac{1-\gamma\eta^{-1}}{1-\gamma})[\frac{\Phi(m_t)}{m_t}]$$

$$+\frac{\Lambda_1^{-1} m_t^{(1-\gamma\eta^{-1})/(1-\gamma)}(k_t^B)^{-\omega/(1-\gamma)}}{\sigma(1-\tau)\{[\beta + \chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t]\}^2}\times$$

$$\Big\{[\beta + \chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t]\chi_2 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2}[\phi m_t^{\phi-1}\theta_t + m_t^{\phi}(\frac{d\theta_t}{dm_t})]$$

$$-[1 + \chi_2 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t]\chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2}[\phi m_t^{\phi-1}\theta_t + m_t^{\phi}(\frac{d\theta_t}{dm_t})]\Big\},$$

where, from (A14) $d\theta_t/dm_t > 0$.

The sign of the first term in this expression is positive if $\gamma < \eta$, as assumed in the text. The sign of the term in curly brackets is determined by the sign of

$$\chi_2\{\beta + \chi_1 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t\} - \chi_1\{1 + \chi_2 (k_t^B)^{\kappa_1}(k_t^A)^{\kappa_2} m_t^{\phi}\theta_t\},$$

which simplifies to $\chi_2\beta - \chi_1$. As noted earlier, $\chi_2 > \chi_1$, but because $\beta < 1$, the sign of this expression is in general ambiguous. Thus, the sign of dm_{t+1}/dm_t, and whether $|dm_{t+1}/dm_t| \gtrless 1$, cannot be determined a priori when $\phi > 0$.

Note also that

$$\frac{dm_{t+1}}{d\theta_t} = \frac{\Lambda_1^{-1} m_t^{(1-\gamma\eta^{-1})/(1-\gamma)} (k_t^B)^{-\omega/(1-\gamma)}}{\sigma(1-\tau)\{[\beta + \chi_1 (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^\phi \theta_t]\}^2} \times$$

$$\Big\{[\beta + \chi_1 (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^\phi \theta_t] \chi_2 (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^\phi$$

$$-[1 + \chi_2 (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^\phi \theta_t] \chi_1 (k_t^B)^{\kappa_1} (k_t^A)^{\kappa_2} m_t^\phi \Big\},$$

whose sign therefore depends also on $\chi_2\beta - \chi_1$. Thus, when the transition function reached m_C it may become either steeper or flatter.

To determine the public-private capital ratios, note that (23) and (25),

$$K_{t+1}^h = \upsilon_h \tau [w_t^Y N_t^Y + w_t^R (1-\varepsilon) N_t^R], \quad h = A, B$$

Combining this expression with (A15) yields

$$k_{t+1}^h = \frac{\upsilon_h \tau}{\sigma(1-\tau)} = J_h. \quad h = A, B \tag{A18}$$

To determine the growth rate, note that from (A7), (A16), and (A18),

$$\frac{Y_{t+1}}{Y_t} = \frac{\Lambda_2 J_B^{\omega/(1-\gamma)}}{m_{t+1}^{-\gamma(\eta^{-1}-1)/(1-\gamma)}} (\beta + \chi_1 J_B^{\kappa_1} J_A^{\kappa_2} m_t^\phi \theta_t), \tag{A19}$$

where $\Lambda_2 = \Lambda_1 \sigma(1-\tau)$. Equation (A19) yields the steady-state solution (36) in the text.

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Figure 1
Steady-State Equilibrium in m-θ Space:
Concave MM

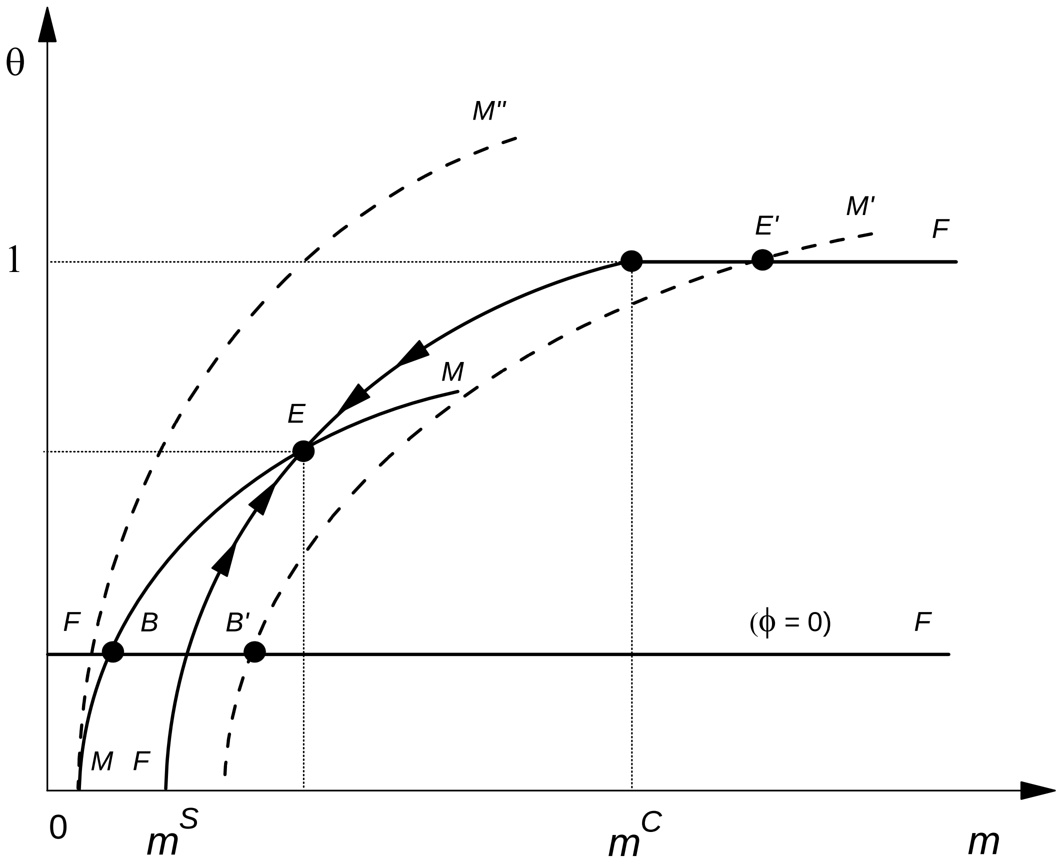

Figure 2

Steady-State Equilibrium in m-θ Space:
Concave-Convex *MM*

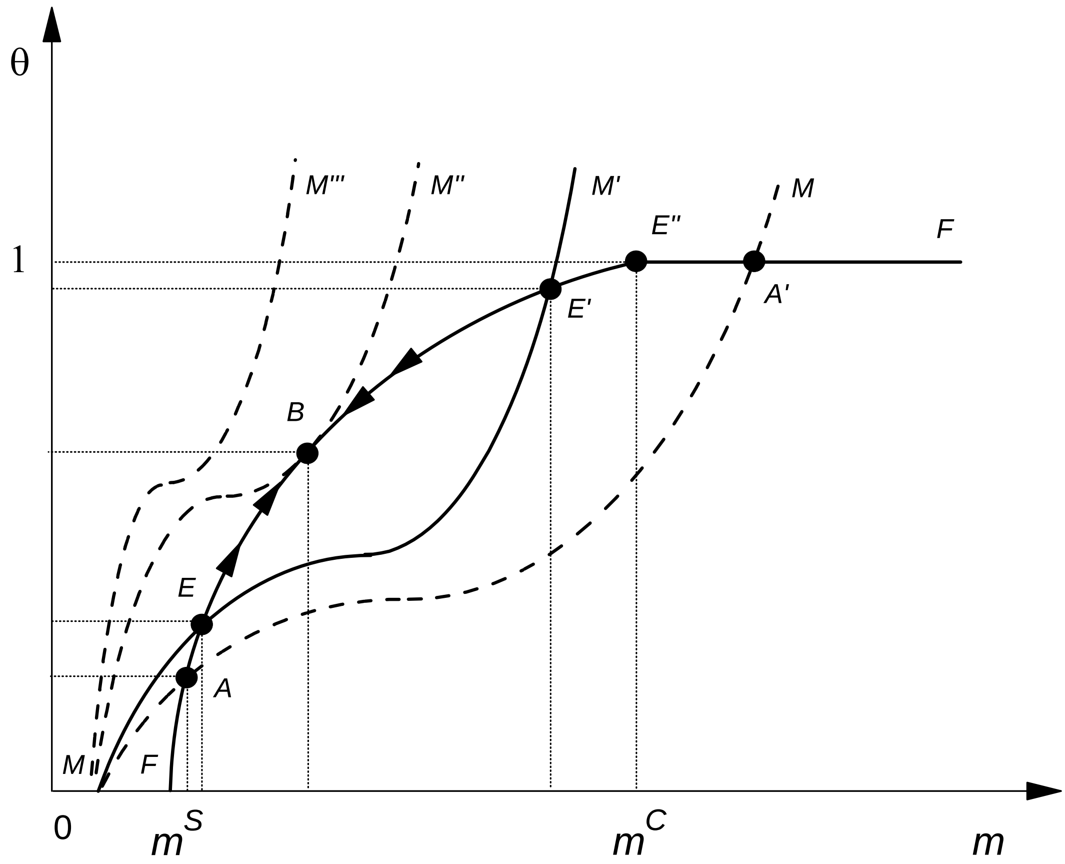

Figure 3
Dynamics of the Knowledge-Capital Ratio
with $\phi = 0$ and $\eta > \gamma$

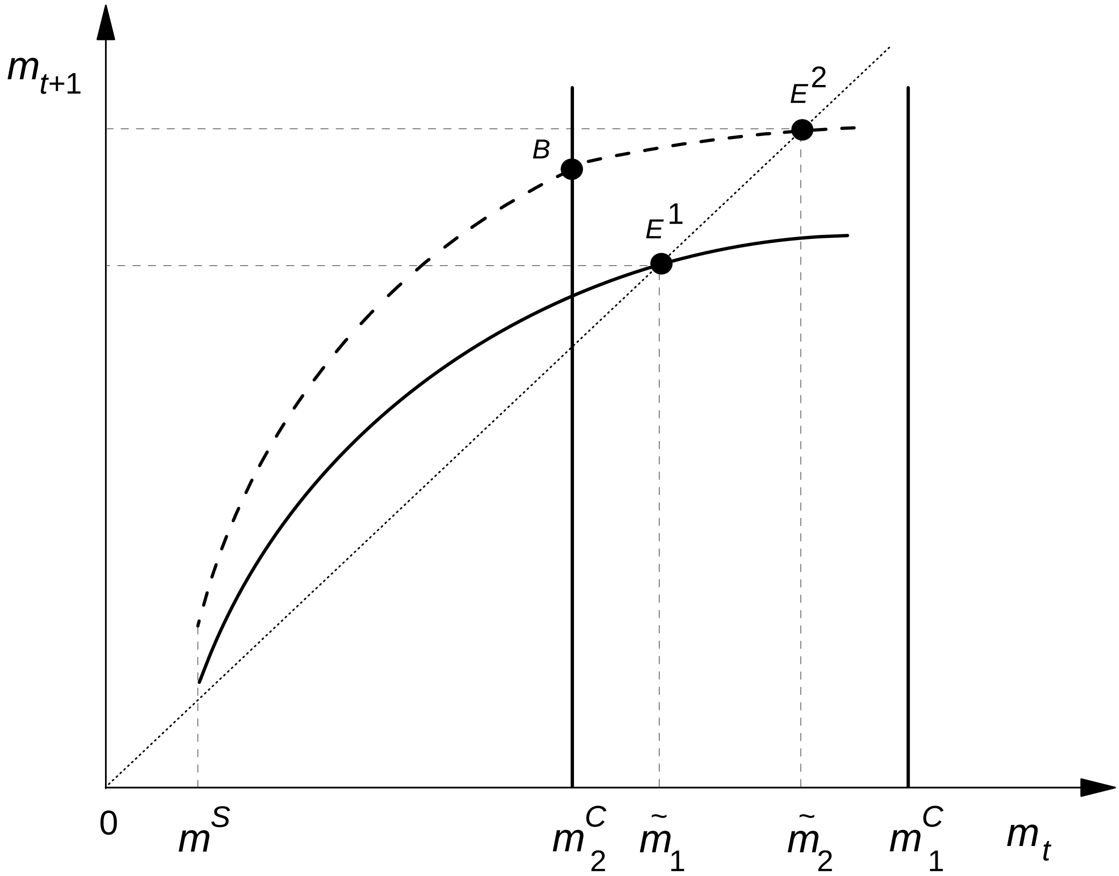

Figure 4
Knowledge Network Externality and Multiple Equilibria

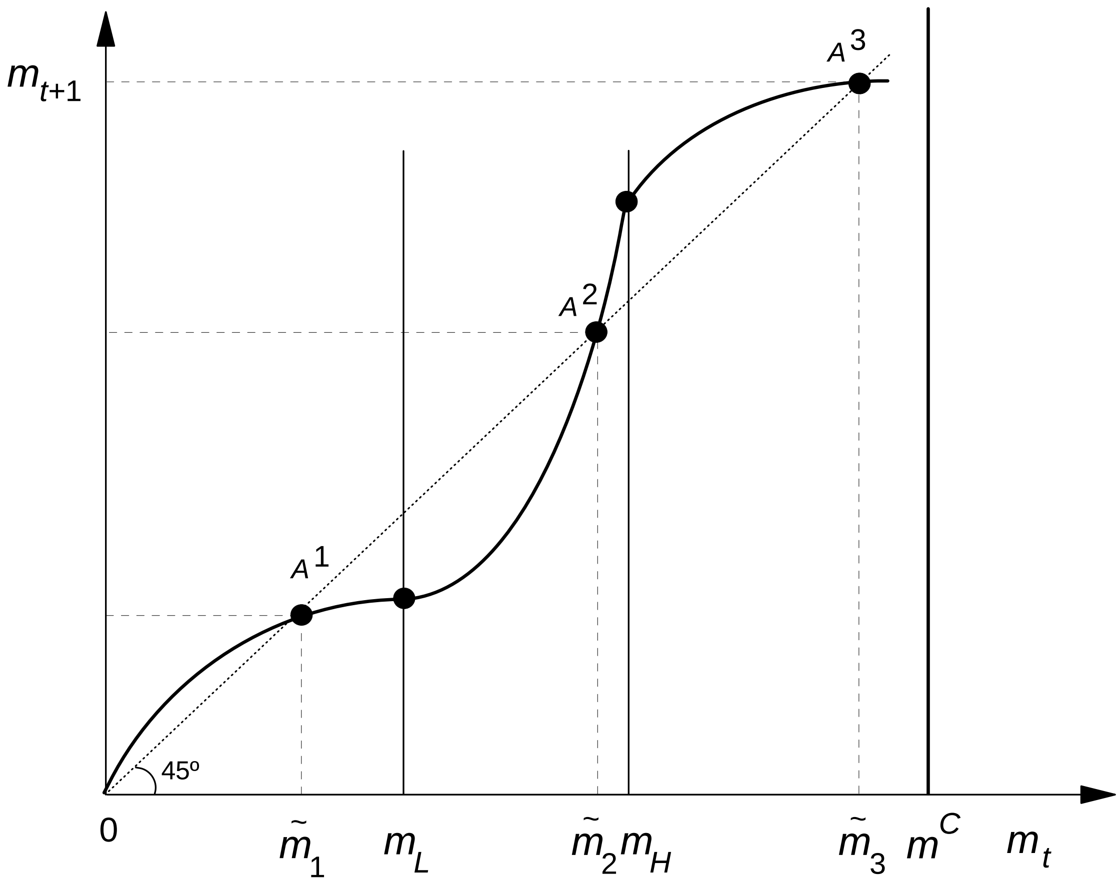

Figure 5
Escaping from a Middle-Income Growth Trap