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The Moroccan New Keynesian Phillips Curve

A Structural Econometric Analysis

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Abstract

The Phillips curve is central to discussions of inflation dynamics and monetary policy. In particular, the New Keynesian Phillips Curve is a valuable tool to describe how past inflation, expected future inflation, and real marginal cost or an output gap drive the current inflation rate. However, economists have had difficulty applying the New Keynesian Phillips Curve to real-world data due to empirical limitations. This paper overcomes these limitations by using an identification-robust estimation method called the Tikhonov Jackknife instrumental variables estimator. Data from Morocco are used to examine the ability of the New Keynesian Phillips Curve to explain Moroccan inflation dynamics. The analysis finds that by adding more information to the hybrid version of the New Keynesian Phillips Curve model by increasing the number of moment conditions, the inflation dynamics in Morocco can be well-described by the New Keynesian Phillips Curve. This framework suggests that the New Keynesian Phillips Curve would be a strong candidate for short-run inflation forecasting.

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The Moroccan New Keynesian Phillips Curve: A Structural Econometric Analysis[†]

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1 Introduction

An important issue in monetary policy models is the short-run dynamics of inflation, especially for how central banks should react to developments in the economy while maintaining inflation targets. In this respect, important advances have been made during the last two decades in the theoretical modeling of inflation dynamics. Much of the modern analysis of inflation is based on the New Keynesian Phillips Curve; see Galı and Gertler (1999) (GG henceforth). This model of inflation dynamics is grounded in an optimizing framework where imperfectly competitive firms are constrained by costly price adjustments.

The New Keynesian Phillips curve (NKPC) is a forward-looking model of inflation dynamics, where short-run inflation dynamics are driven by the expected discounted stream of real marginal costs. However, given the statistical failure of the basic NKPC formulation when confronted with data, the curve has since evolved into its more empirically viable hybrid form. In particular, adding lagged inflation to the model corrects the signs of estimated coefficients (see Fuhrer and Moore (1995), Fuhrer (1997) and Roberts (1997)).

GG estimate the hybrid version of this model, in which real marginal cost is the labor income share and the parameters are functions of three key structural parameters: the fraction of backward-looking price-setters, the average duration that an individual price is fixed, and a discount factor. Using data for the U.S, they find that the real marginal costs are statistically significant and the hybrid specification of the NKPC outperforms the purely forward-looking version of the NKPC (without a lag of inflation in the dynamics). Indeed, the popularity of the NKPC models comes in large part from the fact that the model is supported by empirical data.

But even as the popularity and usage of the NKPC has grown, its empirical identifiability has been criticized. Many authors argue that the above results are unreliable because they are derived using methods such as the generalized method of moments (GMM), which are not robust to identification problems, also known as the weak instrument problem; see Stock et al. (2002), Dufour (1997), Dufour (2003), Kleibergen and Mavroeidis (2009) and Dufour et al. (2006). In fact, the identification of coefficients for endogenous variables in linear structural equations is achieved using instrumental variables that are assumed to be correlated with the right-hand-side endogenous variables (strong) and uncorrelated with the structural error (valid). However, when instruments are weak, i.e., when the correlations between an instrument and the endogenous variables is low, conventional estimation procedures can be misleading. The problem of weak instruments or weak identification has received considerable attention in both the theoretical and applied econometric literature. Empirical examples include include Angrist and Krueger (1991), who measure returns to schooling, and Eichenbaum, Hansen, and Singleton (1988), who consider consumption asset pricing models.

Morocco has generally experienced low and stable inflation over the last two decades. This period coincided with domestic macroeconomic stability and lower global inflation. However, these inflation dynamics are expected to change, following the fact that Moroccan authorities have started in 2018 the reform of its exchange rate regime in the respective of adopting inflation targeting. In this paper, we estimate the NKPC curve for the Moroccan case, in light of recent econometric findings related to instruments. Our aim is to produce more reliable estimates based on identification-robust methods.

The problem of weak instruments is due to a very small concentration parameter, which is a measure of the strength of the instruments or moment conditions. To boost the concentration parameter, we increase the number of moment conditions to a large number (from 4 to 20). However, an excessive number of moments may create bias.

Our estimation strategy differs from the other related papers in an important way. The small-sample bias is addressed using the Tikhonov Jackknife instrumental variable estimator (TJIVE) developed by Hansen and Kozbur (2014), which can be viewed as a regularized version of the Jackknife estimator. This estimator has better finite sample properties than the GMM estimator used by GG. Carrasco and Doukali (2017) extend their work by considering Landweber-Fridman and principal components regularization schemes. They also provide a data-driven method for selecting the regularization parameter based on an expansion of the mean-squared error.

Recently, Barnichon and Mesters (2018) estimate the NKPC for the U.S. by applying an instrument selection method based on the least absolute shrinkage and selection operator (LASSO) method. The regularization approach, that we consider in this paper, does not rely on variable selection (such as the LASSO method). Therefore, it does not require ex ante knowledge about the ordering of moments or instruments. This ensures that all available moments are used (without discarding any a priori) in an efficient way, even if there are weak instruments. In fact, the regularization method allows bias to be reduced in the presence of many moments by solving the problem of the singularity of the covariance matrix of moments. The use of many moments first increases the bias, but when the regularization is introduced, this bias shrinks.

An important contribution of this paper is the implementation of a new overidentifying restrictions J test proposed by Chao et al. (2014), which is robust to the presence of many moments. The issue with weak identification is that various test statistics deteriorate. Indeed, the conventional J test for overidentifying restrictions performs poorly when the number of moments grows (see Kunitomo, Morimune, and Tsukuda (1983) and Burnside and Eichenbaum (1996)). To address this problem, we use Chao et al. (2014)'s test, which is a new version of the J test that is robust to the presence of many instruments and heteroskedasticity. Their test is based on subtracting out the diagonal terms in the numerator of the test statistic.

Our estimation strategy leads to the following conclusions. First, our results show that the hybrid version of the NKPC is dominant. Second, the J test proposed by Chao et al. (2014) rejects the forward-looking specification of the NKPC curve and generally accepts the hybrid form. Third, our results are robust to the inclusion of additional lags of inflation and the output gap. In effect, the TJIVE, which is robust to the number of instruments, allows us to reduce the bias in the many instruments setting and ensures that all available instruments are used efficiently, even if there are weak instruments that are not discarded a priori.

This paper also assesses the usefulness of the Phillips curve for inflation forecasting in a small open economy. Many studies support the use of the Phillips curve for forecasting. For instance, Stock and Watson (1999) use generalized Phillips curve forecasts to forecast the inflation rate one year ahead. Stock and Watson (2008) compare Phillips curve forecasts to several multivariate specifications of forecasting models and find a good Phillips curve performance for the U.S.

Moroccan monetary authorities expressed an intention to move to inflation targeting[1] over the next few years. This will require a formulation of a monetary policy function a la the Taylor rule, to stabilize inflation around its target and the output gap. In this setting, forecasting inflation will become

[1] An inflation-targeting regime is an institutional arrangement in which the central bank's mandate is to target a defined medium-term inflation rate that is compatible with macroeconomic stability. The main policy instrument generally used in this setup is the official policy interest rate, which is adjusted whenever the projected inflation rate over the forecast horizon significantly deviates from the central bank's stated inflation target.

crucial for policymakers as well as for the public, which tries to understand and react to central banks' decisions. An important implication of this paper is that the Moroccan central bank should consider the Phillips curve model, including the New Keynesian feature, as a potential strong candidate for inflation forecasting when it fully transitions to its inflation-targeting regime.

The structure of this paper is organized as follows. Section 2 presents the theoretical framework underpinning the NKPC. Section 3 describes the specific model and the methodology used in this paper. Section 4 presents the new J test. Section 5 presents our empirical results. Section 6 provides some policy lessons. Section 7 concludes.

2 The New Keynesian Phillips Curve

2.1 Specifications

In this section, we present the hybrid version of the NKPC. This hybrid specification can be derived from the microfoundations framework where firms evolve in a monopolistically competitive environment with price stickiness (firms cannot adjust their prices at certain times). Following GG, each firm, in any given period, may change its price with a fixed probability of $1-\theta$, and its price will be kept unchanged or proportional to trend inflation with probability θ. Those probabilities are independent of the firm's price history. In such an environment, profile-maximization and rational expectations lead to the following hybrid NKPC equation for inflation, π_t:

$$\pi_t = \lambda mc_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} \tag{2.1}$$

where

$$\lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta+\omega-\omega\theta+\omega\beta\theta} \tag{2.2}$$

$$\gamma_f = \frac{\beta\theta}{\theta+\omega-\omega\theta+\omega\beta\theta} \tag{2.3}$$

$$\gamma_b = \frac{\omega}{\theta + \omega - \omega\theta + \omega\beta\theta} \tag{2.4}$$

where π_t is the inflation rate at time t, mc_t is the marginal cost and $E_t\pi_{t+1}$ is the expectation of future inflation conditional on the information set at time t. The parameter γ_f determines the forward-looking component of inflation and γ_b determines its backward-looking part, β is the subjective discount rate and ω is the proportion of firms that use a backward-looking rule of thumb. Equation 2.1 is usually referred to as the "semi-structural" specification corresponding to a deeper microfounded structural model. GG estimates a version of this model in which the real marginal costs are the labor share. Using data for the U.S., they find that the real marginal costs are statistically significant and inflation dynamics are predominantly forward-looking. They also find that γ_b is statistically significant but quantitatively small relative to γ_f.

2.2 Measure of marginal cost

Since the NKPC curve provides a relationship between marginal costs and inflation, the measurement of marginal costs is important for the identification of the NKPC parameters. In fact, when applying the NKPC to data, a first problem relates to the choice of an appropriate proxy for mc_t. For example, Galı and Gertler (1999) emphasize the importance of using direct measures of the real marginal cost, such as the labor income share, whereas Leith and Malley (2007) use a proxy based on the cost of intermediate goods (which corresponds to the largest determinant of the total cost of production). Guay et al. (2004) derive the NKPC when firms use alternative production functions. Others consider a measure of marginal cost based on the assumption of a Cobb-Douglas technology: $Y_t = K_t^{\nu}(A_t H_t)^{(1-\nu)}$, where Y_t is the output, K_t is the capital stock, A_t is labor-augmenting technology, H_t is hours worked, and ν is the output elasticity of capital. Real marginal cost is then given by $S_t/(1-\nu)$, where $S_t = \frac{W_t H_t}{P_t Y_t}$ is the labor income share, W_t is the nominal wage, and P_t is the price level. In a log-linear deviation from the steady state, the real marginal costs are given by: $mc_t = s_t = w_t + h_t - p_t - y_t$. These studies generally argue that these corrections do not affect the qualitative nature of the results discussed below.

In this paper, we consider a second proxy of marginal costs, which is the output gap as the relevant indicator of real economic activity. The output gap is defined as the difference between the actual output of the economy and its potential output. The output gap is an important variable for monetary policy, as it is a key source of inflation pressure in an economy. However, measuring this key variable is no easy task because unlike actual output, the level of potential output (and hence the output gap) cannot be observed directly, and so cannot be measured precisely. We measure the output gap using the Hodrick-Prescott filter. In this measure, the output gap is a combination of lags, leads, and contemporaneous values of output.

3 Estimation issues

3.1 Standard GMM approach

GG use the standard GMM estimator developed by Hansen (1982) to estimate the hybrid version of the NKPC. The reduced form can be written as:

$$\pi_t = \lambda mc_t + \gamma_f \pi_{t+1} + \gamma_b \pi_{t-1} + \varepsilon_{t+1} \tag{3.1}$$

where $\varepsilon_{t+1} = \pi_{t+1} - E_t \pi_{t+1}$ is the error term orthogonal to the information set in period t. We can rewrite the above NKPC model in terms of the orthogonality conditions:

$$E_t[(\pi_t - \lambda mc_t - \gamma_f \pi_{t+1} - \gamma_b \pi_{t-1}) z_t] = 0$$

The vector of instruments z_t includes variables that are orthogonal to ε_{t+1}, allowing for GMM estimation. GG choose a small number of lags for instruments to minimize the potential estimation bias that can arise in small samples due to the number of overidentifying restrictions. It is well known that the standard GMM estimator suffers from a small-sample bias in the presence of endogeneity, which is increased dramatically when many instruments are used and/or when the instruments are weakly correlated with the endogenous variables; see Yogo (2004).

In the next section, we present an estimation procedure that remains valid in the presence of many instruments. The strategy uses the Jackknife method to estimate first stage predictions of the endogenous variables. The chief contribution of this procedure is the use of ridge regression at each iteration of the Jackknife. The advantage of regularization is that all available instruments can be used without discarding any a priori.

3.2 Estimation strategy

Several authors argue that the above results are unreliable because they are derived using methods that are not robust to weak instrument problems (see Canova and Sala (2009), Mavroeidis (2005) and Nason and Smith (2008)). As we explain below, the weak instrument problem arises if marginal costs have limited dynamics or if their coefficients are close to zero. In other words, a flat NKPC lacks the necessary exogenous variation in inflation forecasts. More precisely, the weakness of the instruments is characterized in linear instrumental variables (IV) regression models by a unitless measure known as the "concentration parameter" (see, for example, Phillips (1983) and Rothenberg (1984)). Hence, adding more instruments is a way to boost the concentration parameter. Where do you find these new instruments? If you already have exogenous instruments, it is possible to interact them, as Angrist and Krueger (1991) do in estimating returns from schooling. It is also possible to take higher-order powers of the same instruments, as in Dagenais and Dagenais (1997).

In macroeconomic models, the use of lag variables is usually a source of many instruments or moment conditions. However, in finite samples, the inclusion of an excessive number of moments may be harmful. To address this issue, we consider the TJIVE developed by Hansen and Kozbur (2014), because of its useful properties relative to other existing IV competing estimators in the presence of many (possibly weak) instruments. We use this alternative estimation method for two reasons. First, the TJIVE does not depend on the number of instruments used in the reduced equation. Second, this leading regularized estimator performs very well (i.e. is nearly median unbiased) even in the case of relatively weak instruments (see Carrasco and Doukali (2017) and Hansen and Kozbur (2014)).

We now present our estimation procedure. We note that, unlike the standard GMM estimator, the number of instruments is not restricted, and instruments are allowed to be weak.

We can rewrite Equation 3.1 as :

$$y_t = X_t'\delta + \varepsilon_t \tag{3.2}$$

$$X_t = \Upsilon_t + u_t \tag{3.3}$$

where $t = 1,\ldots,T$. The vector of interest is $\delta = [\lambda, \gamma_f, \gamma_b]$. $X_t = [mc_t, \pi_{t+1}, \pi_{t-1}]$ is the vector of the exogenous variables. y_t is the inflation rate. The vector Υ_t is the optimal instrument. The estimation will be based on a sequence of instruments $Z_t = Z(\tau; \nu_t)$, where ν_t is a vector of exogenous variables and τ is an index taking countable values.

First, we recall the expression of the classical Jackknife estimator (JIVE):

$$\begin{aligned} \hat{\delta} &= (\hat{\Upsilon}'X)^{-1}(\hat{\Upsilon}'Y) & (3.4) \\ &= (\sum_{t=1}^{n} \hat{\Upsilon}_t X_t')^{-1} \sum_{i=t}^{n} \hat{\Upsilon}_t y_t & (3.5) \end{aligned}$$

The leave-one-out estimator, $\hat{\Upsilon}_t$, is defined as $\hat{\Upsilon}_t = Z_t'\hat{\mu}_{-t}$, where $\hat{\mu}_{-t} = (Z'Z - Z_tZ_t')^{-1}(Z'X - Z_tX_t')$ is the ordinary least squares (OLS) coefficient from running a regression of X on Z using all but the t^{th} observation.

The JIVE can alternatively be written as:

$$\hat{\delta} = (\sum_{t=1}^{n} \hat{\mu}_{-t}' Z_t X_t')^{-1} \sum_{i=1}^{n} \hat{\mu}_{-t}' Z_t y_t \tag{3.6}$$

with

$$\hat{\mu}_{-t}' Z_t = (X'Z(Z'Z)^{-1}Z_t - P_{tt}X_t)/(1 - P_{tt}) = \textstyle\sum_{s\neq t}^{n} P_{ts}X_s/(1 - P_{tt})$$

where P is an $n \times n$ projection matrix defined as $P = Z(Z'Z)^{-1}Z'$, and P_{ts} denotes the *(t,s)*th element of P.

Then, the JIVE estimator is given by:

$$\hat{\delta} = \hat{H}^{-1} \sum_{t\neq s}^{n} X_t P_{ts} (1 - P_{ss})^{-1} y_s,$$

where $\hat{H} = \sum_{t \neq s}^{n} X_t P_{ts} (1 - P_{ss})^{-1} X_s'$, and $\sum_{t \neq s}$ denotes the double sum $\sum_t \sum_{s \neq t}$. When the number of the instruments is large, the inverse of Z'Z needs to be regularized because it is singular or nearly singular.

The expression of the TJIVE, $\hat{\delta}$, is:

$$\hat{\delta} = \hat{H}^{-1} \sum_{t \neq s}^{n} X_i P_{ts}^{\alpha} (1 - P_{ss}^{\alpha})^{-1} y_s, \quad (3.7)$$

$$\hat{H} = \sum_{t \neq s}^{n} X_t P_{st}^{\alpha} (1 - P_{ss}^{\alpha})^{-1} X_s' \quad (3.8)$$

where P^{α} is an $n \times n$ matrix after regularization defined as:

$$P^{\alpha} = Z(Z'Z + \alpha I)^{-1} Z',$$

and P_{ts}^{α} denotes the (t,s)th element of P^{α}. The TJIVE depends on a regularization term, α. We select α that minimizes the mean squared error (MSE) as in Carrasco and Doukali (2017).

In section 5, we present estimates of the reduced-form parameters for the hybrid and forward-looking versions using classical GMM estimators and TJIVEs. First, we use the four moment conditions set, before considering our proposed set of moment conditions by increasing the number of moments in a robustness analysis. We consider the following specification:

$$E_t[(\pi_t - \lambda mc_t - \gamma_f \pi_{t+1} - \gamma_b \pi_{t-1}) z_t] = 0$$

As noted earlier, one important issue is the weakness of instruments used to estimate the NKPC. For instance, GMM estimates are unreliable because they are not robust to the weak instrument problem. To strengthen the instruments, we increase the number of instruments by allowing more lag variables. We use the set of instruments $\{\pi_{t-i}\}_{i=1}^{k=10}$ and $\{mc_{t-i}\}_{i=1}^{k=10}$, plus a constant term. According to Hansen et al. (2008, 403), the concentration parameter is a better indication of the potential weak instrument problem than the F-statistic. Since the increase of the number of instruments improves efficiency and the regularized Jackknife corrects for the bias due to the many instruments problem, we expect to obtain reliable point estimates.

4 Test statistic

Another important issue in our estimation strategy is testing overidentifying restrictions. Many empirical studies are used to implement the classical J test, which can be seen also as a specification test for the linear instrumental variables regression (see Hansen (1982)). If the model is correctly specified, all the moment conditions (including the overidentifying restrictions) should be close to zero. However, it was shown that the conventional J test for overidentifying restrictions performs poorly when the moments condition is increased. To deal with this, we use Chao et al. (2014)'s test, which is a new version of the J test that is robust to the many moments condition and heteroskedasticity. Their test is based on subtracting out the diagonal terms in the numerator of the test statistic. Their proposed test corrects rejection frequency as long as the number of moments increases. It is also correct under homoskedasticity with a fixed number of moments. Their test statistic takes the form:

$$J_{CHNSW} = \frac{\hat{\varepsilon}' P \hat{\varepsilon} - \sum_t^n P_{tt} \hat{\varepsilon}_t^2}{\sqrt{\hat{V}}} + L \tag{4.1}$$

with

$$\hat{V} = \frac{\hat{\varepsilon}(2)' P(2) \hat{\varepsilon}(2) - \sum_t^n P_{tt}^2 \hat{\varepsilon}_t^4}{tr(P)} = \frac{\sum_{t \neq s}^n \hat{\varepsilon}_t^2 P_{ts}^2 \hat{\varepsilon}_s^2}{L}$$

where L is the number of moments conditions, P is the projection matrix, $\hat{\varepsilon}_t = y_t - W_t' \hat{\delta}$, $\hat{\varepsilon}(2) = (\hat{\varepsilon}_1^2, ..., \hat{\varepsilon}_n^2)$, $P(2)$ is the n-dimensional square matrix where component t,sth is P_{ts}^2. Note that the numerator of the test statistic, $\sum_{t \neq s}^n \hat{\varepsilon}_s P_{ts} \hat{\varepsilon}_s$, is the numerator of the traditional Sargan test without the observation i. The denominator is a heteroskedastic-consistent estimator of the variance of $\sum_{t \neq s}^n \hat{\varepsilon}_t P_{ts} \hat{\varepsilon}_s$. The test rejects the null hypothesis when J_{CHNSW} is greater than the critical value of a chi-squared distribution with $L - p$ degrees of freedom.

5 Empirical results

As noted, a natural response to improve the estimation of the NKPC and deal with the identification problem is to add more variables to the instrument set, but there are limits to how many instruments one can use. This is because of the so called "many instruments" problem, which biases the GMM estimator. Andrews and Stock (2007) showed that, provided the number of instruments is not too large relative to the sample size, the identification-robust statistics remain size-correct in the instrumental variables regression model with many weak instruments. But again, substantial size distortion can arise in finite samples. To address this issue, we consider the TJIVE, which is robust to the many instruments problem, and compare it to the GMM method.

In this section, we report the results for the forward-looking NKPC and the hybrid NKPC. We report the estimators corresponding to the GMM approach and the TJIVE method. As a first step, we use a small number of instrument sets. Second, we increase the number of instruments from 4 to 20 by allowing more lags variables in a robustness analysis. To do so, we conduct estimations with the following sets of instruments: [1] two lags of inflation and two lags of the output gap (just-identified case), [2] four lags of inflation and four lags of the output gap, and [3] 10 lags of inflation and 10 lags of the output gap. We examine how informative additional lags of inflation and the output gap are in the NKPC. Finally, as previously mentioned, the TJIVE considered in this paper depends on a tuning parameter, α, which needs to be selected. We use a data-driven method to select the regularization parameter α; by minimizing an estimator of the approximate MSE (see formula (5.3) of Carrasco and Doukali (2017)). Estimation results are reported in Tables 1 and 2.

The point estimates we obtain are comparable to those found in many other studies that use a GMM approach when the instruments set is small. Also, the results are similar for the both the forward-looking and hybrid NKPC in the just-identified case (small number of instruments). Nevertheless, many authors have argued that the above results are unreliable because they are derived using methods such as GMM that are not robust to identification problems. These point estimates (when L is small) are biased and not informative, suggesting that the parameters of the curve are indeed not well-identified.

To circumvent the difficulties associated with weak instruments, we increase the number of the instruments to boost the concentration parameter, which is a measure of the strength of the instruments or moment conditions. However, an excessive number of moments may induce a bias. The small-sample bias is addressed by using the TJIVE, which is robust to identification problems.

Interestingly, the results based on the TJIVE are more encouraging for the hybrid NKPC when we increase the number of instruments. In fact, in accounting for inflation dynamics, the forward-looking component is larger than the backward-looking component. In effect, the reduced-form coefficients γ_f and γ_b are significantly different from zero. We also note that the quantitative importance of the backward-looking component for inflation dynamics is not negligible, even if the forward-looking component remains dominant in the dynamics of inflation. The coefficient of the output gap is statistically significant and has the correct sign. The empirical evidence is found when the number of instruments is large for the hybrid NKPC.

The final column in Tables 1 and 2 shows Chao et al. (2014)'s test of the overidentifying restrictions. We find that the J statistics is larger than chi-square critical value, which means that the null hypothesis is rejected for the forward-looking NKPC. However, the J test statistic is smaller than the chi-squared critical values when $L = 8$ and $L = 20$ for the hybrid NKPC, so we can conclude that the model is correctly specified.

First, our results show that the hybrid version of the NKPC is dominant. Second, the J test suggested by Chao et al. (2014) rejects the forward-looking specification of the NKPC curve and generally accepts the hybrid form. Third, our results are robust to the inclusion of additional lags of inflation and the output gap. In effect, the TJIVE, which is robust to the number of instruments, allows us to reduce the bias in the presence of many instruments, and ensures that all available instruments are used in an efficient way (without discarding any a priori), even if there are weak instruments.

Table 1: Forward-Looking NKPC

Method	Instruments	λ	γ_f	J_{CHNSW}
GMM	$L = 4$	0.24	0.883	17.09
		[0.20]	[0.19]	
	$L = 8$	0,312	0.913	23.28
		[0.25]	[0.31]	
	$L = 20$	0.37	0.950	36.21
		[0.28]	[0.32]	
$TJIVE$	$L = 4$	0.23	0.845	16.61
		[0.14]	[0.16]	
	$L = 8$	0.27	0.734	20.50
		[0.12]	[0.09]	
	$L = 20$	0.28	0.729	32.40
		[0.05]	[0.06]	

The p-values appear in brackets for the null hypothesis that the estimate is equal to zero.

Table 2: Hybrid NKPC

Method	Instruments	λ	γ_f	γ_b	J_{CHNSW}
GMM	$L=4$	0.174	0.403	0.350	13.40
		[0.31]	[0.10]	[0.12]	
	$L=8$	0.350	0.813	0.444	17.15*
		[0.23]	[0.17]	[0.15]	
	$L=20$	0.380	0.983	0.521	19.32
		[0.27]	[0.16]	[0.12]	
TJIVE	$L=4$	0.112	0.494	0.351	12.45
		[0.08]	[0.17]	[0.12]	
	$L=8$	0.12	0.674	0.277	10.25*
		[0.05]	[0.03]	[0.01]	
	$L=20$	0.151	0.656	0.320	15.2*
		[0.04]	[0.002]	[0.001]	

The p-values appear in brackets for the null hypothesis that the estimate is equal to zero. * means the model is correctly specified (at the 5% level).

6 Discussion and policy implications

In 2016, Bank Al-Maghrib (the Moroccan central bank) started to use a new forecasting and policy analysis system (FPAS) called the Moroccan Quarterly Projection Model (QPM). The goal of the QPM is to improve monetary policy decisions to align Bank Al-Maghreb with central banking best practices. Additionally, Morocco is expected to transition to a full-fledged inflation targeting regime in the near future. This will require monetary policy that stabilizes inflation around its target and the output gap, as outlined in the Taylor rule. Under such a regime, forecasting inflation is crucial for a central bank, as well as for the public, which tries to understand and react to the central bank's decisions. Modeling inflation is therefore a core task for inflation-targeting central banks.

While the literature is divided on the usefulness of the Phillips curve in forecasting inflation, several papers find that different forms of the Phillips curves (including New Keynesian versions, which include indicators for real economic activity, past inflation and future inflation) forecast inflation well. For instance, Stock and Watson (2008) compare Phillips curve inflation forecasts to several multivariate specifications of forecasting models and find that the Phillips curve performs well in forecasting U.S. inflation. Recently, Gabrielyan (2018) investigates the forecasting ability of the Phillips curve for headline inflation for Sweden, Canada and New Zealand, three countries that used an inflation-targeting regime from 1983-2016. She finds that Phillips curve models can improve inflation forecasts against the random walk and autoregressive models' benchmarks when the central bank is explicitly targeting inflation. However, the results from earlier periods are not homogeneous across various econometric specifications and different sample periods. The latter finding suggests that the Phillips curve is more appropriate for inflation forecasting when a central bank has already adopted an inflation-targeting regime.

An important implication is that Bank Al-Maghrib should consider the Phillips curve model, including the New Keynesian version discussed in this paper, as a potential strong candidate for inflation forecasting when it fully transitions to an inflation-targeting regime. However, we emphasize that this model is not a pure forecasting device. Rather, it is a tool that helps to structure monetary policy discussion, identifies the impact of key activities and inflation drivers (e.g. the output gap), and focuses on a forward-looking perspective.

7 Conclusion

This paper discusses the identification-robust many and possibly weak moments estimation of the parameters of the New Keynesian Phillips curve, and applies this method to Moroccan data. The diagnostic criteria for weak identification, such as the concentration parameter, indicate that existing econometrics techniques perform poorly. The approach in this paper addresses several important econometrics issues. First, we use a regularization schema (TJIVE) to solve the problem of many weak instruments when we include more variables in the moments set. The use of many moments increases bias, but when regularization is introduced, the bias shrinks. Second, we implement a new J test developed by Chao et al. (2014) to test the null hypothesis that the model is correctly specified. While our focus is on obtaining consistent coefficient estimates of the New Keynesian Phillips curve and testing the hypothesis that the model is well specified, our model provides a solid framework for developing near-term inflation projections.

Nevertheless, our analysis could be improved. Because of data limitations, we are forced to take the output gap as a proxy for the real marginal cost. An important extension would be to develop efficient algorithms for simulating the Bayesian posterior to correct for the proxy of the real marginal cost.

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Appendix

A. Data sources

All data series are quarterly, beginning in 1998:Q1 and ending in 2016:Q4. Data on gross domestic product and inflation are from national statistical office.

The output gap is the deviation of real GDP from its steady state. We give a measure of the output gap using the Hodrick-Prescott filter. In this measure, the output gap is a combination of lags, leads, and contemporaneous values of output.

B. Presentation of the Tikhonov regularization

We consider the general case where the estimation is based on a sequence of instruments $Z_i = Z(\tau;\nu_i)$, with $\tau \in N$. Assume that τ lies in a space Ξ ($\Xi = \{1,..,L\}$ or $\Xi = \mathbb{N}$) and let π be a positive measure on Ξ. Let K be the covariance operator for instruments from $L^2(\pi)$ to $L^2(\pi)$ such that:

$$(Kg)(\tau) = \sum_{l=1}^{L} E(Z(\tau,\nu_i)Z(\tau_l,\nu_i))g(\tau_l)\pi(\tau_l).$$

where $L^2(\pi)$ denotes the Hilbert space of square integrable functions with respect to π. K is supposed to be a nuclear operator, which means that its trace is finite. Let λ_j and ψ_j, $j = 1...$ be the eigenvalues (ordered in decreasing order) and the orthogonal eigenfunctions of K, respectively. The operator can be estimated by K_n defined as:

$$K_n : L^2(\pi) \to L^2(\pi)$$
$$(K_n g)(\tau) = \sum_{l=1}^{L} \frac{1}{n} \sum_{i=1}^{n} (Z(\tau,\nu_i)Z(\tau_l,\nu_i))g(\tau_l)\pi(\tau_l).$$

If the number of instruments L is large relative to n, inverting the operator K is considered as an ill-posed problem, which means that the inverse is not continuous and its sample counterpart, K_n, is singular or nearly singular. To solve this problem, we need to stabilize the inverse of K_n using regularization. A regularized inverse of an operator K is defined as $R_\alpha : L^2(\pi) \to L^2(\pi)$, such that

$lim_{\alpha\to 0} R_\alpha K\rho = \rho, \forall\rho \in L^2(\pi)$, where α is the regularization parameter (see Kress (1999) and Carrasco, Florens, and Renault (2007)).

Tikhonov regularization

We consider the Tikhonov regularization scheme

$$
\begin{aligned}
(K^\alpha)^{-1} &= (K^2 + \alpha I)^{-1} K. \\
(K^\alpha)^{-1} r &= \sum_{j=1}^{\infty} \frac{\lambda_j}{\lambda_j^2 + \alpha} \langle r, \psi_j \rangle \psi_j.
\end{aligned}
$$

where $\alpha > 0$ and I is the identity operator. For the asymptotic efficiency, α has to go to zero at a certain rate. The Tikhonov regularization is related to ridge regularization. The ridge method was first proposed in the presence of many regressors. The aim was to stabilize the inverse of XX' by replacing XX' by $XX' + \alpha I$. However, this was done at the expense of bias relative to the OLS estimator. In the IV regression, the two-stage least squares estimator is already biased, and the use of many instruments usually increases this bias. The implementation of the Tikhonov regularization and the selection of an appropriate ridge parameter for the first-step regression helps to reduce this bias.
Let $(K_n^\alpha)^{-1}$ be the regularized inverse of K_n and P^α be an $n \times n$ matrix as defined in Carrasco (2012) by $P^\alpha = T(K_n^\alpha)^{-1}T^*$, where $T : L^2(\pi) \to R^n$ with
$Tg = (< Z_1, g >, < Z_2, g >', ..., < Z_n, g >')'$
and $T^* : R^n \to L^2(\pi)$, with
$T^*v = \frac{1}{n}\sum_j^n Z_j v_j$
such that $K_n = T^*T$ and TT^* is an $n \times n$ matrix with typical element $\frac{<Z_i, Z_j>}{n}$. Let $\hat{\phi}_j$, $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq ..... \geq 0$, $j = 1, 2, ...$ be the orthonormalized eigenfunctions and eigenvalues of K_n, and ψ_j be the eigenfunctions of TT^*.
We then have $T\hat{\phi}_j = \sqrt{\lambda}_j \psi_j$ and $T^*\psi_j = \sqrt{\lambda}_j \hat{\phi}_j$. For $v \in R^n$, $P^\alpha v = \sum_j^\infty q(\alpha, \lambda_j^2) < v, \psi_j > \psi_j$ where $q(\alpha, \lambda_j^2) = \frac{\lambda_j^2}{\lambda_j^2 + \alpha}$.

Note that the case when $\alpha = 0$ corresponds to no regularization. Thus, we have $q(\alpha, \lambda_j^2) = 1$ and $P^0 = Z(Z'Z)^{-1}Z'$.