From Firm Productivity Dynamics to Aggregate Efficiency

Bernabe Lopez-Martin

I construct a quantitative framework to evaluate how financial constraints can reduce productivity growth at the firm level and result in lower aggregate productivity. I consider a model where firms are able to invest in innovation in order to increase their productivity, or knowledge capital. This investment is a costly and uncertain enterprise. As the capacity to obtain external funds is diminished, resources allocated to this effort will be reduced due to different mechanisms at work. First, the return of this investment in the case of success may be diminished by the inability to quickly increase production capacity if the credit necessary to do so is scarcer (i.e., if entrepreneurs cannot rent the optimal level of physical capital). Second, financial constraints reduce profits obtained by entrepreneurs and therefore the amount of assets they are able to accumulate in every period. Finally, financially underdeveloped economies will be characterized by a lower average ability of entrepreneurs. This is due to the lower equilibrium wage in the economy, which results in a larger mass of individuals opting to set up firms. In the margin, these individuals tend to have lower ability to manage a firm and relatively low prospects of generating firm productivity growth through innovation. JEL codes: O11, O16, O30, O4, E23

I. Quantitative Model

The model builds upon the frameworks of occupational choice and heterogeneous entrepreneurial ability of Lucas (1978) and industry dynamics of Hopenhayn.
There is a continuum of individuals who possess heterogeneous innate entrepreneurial ability,\(^1\) with some probability they have an opportunity to become entrepreneurs and establish a firm. All individuals earn the same wage as workers, since there is no heterogeneity in their effective units of labor and workers are perfectly mobile across firms. Entrepreneurs can allocate resources to investment in technology through a controlled stochastic process (Pakes and McGuire 1994; Klette and Kortum 2004; Farias et al. 2012; Doraszelski and Jaumandreu 2013): entrepreneurs decide every period the amount of resources invested in innovation, which determines the probability of an increase in firm productivity.

**Preferences**

Time is discrete and a period, indexed by \(t\), represents a year. Individuals value the consumption of the final good, denoted \(c_t\), through lifetime and intratemporal preferences represented as follows:

\[
U = \mathbb{E} \left[ \sum_{t=0}^{\infty} (\beta(1-\mu))^t u(c_t) \right] \quad \text{and} \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}
\]

where \(\beta\) is the discount factor, \(\sigma\) is the coefficient that governs risk aversion. The probability that an individual dies in any given period is \(\mu\), so that the effective discount factor is \(\beta(1-\mu)\). When an individual dies, he is immediately replaced by another individual with the same entrepreneurial ability \(\varphi\) so that the mass of individuals and their distribution over ability is constant.

**Production Technology**

Production of the final good is carried out by single establishment firms and each firm is managed by its owner. Individuals possess innate and permanent entrepreneurial ability \(\varphi\), received according to a distribution \(h(\varphi)\). Entrepreneurs are able to accumulate knowledge capital denominated \(n\) (through a process described below) and have access to a decreasing returns to scale production technology with respect to capital and labor:

\[
q = (\varphi n)^{1-v} f(k,l)^v \quad \text{with} \quad f(k,l) = k^\alpha l^{1-\alpha},
\]

where \(k\) is capital and \(l\) is labor used in production. Following Lucas (1978), \(v \in (0,1)\) is the span-of-control parameter that determines the decreasing returns to scale in the production technology.

\(^1\) Differences in management quality are an important determinant of productivity differences across firms (see Bartelsman and Doms 2000; Foster et al. 2001; Syverson 2011).
Innovation Technology

Entrepreneurs can invest in the innovation good $x$ to increase the stock of knowledge capital. Three outcomes are possible every period, depending on the amount of investment in the innovation good in the previous period: knowledge capital may increase by a proportion $\Delta$, it may remain constant, or decrease by $\Delta$. Knowledge capital is defined on the grid $\{n, n(1 + \Delta), n(1 + \Delta)^2, \ldots, n\}$, where $n$ and $\bar{n}$ are the lowest and highest possible levels of knowledge capital, respectively. The probability of a successful outcome is given by:

$$P(n' = n(1 + \Delta)|n, x) = (1 - \gamma)\frac{(1 - \lambda)a(x/n)}{1 + a(x/n)} + \gamma. \quad (3a)$$

There are diminishing returns to innovation investment $x$. Fixing a probability of success in innovation, $P(n(1 + \Delta)|s, x)$, the necessary investment in innovation goods $x$ to increase the productivity of the firm by a fixed percentage is proportional to knowledge capital $n$. The probability of a negative outcome is given by:

$$P(n' = n/(1 + \Delta)|n, x) = \frac{(1 - \gamma)\lambda}{1 + a(x/n)}. \quad (3b)$$

Knowledge capital summarizes the history of investment and success in innovations and governs the size of the firm (Klette and Kortum 2004). Furthermore, it is lost when the firm closes, regardless of whether exit is due to an exogenous exit shock or the entrepreneur finds it optimal to close the firm. Finally, knowledge capital is assumed to be firm-specific and there is no market for its trade.

Workers

We group state variables by letting $s = (\varphi, n, b)$, where $\varphi$ is the individual’s permanent entrepreneurial ability, $b$ are financial assets and $n$ is knowledge capital. Additionally, $z \in \{e, w\}$ denotes whether the individual is an entrepreneur or a worker, respectively. The problem of the worker amounts to a savings decision.


3. The model can be extended to consider unbounded knowledge capital, which would require additional conditions to guarantee a well defined dynamic program and convergence in the stationary distribution (see Atkeson and Burstein, 2010). We select the upper bound so that a negligible amount of firms reach this level.

4. It can be verified that optimal labor and capital inputs, output and profit are proportional to knowledge capital $n$ under the production function previously specified in the case of no financial restrictions.

and determining the conditions under which it is optimal to establish a firm:

\[
v_w(s) = \max_{\{c,b\geq 0\}} u(c) + \beta(1 - \mu) \sum_{\{e'\}} Q(e') v(s')
\]

\[
\text{s.t. } c + b' = w + (1 + r)b \quad \text{and} \quad n = n_w
\]

At the beginning of each period, workers face their occupational choice \(v(s) = \max\{v_e(\cdot, n_w, b), v_w(s)\}\), where the value of becoming an entrepreneur is represented by \(v_e(\cdot)\). The worker is free to continue in the labor market and earn a wage \(w\) every period, or become an entrepreneur if he is given an opportunity to start a firm with probability \(Q(e' = 1) = \theta\). When individuals re-enter the labor market, their knowledge capital is reset to \(n_w\), which follows from the interpretation that knowledge capital represents an intangible asset embedded in the firm. All new-born individuals receive an entrepreneurial ability \(\varphi\) from the distribution \(h(\varphi)\) and initial assets \(b\). In this setup occupational choice depends on the ability of the individual as an entrepreneur but may also depend on financial wealth, necessary to reach a sufficiently profitable scale of production when financial constraints are present.

**Entrepreneurs**

Given the level of knowledge capital and financial wealth, profits for an entrepreneur are given by:

\[
\pi(s) = \max_{\{k,l\}} q - (r + \delta)k - wl \quad \text{s.t. } k \leq \bar{k}(s),
\]

where the choice of capital input is restricted by an endogenous collateral constraint, which we describe below. Additionally, the entrepreneur may return to the labor market in any period:

\[
v_e(s) = \max_{\{c,x,b\geq 0\}} u(c) + \beta(1 - \mu) \sum_{\{n'\}} P(n' \mid n, x) \max\{v_w(s'), v_e(s')\}
\]

\[
\text{s.t. } c + b' + x = \pi(s) + (1 + r)
\]

The entrepreneur is able to invest in the knowledge capital of the firm as long as the firm is active, but is lost if the individual decides to return to the labor market.

**Financial Markets: Endogenous Collateral Constraints**

In specifying the collateral constraints we follow Amaral and Quintin (2010) and Buera et al. (2011). At the beginning of a period the entrepreneur makes a deposit \(b\) and rents capital \(k\) from a financial intermediary. At the end of the period, the entrepreneur receives his deposit earning interest rate \(r\) and pays the
cost of capital rental at the total rate of \( r + \delta \). Borrowing and capital rental are realized within a given period and the assets of the individual are restricted to be positive \( b \geq 0 \) in all periods.

Entrepreneurs may reneg on financial contracts after production has taken place. If this occurs, the entrepreneur keeps a fraction \( (1 - \psi) \) of the undepreciated capital and the revenue net of labor payments. The punishment for default is the loss of the financial assets deposited with the financial intermediary \( b \). Entrepreneurs regain access to financial markets in the following period. This implies that a static condition determines enforceable allocations, allowing for the consideration of financial constraints in a tractable manner. In this setup parameter \( \psi \) indexes enforcement of financial contracts in the economy, which encompasses economies with no credit when \( \psi = 0 \) and perfect credit markets if \( \psi = 1 \).

The analysis is restricted to financial contracts that are incentive-compatible, there is no default in equilibrium. Effectively, imperfect enforcement of financial contracts determines an upper bound \( k(s) \) on the amount of capital that entrepreneurs are able to borrow. Mathematically the financial constraint can be described as follows. In the case of no-default the entrepreneur receives profits plus interest rate income from financial assets:

\[
\max_{\{l\}} q - wl - (r + \delta)k - x + (1 + r)b, \tag{6}
\]

while in the case of default the entrepreneur would receive (off-equilibrium):

\[
\max_{\{l\}} (1 - \psi)(q - wl + (1 - \delta)k) - x. \tag{7}
\]

Capital rental is said to be enforceable if and only if it satisfies \( (6) \geq (7) \). Note that \( x \) does not distort the bound of enforceable capital. The borrowing limit is increasing in financial wealth since the loss of collateral is greater in the case of default. It is also increasing in firm productivity, as only a share of output is kept in the case of default (see Amaral and Quintin 2010; Buera et al. 2011).

**Equilibrium**

The state space is given by \((\varphi, n, b, z)\), we previously defined \( s = (\varphi, n, b) \) and \( z \in \{e, w\} \). Given an interest rate \( r \), an open economy stationary competitive equilibrium consists of: quantities and production inputs \( \{q(s), l(s), k(s)\} \) and profits \( \{\pi(s)\} \), policy functions for financial asset accumulation \( \{b'(s, z)\} \), policy function for investment in the innovation good \( \{x(s)\} \), a wage \( w \), and an invariant measure \( M(s, z) \) of individuals over the state space, such that: workers and entrepreneurs solve the dynamic problems \( \{v(s), v_e(s), v_w(s)\} \) and profit maximization, the market clearing condition in the labor market holds (the demand of labor by entrepreneurs equals the mass of workers), the measure \( M(s, z) \) is
consistent with policy functions, optimal decision rules and the stochastic process for knowledge capital.

II. Parameters

The parameters of the model are divided into two groups: a first group of standard parameters taken from the literature (see table 1), a second group of parameters that are set to match key features of the US economy (the baseline economy, table 2).

**Predetermined Parameters**

The predetermined parameters are taken from different models of firm dynamics: Atkeson and Kehoe (2005), Amaral and Quintin (2010), Buera et al. (2011),

<table>
<thead>
<tr>
<th>Description of parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective discount factor</td>
<td>$\beta(1 - \mu)$</td>
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<tr>
<td>Risk aversion</td>
<td>$\sigma$</td>
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<tr>
<td>Interest rate (open economy assumption)</td>
<td>$r$</td>
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<tr>
<td>Span-of-control</td>
<td>$v$</td>
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<tr>
<td>Income share of capital</td>
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<tr>
<td>Capital depreciation rate</td>
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<tr>
<td>Innovation technology</td>
<td>$\lambda$</td>
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<td>0.21</td>
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<td>Initial knowledge capital</td>
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<td>$\Delta$</td>
<td>0.38</td>
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<td>$\phi$</td>
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**Table 1. Predetermined Parameters**

**Table 2. Calibrated Parameters and Target Statistics**

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<table>
<thead>
<tr>
<th>Target statistics</th>
<th>data</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total firm entry (and exit) rate</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Exit rate firms 20+ workers</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Std. deviation growth rates</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Relative size firms 25–30/1–5 years (survivors)</td>
<td>2.69</td>
<td>2.62</td>
</tr>
<tr>
<td>Employment at firms w/500+ workers</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Knowledge capital investment/total output (%)</td>
<td>4.40</td>
<td>4.70</td>
</tr>
<tr>
<td>Private credit/output</td>
<td>2.30</td>
<td>2.34</td>
</tr>
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Midrigan and Xu (2014), Restuccia and Rogerson (2008), among many others. The values of two of the innovation technology parameters, $a$ and $\lambda$, are taken from Farias, Saure, and Weintraub (2012).

**Calibrated Parameters**

In the model, the total exit rate includes the entrepreneurs that decide to close their firms and those that exit due to an exogenous shock. The exogenous exit rate $\mu$ is set to match the exit rate of firms with more than 20 employees for the US total business sector (see fig. 1.2 in Bartelsman et al. 2009). Entrepreneurial ability is distributed according to a discretized Pareto distribution with nine possible values; its parameter is closely related to the size distribution of firms in the model; our target is the share of employment in firms with more than 500 workers. Parameter $\theta$ represents the probability that a worker receives an opportunity to set up a firm; this parameter is linked to the total entry rate of firms (in a stationary equilibrium the total and entry rates are equalized).

The technology accumulation parameters mainly determine the life-cycle growth of firms. I target the average size of firms that are 25–30 years relative to firms that are younger than 5 years (Hsieh and Klenow 2014). Additionally, McGrattan and Prescott (2010) estimate the ratio of business intangible investment to total adjusted domestic product to be 0.044 for the United States. Parameter $\Delta$ governs the size of the fluctuations in productivity. We follow Atkeson and Burstein (2010), albeit with a different specification for the accumulation of knowledge capital, and target the standard deviation of the growth rate of employment of large firms in the model, taking the statistic computed for publicly traded firms in the United States for the period 1980–2001 from Davis et al. (2007).

Parameter $\psi$ represents financial development and the access to credit for firms. As is standard in the literature, we target the ratio of private credit provided by financial institutions and private bond markets over GDP (Beck et al. 2009). The value for the US results in an economy with perfect financial markets in the model: the average of the ratio for the years between 1993 and 2001 is 2.30 (this covers the period of the data used to impute firm life-cycle growth in Hsieh and Klenow 2014).

### III. Quantitative Analysis

The main purpose of our model is to help us understand the channels through which financial development affects aggregate productivity in a model with endogenous firm level productivity. In the presented framework, financial constraints affect the growth of productivity at the firm level and can potentially explain firm life-cycle productivity growth differences between developed and developing economies.

The quantitative exercise consists in reducing parameter $\psi$ to approximately match the volume of credit relative to output in a developing economy such as
Mexico. The main results for aggregate statistics are presented in Table 3: the fall in total output is 36.4 percent, explained by a fall in total aggregate productivity of 10.3 percent and a decrease in capital to approximately one third of the developed economy. To gain insight on the sources of the productivity loss we can decompose the weighted average firm level productivity as follows:

\[
\bar{\omega} = \sum_{\{i\}} \omega_i \cdot \left( \frac{q_i}{Q} \right) = \bar{\omega} + \text{cov}(\omega, q/Q) \quad \text{with} \quad \omega = (\varphi n)^{1-v},
\]

where \(\bar{\omega}\) is average firm level productivity weighted by the output share of the firm \(q_i/Q\) and \(\bar{\omega}\) is the arithmetic mean of firm level productivity. This decomposition provides a measure of the efficiency of resource allocation in the economy: the covariance term reveals to what extent more productive firms in the economy have a larger share of output (see D’Erasmo et al. 2012). As can be seen in this model, both the weighted and the unweighted averages for firm productivity are lower in the economy with less access to credit as well as the covariance term (results in Table 4).

The impact on firm life-cycle productivity growth is considerable: in the developed economy the ratio of productivity for firms 25–30 years of age relative to the same firms when they were 1–5 years of age is 2.75, while this ratio is reduced to almost half to 1.31 in the less financially developed economy.

5. It can be shown that in this type of models TFP is obtained from the equations \(Q = A(K^n L^{1-n})\), where \(L\) is the mass of workers (see Midrigan and Xu 2014).
IV. Conclusion

The stylized equilibrium framework proposed provides a link between financial constraints, productivity growth at the firm level, and aggregate productivity. This can contribute to our understanding of the lower life-cycle productivity growth of firms in developing economies as recently emphasized in the literature (Hsieh and Klenow 2014). The proposed framework can be extended along several dimensions. In ongoing research, I explore a model which introduces the informal sector. Through the general equilibrium mechanism described previously, lower wages lead to an increase in the number of low-productivity low-growth firms in the informal sector which can potentially amplify the negative impact of financial constraints on the aggregate economy.

References


