WPS6091

Policy Research Working Paper 6091

Turkish Treasury Simulation Model for Debt Strategy Analysis

Emre Balibek
Hamdi Alper Memis

The World Bank
Financial Advisory and Banking
June 2012

Policy Research Working Paper 6091

Abstract

Governments raise funds to meet their financing needs using a range of fixed income securities and loans with different maturities, interest rates, and exchange rate structures. Public debt managers need to consider various policy objectives when deciding on the structure of the public liability portfolio. This paper describes a simulation model developed at the Turkish Treasury to assist the decision-making process in debt strategy formulation. The model is used to analyze the medium and long-term consequences of alternative debt management strategies in terms of cost and risk characteristics, and provides key inputs to decision making.

This paper is a product of the Turkish Treasury, supported by the Financial Advisory and Banking Department, World Bank. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The authors may be contacted at emre.balibek@treasury.gov.tr and alper.memis@treasury.gov.tr.

Turkish Treasury Simulation Model for Debt Strategy Analysis[1]

Emre BALIBEK, PhD[2] and Hamdi Alper MEMIS[3]

JEL Classification: H63, G32, G17

Key words: Public debt management, risk management, simulation models, financial markets

[1] This work was extensively supported by the World Bank through technical assistance during 2006-2007. The ideas expressed in this paper are only those of the authors and do not necessarily reflect the views and policies of the Turkish Treasury. No responsibility for them should be attributed to Turkish Treasury. In this paper, aspects of the model are described to the extent that confidentiality policies of the Turkish Treasury allow.

[2] Deputy General Director, Risk Management Unit, Turkish Treasury (emre.balibek@treasury.gov.tr)

[3] Senior Associate, Market Risk Management Department, Turkish Treasury (alper.memis@treasury.gov.tr)

Acknowledgements

We would like to express our gratitude to the World Bank Treasury, particularly Lars Jessen, for continuous support, guidance, useful comments and helpful discussions during the development stage of the model introduced in this paper. We would also like to acknowledge Prof. Dr. Burak Saltoğlu of Bogazici University and David Jamieson Bolder of Bank for International Settlements (who worked with us during his tenure at the Bank of Canada) for their assistance in building the model. We are very grateful to our colleagues Fatoş Koç and H.Hakan Yavuz for their invaluable contributions and support during all stages of our work and to our general director, Dr. M.Coşkun Cangöz, not only for being the motivating force behind the project, but also for sharing his comments on this paper. Last but not the least, we would like to thank Prof. Dr. Murat Köksalan of Middle East Technical University, Cigdem Aslan, Antonio Velandia, Rodrigo Cabral, Eriko Togo, Alex Sienaert and Farah Hussain of the World Bank and Myrvin Anthony of the International Monetary Fund for their feedback on the final draft of this paper. All thanks are without implication and we retain full responsibility for any omissions or errors.

Acronyms and Abbreviations

ATR	Average Time to Re-fixing
CIR	Cox, Ingersoll and Ross Model
CKLS	Chan, Karolyi, Longstaff and Sanders Model
CPI	Consumer Price Index
DM	Decision Maker
GDP	Gross Domestic Product
GUI	Graphical User Interface
IMF	International Monetary Fund
OECD	Organization for Economic Cooperation and Development
PDM	Public Debt Management
TDSM	Turkish Debt Simulation Model
TRY	Turkish Lira
USD	US Dollar
VaR	Value at Risk

1. Introduction

Public debt managers borrow funds to finance the cash needs of the government and to refinance maturing debt; and they issue a range of borrowing instruments with different maturities, interest rates, and exchange rate structures. The choice of instruments to be issued depends on the objectives of public debt management (PDM), as well as other factors such as the local and international market environment, the governments' credit rating, and macroeconomic environment. Formulation of debt issuance strategies, i.e., determining the portfolio composition of government debt, is a complicated process since the objectives of PDM usually conflict with each other and there is uncertainty regarding the movements of variables that affect the outcomes of the decisions made.

One of the basic objectives of PDM is to borrow at the lowest possible cost. The risks associated with the debt portfolio should also be considered to avoid any adverse effect on debt service. A country's financial stability is affected by the health of public sector balances and the sustainability of debt service; therefore distress in debt management, such as difficulties in fulfilling liabilities on a timely basis or excessive costs on the debt, has spill-over effects on the entire economy. Hence, debt managers should form sound debt management policies that reflect the government's cost and risk preferences together with other debt management goals (IMF and the World Bank, 2003).

One of the major risks that public debt managers are concerned with is "market risk", which is defined as the risk of an increase in the cost of debt service due to fluctuations in market conditions (IMF and the World Bank, 2001). In financial markets, there generally exists a trade-off between return and risk. As a result, for a portfolio manager, achieving a higher return requires investing in riskier assets. Since the government is the issuer of financial assets (securities), from the government's perspective, the trade-off is between creating a low-cost portfolio and containing the associated risks.

Frequently, in financial markets, due to the uncertainty perceived for future periods, the yield curve is upward sloping, and thus short-term interest rates are usually lower than those for longer term. In such a context, a trivial choice for minimizing costs is to issue short-term debt to make use of lower interest rates. However, a portfolio consisting of short-term instruments (or instruments indexed to short-term rates) will result in the need to renew (or re-fix the interest on) debt more frequently, which in turn induces high sensitivity to developments in financial markets since future interest rates are unknown and the shape of the yield curve varies through time. In addition to increasing exposure to changes in interest rates, dependence on short-term debt may increase the issuer's roll-over risk.

Similar to other financial decision-making problems, an important characteristic of the public debt management strategy problem is that decisions are made under uncertainty. There is no perfect information regarding the future states of relevant financial variables, such as interest and exchange rates. For example, a debt management office may issue floating rate bonds anticipating a decline in interest rates in order to reduce cost. However, the actual cost of this strategy depends on the interest rate realizations until the redemptions of these bonds. On the other hand, issuing fixed rate securities also carries a risk, that of locking in high rates in an environment of declining rates. A similar argument also applies for decisions regarding whether or not to issue in foreign currency, if the country is implementing a floating rate currency regime. In summary, uncertainty about the future raises the need to consider risk objectives.

Thus, formulation of borrowing strategies, taking into consideration the dilemma between attaining a minimum cost portfolio and diminishing the associated risks, is an important decision-making problem. In order to deal with this dilemma and to make better informed decisions, debt managers use several analytical tools, ranging from deterministic models to stochastic approaches. In deterministic scenario-based methods, cost and risk is analyzed under a handful of future market scenarios defined by debt managers, or other policy makers. Debt management offices have also adapted simulation modeling techniques that were pioneered by private financial institutions. The stochastic simulation models of PDM offices are generally derived from the "Value at Risk" (VaR) concept used by banks and other financial firms. VaR models provide an estimate of the maximum expected loss of market value that the assets may suffer within a given period at a certain confidence interval. The results from VaR models can easily be communicated to policy makers, and therefore they became widely popular in the mid-1990s.

While the focus of VaR is on market value, the focus of a public debt manager is on the cost of the debt. For sovereigns, therefore, the approach is modified as a "Cost-at-Risk" or "Cash-Flow-at-Risk" model.[4] These models provide an estimate of the variability of debt service

[4] One major difference between VaR model and its adaptations to public debt management is the time horizon used in decision making and modeling. Although, VaR is used for short-term decision making, Cost-at-Risk type models are used to analyze medium-long term consequences of strategy alternatives. This difference in the time horizon makes the task for debt managers much more complex, as they will have to make forecasts/simulations for longer periods.

costs and/or cash-flows, and enable debt managers to test cost and risk performances of alternative debt management strategies under various simulation scenarios for interest and exchange rates, and to identify and understand the trade-offs.

This paper presents a description of the stochastic simulation model developed by the Turkish Treasury. In the following sections, we provide a description of the model, elaborate on how the model was developed, and discuss issues with regard to implementation. The main aim of this paper is not only to contribute to literature in this field, but also to invite comments from researchers and practitioners, which will be invaluable for further development of the model.

This paper is organized in six sections. Section two begins with a discussion on conceptual aspects of model building and simulation modeling, and also provides the general structure of sovereign debt strategy models. In Section three, we discuss the evolution of the conceptual framework of the Turkish Treasury simulation model. We focus particularly on the cost and risk concepts, horizon and granularity choices, as well as debt metrics used in strategy formulation framework. In Section four, we describe the technical aspects of the model. Section five discusses the analysis performed within the experimental framework and in Section six we conclude and indicate prospects for future work.

2. Models for Debt Management Strategy Analysis: Why Do We Need a Model?

A "system" is defined as "a group of objects that are joined together in some regular interaction or interdependence toward accomplishment of some purpose" (Banks and Carson, 1984, p.6). The links and interactions between components of a system are generally complex and in many real life cases can be beyond limits of direct comprehension. Therefore, a tool is needed to understand and study the details of functioning of a system. A model is developed as a simplified substitute for a complex reality, to represent a particular system where all details of the system under focus are not required to reach the purpose of analysis.

Mitchell (1993) considers two main perspectives while analyzing models: models as device for predicting the output from a real system and models as a statement of collection of beliefs about the real world that are considered relevant. These two meanings correspond to two main uses of building models: assisting decision making as a device for experimentation to make predictions and securing comprehension of the relationships between system elements.

In mathematical models, the system is represented by symbolic notations and equations and it can be solved and analyzed by mathematical methods. However, in real life, many problems are too complex to be solved mathematically and in these instances, other tools such as simulation are needed.

Simulation is “the imitation of the operation of a real-world process or system over time” (Banks and Carson, 1984, p.2). It helps us compress the functioning of the system in time and space and enables the researcher to analyze the interactions that would not otherwise be observable. In the case of debt management, for example, one needs to wait until maturity to be able to detect the final cost of a floating rate note. A computer-based simulation model generates an artificial account of the life of a bond at a moment in time and enables analysts to evaluate its possible costs under a set of scenarios.

Once built and validated, a simulation model can be used to answer “what-if” questions about the real system. Simulation modeling can not only help the modeller predict current system behavior considering a set of future scenarios, but can also be used as a design tool by enabling the analysts to make modifications in the system. That is, potential changes in the system components and/or their interactions can be simulated to analyze their possible impacts on system performance. A simulation model, once set up properly, can be used repeatedly to analyze suggested set-ups or strategies in a short period of time at a relatively low cost. Therefore, simulation modeling is a powerful tool that enables the investigation and experimentation of a complex system.

2.1 Overview of Debt Strategy Models

In the context of government debt management, scenario analysis based on a limited number of deterministic scenarios and more advanced simulation models are used to support the decision-making process in strategy formulation. Many countries use simple models based on deterministic scenarios (IMF and the World Bank, 2003 and Jensen and Risbjerg, 2005). In these models, there exists a base-line scenario of relevant variables, and risk is measured by the variability of costs under a limited number of user-defined scenarios of shocks to the baseline scenario (Velandia, 2002). The deterministic model outcomes are typically easier to understand and interpret compared to stochastic simulation models. Therefore, developing deterministic models can be seen as a simple and efficient method in cost-risk modeling of public debt, since they are also easy to communicate to policy makers. Despite these advantages, “as only a limited number of scenarios are generated in deterministic models, it is not possible to achieve a direct quantification of the risk by relating potential outcomes to their likelihood” (Risbjerg and Holmlund, 2005, p.46). Furthermore, determining the size and directions of shocks to the financial or macroeconomic variables can be quite arbitrary and the relationship of these variables may not be captured well under this approach.

Stochastic simulation models have also been implemented or are under development by a number of debt management offices in recent years in order to analyze the complex systems of debt strategy formulation and to evaluate alternative policy designs, i.e., different portfolio compositions. These tools, generally referred to as "Cost-at-Risk" or "Cash-Flow-at-Risk" models, allows comparison of alternative debt management strategies under various simulation scenarios. The performance analysis of different settings with respect to various policy objectives are then used to assist decision makers. In stochastic simulation models, cost and risk depend on the evolution of stochastic variables that are typically generated by a model, the parameters of which will be based on the historical paths of the variables.[5] In these models, hundreds or thousands of alternative scenarios for future market rates are generated, allowing the generation of statistical distributions. The relationships among financial variables can be maintained in simulated paths through the use of several analytical techniques. The key challenges in stochastic modeling include the complexity of the structure, the distributional assumptions of financial variables, and the stability of estimated relationships (Anderson, 2011).

Given their pros and cons, deterministic and stochastic techniques are not necessarily substitutes for each other, nor should simulation be interpreted as a required further step in debt strategy modeling. These approaches are similar in essence, with differences in structural forms, and both can provide useful input for decision makers. The choice between methods should be based on available resources, the degree of detail needed for analysis, and other country-specific circumstances. It is also possible to employ deterministic and stochastic models as complementary tools.

Other than these models, which are practically used in several debt offices, optimization approaches have also been proposed for PDM strategy formulation in recent years. Bolder and Rubin (2007) and Bolder and Deeley (2011) attempt to combine simulation and optimization approaches with the aim of approximating the debt management objective function through simulations by using function approximation algorithms and then to optimize this approximation. Balibek and Köksalan (2010) and Consiglio and Stanio (2010) suggest employment of stochastic programming models to formulate an issuance strategy. However,

[5] In some economies, the past may not always be a good predictor for the future, so that debt managers resort to alternative methods in generating scenarios.

to the best of our knowledge, optimization models have not yet been put into practical use due to challenges with regard to their complex structure, which impedes interpretation and maintenance.

2.2 Simulation Models for Public Debt Management

A detailed description of government debt simulation models is provided by Risbjerg and Holmlund (2005). The general structure contains inputs such as the initial debt portfolio, assumptions about future borrowing requirements and the strategy; a scenario generator that creates various paths for macroeconomic/financial variables; and a computation engine that calculates the issuance amounts and cash flows of the instruments. Debt service costs, associated risks and other key indicators are computed as outputs. The authors classify simulation models into several categories: in terms of the scope of the modelled variables and with regard to the modeling approach. Based on the variables included in the model, simulation models can be categorized in two classes. In one class of models, the analysts only represent the evolution of financial variables, mainly the interest rate, as endogenous, and treat macroeconomic variables such as the growth rate and/or primary budget balance as exogenous. In these applications, the model is run under various scenarios, generated for the interest rate, based on growth and primary balance projections provided externally.

Another approach is to develop a model of the economy in general, and model the co-movement of interest rates, the business cycle, the primary surplus, etc. This method enables the analysts to consider the relationship between financial and macroeconomic variables. The simulation model developed at the Swedish National Debt Office (Bergström et al. 2002), where the evolutions of the economic growth rate, inflation, the real exchange rate, and short- and long-term interest rates are represented, and the Canadian model (Bolder, 2002 and 2003 and Bolder and Deeley, 2011) in which the business cycle, the interest rate, and the government financial position are modelled together, are two advanced examples in this category. The United Kingdom Debt Management Office developed a simulation model (Pick and Anthony, 2006) that consists of a macroeconomic model (in which output gap, primary net financing requirement, inflation rates and short-term interest rate are modelled as inter-related equations), yield curve models, and a debt strategy component. In practice, there are relatively few examples of the use of macroeconomic models by debt management offices (Risbjerg and Holmlund, 2005), since maintaining and improving these models is quite challenging due to their highly complex nature.

On the other hand, the Danish model (Danmarks Nationalbank, 2005), which only considers the movement of interest rates, can be classified in the first group. Austrian and Portuguese models can be also classified in this category (Kocher and Nebenführ, 2005 and Granger, 2005). The Austrian model uses monthly updated projections made by the Ministry of Finance for the future path of primary budget balances and generates future financial market rates and prices through Monte Carlo simulation procedures. The simulation engine of the Portuguese Debt Management Agency computes the portfolio dynamics based on simulation paths for interest rates and a set of reference for macroeconomic scenarios.

The Brazilian model offers a combination of the two groups discussed here. In this model, the scenarios can be generated by two different methods: in the first, financial stochastic models are used for domestic and external interest rates, inflation and real exchange rates, while the second approach uses a macro-structural model to describe main macroeconomic variables (Silva et al., 2010).

As suggested by Risbjerg and Holmlund (2005), another possible classification can be made by examining the approach taken in modeling variables. In the first group of models, the relationship between variables is based on a statistical analysis of their correlations. The other approach is to represent the relationship by structural equations as in the case of the Swedish Model (Bergström et al. 2002).

The Turkish Treasury has been using a debt management strategy simulation model since 2003 to assist decision makers in formulating issuance policies (Turkish Treasury, 2004). As a result of approximately one year of research and development work by a dedicated Treasury team with the help of external consultants, not only was the conceptual framework of the model revised, but the model itself was also moved to a state-of-the art modeling platform Matlab (Turkish Treasury, 2008). The model in its current form provides a powerful and robust analytical framework for strategic analysis of alternative issuance decisions.

In the following sections, we elaborate on the conceptual framework and components of the Turkish Debt Simulation Model (TDSM).

3. Conceptual Framework of TDSM

Problem formulation and conceptual design are essential steps in modeling. The main rationales for developing the strategy model in the Turkish case are to assess the sensitivity of public debt to market movements and to quantify the costs and risks associated with alternative financing strategies, i.e., provide assistance in developing the strategic guidelines.

The definition of cost and risk measures is therefore of vital importance in addressing the intended objectives.

3.1 Cost and Risk Metrics

A clear definition of “cost” and “risk” measures is a precondition for developing a sound debt management strategy (the World Bank and IMF, 2009). In the context of PDM, the cost of debt measure can be defined in many different ways. In general, country-specific factors such as the risk profile of public debt, market conditions, and methods used in measuring and reporting debt are the main drivers determining the relevant cost metric.

Traditionally, interest cost has been the most common measure for cost of borrowing. In other words, cost of borrowing is reflected in the government’s budget in terms of interest expenditures. Governments that employ cash accounting standards record interest expenditures when payments are actually made, while countries that follow accrual accounting standards depict interest expenses as they accrue. Normalized cost figures such as share of nominal interest expenditures in budget items and/or GDP are also used as cost indicators by some debt management offices.

On the other hand, unlike many developed countries, most of the emerging countries issue debt in foreign currencies. Inflation-linked securities are also widely used by a significant number of debt management offices. Therefore, it could be misleading to analyze the cost of the total debt portfolio, including foreign currency bonds and/or inflation-linked bonds, by just taking interest payments into account. The changes in the value of foreign currency debt measured in local currency, due to fluctuations in the exchange rate also affect the cost of debt and the size of the outstanding debt. For inflation-linked bonds the accrued inflation on the nominal value of the bond is paid at maturity date, while interest payments prior to maturity date reflects inflation differential only on real coupon rate. This may result in lower interest payments for inflation-linked bonds compared to fixed coupon bonds for the period prior to maturity. Hence, cost of debt measure(s) should also properly capture the payment structure of inflation-linked debt.

The mark-to-market value of the debt portfolio is a cost indicator used by debt management offices that engage in frequent secondary market activities such as debt buy-backs or bond exchanges. However, for countries that usually redeem bonds at maturity, the marked-to-market value of debt stock is of little relevance. Also, since the market value of the debt does not provide information on cost and risk in relation to the budget, it is not often applied in strategy analysis.

In the Turkish case, until 2007, the “cost of debt” was measured by “interest costs in accrual basis plus changes in debt amortization (principal payments) due to exchange rate movements for foreign currency denominated (or linked) debt”. This enabled to assess the impact of the fluctuations in interest rates as well as exchange rates. However, this cost metric was measured solely for modeling purposes and it was not computed or monitored regularly for statistical or accounting reports. Hence, this created a problem for debt managers in evaluating the cost of the implemented financing strategy.

In order to address this issue and enhance communication with senior management, two different cost concepts were put into effect in 2007. Starting from 2010, a third cost metric has also been included following the increasing share of inflation-linked bonds in the debt stock. As of 2011, the following cost measures are computed:

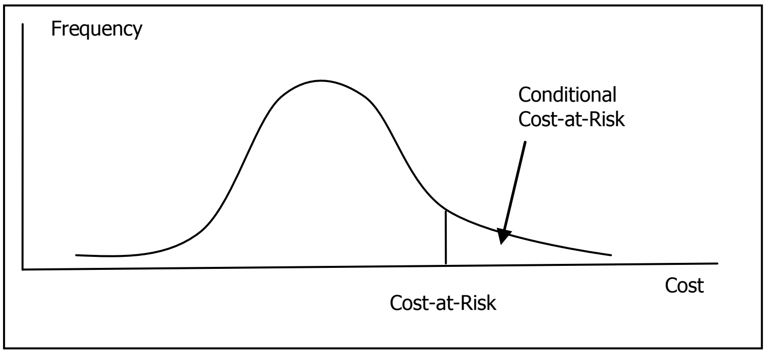
1. **Cash-Based Interest Expenditures:** This cost indicator reflects the interest expenditures in the government budget.
2. **Level of Debt Stock:** According to current national budgetary standards, the changes in the value of foreign currency debt measured in Turkish Lira (TRY), the local currency, are tracked under the principal payments item rather than interest payments. If cash-based interest expenditures were chosen as the sole cost metric, this would lead to an underestimation of the expected cost of a strategy that is based mostly on foreign currency debt. In order to assess the exchange rate risk associated with alternative strategies, a secondary cost indicator, “Level of Debt Stock”, measured in local currency, is also used as a cost measure.
3. **Accrued Inflation-Adjusted Level of Debt Stock:** In 2007, Turkish Treasury started to issue CPI-Indexed bonds, for the first time since the end of the 1990s, with a new design. These bonds became an integral part of Treasury’s financing strategy. CPI-Indexed bonds differ significantly with other instruments in terms of the structure of interest payments and interest rate volatility. While the real coupon payments of these instruments are made in coupon periods, all of the accrued inflation on the nominal value of the bonds is carried until the maturity date. Due to this structure, relying on the traditional cost metrics to compare CPI-Indexed bonds with other financing instruments is misleading. Therefore, in 2010 the Turkish Treasury introduced a new

cost indicator wherein accrued inflation compensation for inflation-indexed bonds is added to level of debt stock.

Given the trade-off between cost and risk of alternative financing instruments, choosing the appropriate risk metric is also very important. From the issuer's perspective, risk can be defined as the likelihood of an increase in the expected cost of the debt portfolio (debt service, interest expenditures, etc.) as a result of fluctuations in relevant variables.

In the Cost-at-Risk methodology of the Turkish Treasury, the upper 95^{th} percentile of the cost distribution was used as the risk indicator between 2003 and 2007. In spite of its extensive use in portfolio management and risk measurement, this approach has some shortcomings. It does not provide any additional information about the level of risk when the chosen confidence level is exceeded. For this reason, the Turkish Treasury adopted in 2007 a more sophisticated metric, namely the Conditional Cost-at-Risk or Tail Cost-at-Risk. The conditional Cost-at-Risk metric focuses on the average of the expected cost values that occur in the tail of the probability distribution rather than a single value in the confidence level determined (Figure 1).

Figure 1: Conditional Cost-at-Risk Approach



3.2 Decision Horizon and Granularity

The length of the decision perspective and the degree of granularity are important determinants of the level of complexity of any model. In Turkey, until 2007, a 3-year period based on monthly figures was chosen as the decision horizon in order to provide parallelism with the budget preparation process. Cost and risk indicators of alternative financing strategies were evaluated in cumulative basis.

In recent years, the average maturity of domestic issuances extended considerably, reaching almost four years as of end-2011 from a level of 9.4 months in 2002. Accordingly the reliance on short-term zero-coupon bonds was reduced with a shift towards fixed-coupon bonds, CPI-indexed bonds and floating rate notes in local currency issuance policies. Therefore, the decision horizon was extended to five years in order to analyze the cost-risk characteristics of these instruments more precisely.

In cost-risk modeling of public debt, many developed debt management offices use longer decision horizons to get a better picture of the medium- and long-term policy responses. Although, the flexible nature of the model enables us to easily extend the decision horizon, volatile historical data regarding macroeconomic and financial variables, mostly attributable to financial crises and regime changes before 2002, and the limited extent of future projections for fiscal data make simulations for horizons greater than 5 years difficult.

The granularity of the model was modified from monthly to quarterly in order to simplify cash-flow modeling. Although a monthly frequency to calculate cash flows is very useful for a highly detailed understanding of government's financing strategy, i.e., annual financing plan, from a longer-term strategic perspective, such a high degree of model granularity leads to a higher degree of complexity and potentially unwieldy calculations.

Typically, debt managers focus on annual cost and risk figures and combine annual observations with averages across years. With the adjusted cost and risk metrics, annual figures are taken into account, instead of using cumulative (or average) figures for the decision horizon. This prevents the possible information loss that may arise from the aggregation approach. Although it may be reasonable for debt management offices that have a long decision horizon to focus on averages, in the Turkish case the relatively short length of the decision horizon creates an incentive to analyze each year's expected debt dynamics separately in a more detailed manner. This helps identify possible concentrations in cost and repayment profiles in specific years.

3.3 Debt Structure Metrics

A main objective of cost-risk modeling is to provide guidance for the formulation of strategic benchmarks. Strategic benchmarks describe the policies designed to reach the desired debt structure and they are typically expressed as numerical targets for key portfolio indicators (IMF and the World Bank, 2001). In other words, strategic benchmarks are generally structured and communicated through the use of key portfolio indicators (debt structure metrics). Although the most common debt structure metrics are the average term-to-maturity, duration and share of maturing debt, countries use a variety of metrics based on their market structures and debt profiles. In addition, countries have different frequencies and bases (average value, end point or mixed) across metrics (OECD, 2010).

In Turkey's case, the strategic benchmarks, and thus the debt metrics, aimed at the composition of financing before 2007. Between 2003 and 2007, the one-to-one correspondence between issuances and the debt portfolio, due to relatively shorter maturity of borrowing, necessitated the management of cash flows rather than debt portfolio figures. In parallel to the extension of maturities, Turkey's strategic benchmarks now focus on the composition of the debt portfolio. The current method allows for a holistic approach to cover outright sales, buy-backs, debt exchanges and possible use of derivative instruments.[6] Therefore, the front offices should also consider the effects of debt exchange and buyback auctions on targets for key portfolio indicators when designing their auction schedules.

The evolution of debt structure metrics are summarized in the following section.

3.3.1 Debt Roll-over Metrics

The main metric used to track re-financing risk was the "average maturity of borrowing" until 2007. Although the strategies implemented in line with this metric proved to be useful in improving the re-financing risk profile of the Treasury, this metric does not take into account the redemption profile in each period and may thus lead to high redemptions in specific periods. The adjusted debt rollover metric, "portion of debt maturing within 12 months," takes into consideration the rollover risk in the short run. Furthermore, since this metric is employed from a multi-year perspective (for each year in the decision horizon) it also helps smooth the redemption profile. Several "maturity" related indicators (i.e., issuance maturities, remaining time to maturity of outstanding debt) are also monitored as complementary measures. Second metric for re-financing risk management is the "level of liquidity buffer." [7] Maintaining a certain level of liquidity buffer enables Treasury to reduce roll-over risk and surpass adverse effects of short-term interest rate volatility. In addition, it provides flexibility during auctions and improves market confidence.[8]

---

[6] Turkish Treasury resorts to debt exchange and buy-back operations, occasionally.

[7] TDSM is not used to determine the desired level of liquidity buffer. Instead, a comprehensive analysis of debt redemption profile, demand for Treasury auctions and availability of non-financing sources is used to set strategic benchmarks regarding the level of liquidity buffer. Then, these targets are used as inputs for the implementation of TDSM.

[8] It should be also noted that excessive or idle cash balances may be costly due to the cost of carry. Given the benefits associated with maintaining a certain level of liquidity buffer, this cost of carry can be considered as an equivalent of the insurance premium.

Table 1: Debt Roll-over Metrics for Liquidity Risk Management

Before 2007	After 2007
Average maturity of borrowing	Portion of debt maturing within 12 months
Level of liquidity buffer	Level of liquidity buffer

3.3.2 Interest Rate Risk Metrics

"Fixed-float share of borrowing" was the primary interest rate risk metric before 2007. However, managing public debt portfolio with respect to a specific fixed-floating rate target is quite challenging. This metric lacks information on interest rate sensitivity and maturity profile of financing instruments. For example, a 6-month Treasury bill and a floating rate note with 6-month coupon payment period are classified differently under this metric whereas both instruments have the same interest rate sensitivity, namely duration of six months at the time of issuance.

In 2007, the interest rate risk metric was modified to "average time to re-fixing" (ATR) in order to identify the exposure of different financing instruments to the changes in interest rates more precisely. Duration and ATR are highly popular among public debt managers due to their ease of use and interpretation (OECD, 2010). These metrics are useful to track the evolution of interest rate exposure over time and for cross country comparisons. However, duration and ATR of a debt portfolio provide information only about the average time at which changes in market interest rates will affect the debt service. Thus they present limited information about the dispersion of the debt stock's interest rate exposure over time and associated re-financing risk. Therefore, these metrics are typically used together with other supplemental risk indicators (Jensen and Risbjerg, 2005). For countries, including Turkey, that issue a varying range of instruments from floating rate notes to long-term fixed coupon bonds, using interest rate metrics that focus more on interest-rate risk dispersion and related re-financing risk is very important.

Therefore, "portion of debt with interest rate re-fixing period of less than 12 months" was chosen as the interest rate risk metric in 2008. In 2011, due to increasing share of CPI-indexed bonds in Treasury's debt portfolio and unconventional payment pattern of these instruments, this metric was re-organized into two sub metrics for a more comprehensive analysis of the interest rate risk exposure of the public debt portfolio: "share of CPI-indexed bonds in the TRY denominated debt stock" and "share of TRY debt stock with interest re-fixing period of less than 12 months, excluding CPI-indexed bonds." The re-classification of interest rate risk metrics provides for examining interest rate sensitivity of debt stock within three sub groups: fixed-rate instruments, floating-rate securities and CPI-indexed bonds, where fixed rate instruments are the ones with average time to re-fixing periods over 12 months. Since the multi-year perspective also applies to this measure, the concentration is not only on the immediate 12 months, but the projections for the following periods are also considered. The aim is to distribute the re-fixing periods evenly over the decision horizon.

Table 2: Interest Rate Risk Metrics

Before 2007	**2008-2010**	**2011 Onwards**
- Fixed-float ratio of borrowing, - Average time to interest re-fixing	- Portion of debt with interest rate re-fixing period of less than 12 months	- Share of CPI-indexed bonds in TRY- denominated debt stock - Share of TRY debt stock with interest re-fixing period of less than 12 months- excluding CPI-indexed bonds

3.3.3 Exchange Rate Risk Metrics

Issuance strategies that rely on foreign currency-denominated debt introduce another facet of market risk for countries. Although foreign currency debt may appear, ex ante, to be cheaper when compared to local currency debt of the same maturity, it could lead to additional costs if the exchange rate depreciates (IMF and the World Bank, 2001). Foreign currency-denominated or indexed debt may deteriorate public debt dynamics by increasing both debt repayments and debt stock levels in a floating exchange rate regime. This type of debt could also be very costly in a fixed exchange rate regime if the regime becomes untenable. Therefore countries that issue debt in foreign currency-denominated or indexed debt should give special attention to exchange rate risk. In addition, countries like Turkey that issue foreign currency debt in multiple currencies should also address risks arising from cross currency movements.

Prior to 2007, the Turkish Treasury used “roll-over ratio of foreign currency borrowing” and “currency composition of foreign currency borrowing” metrics to manage exchange rate risk. Since strategic benchmarks were re-designed for the composition of the debt portfolio after 2007, the exchange rate risk metrics for exchange rate and cross currency risks were modified accordingly for a more holistic approach (Table 3).

Table 3: Exchange Rate Risk Metrics

Before 2007	After 2007
Roll-over ratio of foreign currency borrowing	Currency composition of debt stock (local vs. foreign)
Currency composition of foreign currency borrowing	Currency composition of FX-debt stock

4. Technical Framework of TDSM

Based on the conceptual framework described above, the TDSM was built on the Matlab platform interacting with MS Access, Excel, and Word for data input and output. The TDSM operates within three main modules. The first of these modules, "Debt Stock Database," operates in MS Access, and consists of detailed records of outstanding bond issues. The "Scenario Generator" module produces scenarios for relevant financial variables in the Matlab environment. The "Cash-Flow Engine" module in Matlab subsequently calculates the borrowing requirement based on the payment profile of debt stock, the financial scenarios, and the assumptions about the primary surplus and other cash-flows. The calculated financing need is met using user supplied issuance strategies. The "Cash-Flow Engine" also calculates the overall cost and risk metrics for all strategies iteratively. The following sections elaborate on the technical aspects of TDSM.

4.1 Debt Stock Database

Information on the instruments of the current debt portfolio is held in this database. Each strategy simulation starts with the actual bond portfolio reading from the database, which contains detailed records of the bonds and bills issued by the Treasury in local and international markets.

The configuration of the database reflects the structure of Turkey's central government gross debt stock that has changed considerably over the last ten years. At the end of 2002, local currency debt represented only about 42% of total debt stock and consisted mainly of short-term zero coupon instruments, including treasury bills and floating rate notes. The foreign currency-denominated/indexed debt included bonds and bills issued in local markets, Eurobond issuances in international markets, and a substantial external loan portfolio (multilateral agency, bilateral lender, and other creditor debt). In line with the government's strategic goals, and developments in the Turkish economy, the risk profile of public debt has improved significantly, with increasing maturities in local markets and reduced reliance on foreign currency debt. The share of local currency debt is over 70% now, as of end-2011. Eighty-eight percent of total liabilities are in the form of securities or bonds issued in local and international markets, in contrast with 77% at the end of 2002. The remainder includes external loans.

Given the change in the portfolio structure, the debt database primarily accounts for securities. Each security is represented with an MS Access Database record with identifying attributes such as instrument type, currency, coupon rate etc. The database is updated at the end of each month by using designated queries—delete overdue bonds, add new issuances, and update coupon payment dates—in order to calculate key debt structure metrics.

At the start of the simulation procedure, the debt stock database is checked to ensure that it reflects recent issuances and redemptions, and adjusted if needed. This data is fed into the cash-flow engine module to calculate the debt redemptions arising from outstanding bond issues.

The cash-flow characteristics of the non-securitized loans are not standard and differ significantly from market instruments such as bond and bills. There are also considerable differences among the many loans with regard to their amortization and interest payment schemes. From a modeling perspective, and in terms of a cost-benefit analysis, the time and other resources required to emulate the cash-flows of the loan portfolio in detail would not be justified by their diminishing share in the government's debt stock. Therefore, a different approach is adopted and the cash flows of loans are computed outside the Debt Database and aggregated into clusters of two main foreign currencies: US Dollar (USD) and Euro. The loans denominated in other currencies are represented in the USD cluster using prevailing cross currency rates. The computed payments for each currency group are then fed to the "Cash-Flow Engine" as underlying assumptions.

4.2 Financial Scenario Generation

The cost and risk distributions of alternative strategies depend entirely on the scenario paths created for financial variables. As discussed in Section 2, the general approach in creating these scenarios is to use statistical or structural models. In either case, the scenario generators are calibrated by time series analysis, and therefore the process of creating financial scenarios, is heavily influenced by the quality of the historical data. In countries where the value of past data is questionable from a modeling perspective, one can consider adopting flexible approaches that allow using several models interchangeably, as model performance can vary depending on macroeconomic circumstances. In Turkey's case, the financial scenario generator is constructed as a stand-alone module, so that it is possible to shift conveniently between different approaches without the need to adapt the remaining parts of TDSM.

Financial crises, regime changes and the significant market volatility that Turkey faced before 2002 create stationarity problems in retrospective time-series analysis. Since the country entered a new economic policy environment after the beginning of the millennium, it is difficult for any econometric model, based on long-term historical data, to reasonably describe future financial outcomes. In order to address this problem, the starting point of the historical financial data in input modeling was restricted to the beginning of 2002. Additionally, a forward-looking simulation tool that allows for user calibration was constructed. Another possible solution could be to adopt a deterministic approach and work with user-provided scenarios.

The variety of instruments in multiple currencies poses an additional challenge in scenario generation. The module is extended to cover yield curves not only for local currency bonds, but also for USD- and Euro-denominated international bond issuances.

The financial scenario generator makes use of time-series analysis and imposes random shocks to generate correlated paths for financial variables such as interest and exchange rates. In this module, the main objective is to forecast and simulate variables that directly affect Treasury debt dynamics and thus produce relevant scenario paths.

The variables that are forecasted and simulated in the Cost-at-Risk framework are as follows:

1. Term structure of interest rates

    TRY yield curve

    USD yield curve

    Euro yield curve

2. Exchange rates

    TRY/USD

    TRY/Euro

3. Inflation rate (consumer price index)

A fixed and exogenous set of assumptions are used for other macroeconomic variables such as GDP growth and primary balance based on medium-term program[9] targets of the government.

4.2.1 Model Selection[10]

Short-term interest rates are modelled by using Cox Ingersoll and Ross (CIR, 1985) and Chan, Karolyi, Longstaff and Sanders (CKLS, 1992) models both with mean reversion properties. The risk neutral interest rate process via CIR model can be described as follows:

$$dr = \kappa(\theta - r)dt + \sigma_r r^{\gamma} dZ \qquad (1)$$

where

r: Interest rate level

κ : Speed of return to the average (Speed of Adjustment)

θ : Average long-term interest rate

σ_r: Interest rate volatility

dZ: Wiener process

γ : 0.5

The CKLS model is the generalized version of the CIR model. In other words, in the CKLS model the parameter γ also needs to be estimated whereas in the CIR model this parameter is restricted to 0.5.

CIR and CKLS models are mainly risk-neutral short-term interest rate models, and forecasting and simulating long-term interest rates by using solutions of these models may be misleading. Indeed, the solution of the CIR model proved to understate the term structure in the experiments performed. Particularly, the CIR model is successful in capturing short-term dynamics, but it performs relatively weakly for longer-terms. In order to address this issue, a hybrid model with both mean reversion and dynamic term-structure fitting features is used to estimate and forecast yield curves for TRY, USD and EUR interest rates.

[9] Medium-term program of Turkey shapes public policies and operations on the basis of strategic objectives and directs the resource allocation. Macroeconomic targets of the government are included in this program for the upcoming three years.

[10] The model parameters for short-term interest rates, exchange rates, and inflation rate are estimated by employing the Generalized Method of Moments (Hansen, 1982).

As for the term structure part of the hybrid model, the approach developed by Diebold and Li (2006) is used. The Diebold and Li model uses variations on Nelson-Siegel (1987) exponential components framework to model the term structure of interest rates as a three-dimensional design—level, slope and curvature—evolving dynamically. In the model, yield curves are fitted using the following functional form:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right) \tag{2}$$

where

t: time

τ : time to maturity

The parameter λ_t in the above equation governs the exponential decay rate; small values of λ_t produce slow decay and can fit the curve better at long maturities, while large values of λ_t produce fast decay and can better fit the curve at short maturities. β_{1t}, β_{2t} and β_{3t} are interpreted as three latent dynamic factors where β_{1t} governs the level of yield curve, β_{2t} governs its slope and β_{3t} is closely related to the yield curve curvature (Diebold and Li, 2006).

To implement the hybrid model, the parameters β_1, β_2 and β_3 are estimated first based on historical data, which constitutes the formation period. The second step involves forecasting. β_{2t} and β_{3t} parameters are fixed to β_2 and β_3 based on their estimated values from the formation period. β_{1t}, the variable related to the level, becomes the only varying parameter in generating forecasts and adjusted values for β_{1t} are calculated from the short-term interest rate level forecasts of CIR and CKLS models. The process for calculating adjusted β_{1t} can be described as follows:

$$\beta_{1t}^{'} = L_t - \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) - \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{3}$$

where

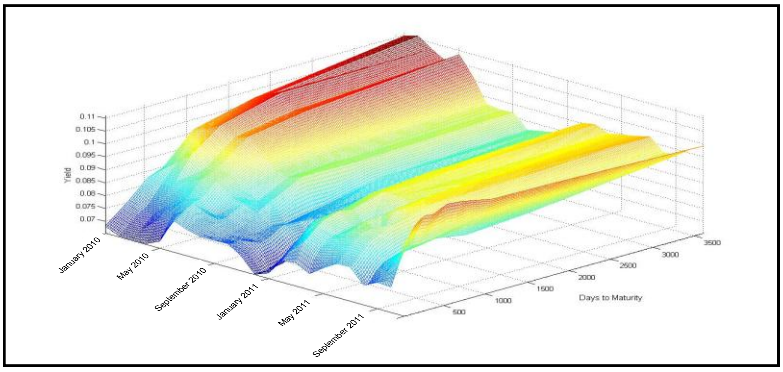
$\beta_{1t}^{'}$: Adjusted β_{1t}

L_t : Simulated forecast level for short term at period t

After $\beta_{1t}^{'}$ is calculated using equation (3), this value is plugged into equation (2) as the new β_{1t} so as to estimate interest rates for all maturity segments. An out-of-sample forecasting experiment displayed that combined hybrid model (CIR or CKLS with Diebold and Li) has superior fit particularly for the longer terms compared to the stand-alone application of CIR and CKLS models.

Dramatic changes in the slopes and curvatures of the term structure of interest rates (from upward sloping to inverted yield curves etc.) due to macroeconomic environment may be the dominant case for some bond markets across the globe. However, the upward sloping yield curves with generally parallel shifts is a more common case for the securities issued by the Turkish Treasury (Graph 1) and thus, level effect is a much more important determinant of the term structures.

Graph 1: Evolution of TRY Yield Curves



Therefore, the hybrid model applied in yield curve estimation is designed to address primarily the level effect. However, in this approach fixing the parameters governing slope and curvature to the estimated values based on the formation period restricts the variety of slope and curvature changes that may be witnessed in bond markets. For this reason, possible changes in the slope of term structure of interest rates are analyzed with a supplemental scenario called "yield curve shifting scenario."[11] This scenario is used within the simulation framework and a discussion on this issue is provided in the following part.

It is a common view in financial markets that the information useful for exchange rate forecasting tends to arrive randomly under a floating exchange rate regime and thus exchange

[11] The experiments performed suggested that the curvatures of the yield curves were quite stable and designing curvature scenarios from formation period had limited effect on yield curves. Hence, this scenario is structured to include only slope changes. However, algorithms can be easily adjusted to cover curvature shocks as well, if needed.

rates follow a random walk.[12] Geometric Brownian motion is a stochastic process based on random walk. In addition, the model specification is consistent with our multivariate simulation setting. The process of a geometric Brownian motion is given below:

$$dS = \mu S dt + \sigma_e S dZ \tag{4}$$

where

S: Exchange rate level

μ: Drift level

σ_e: Exchange rate volatility

dZ: Wiener process

It is assumed that the price index also follows a geometric Brownian motion. This is also a consistent approach within the context of multivariate simulation setting since price level has an important interaction with interest and exchange rates. When pricing inflation-linked debt, the estimations of the Consumer Price Index level from the geometric Brownian motion model is used together with the with real coupon rate estimations. Real coupon rates are estimated via a simple regression model based on the historical relationship between real returns and nominal returns of the comparable maturity fixed-coupon bonds. The estimates from the regression analysis are compared with the real return estimations calculated by using nominal interest rate and inflation rate estimations of the model for a consistency check.

The models used in the scenario generator were chosen among alternatives tested, including other continuous time interest rate models and vector autoregressive approaches. Performances of alternative models are compared on a regular basis. Should there be any structural changes in the macroeconomic environment, the models can be improved or replaced based on performance tests.

4.2.2 Simulation Framework

A key element in debt simulation is to ensure that risk factors (macroeconomic/financial variables) are correlated in each scenario. This requires creating correlated random numbers. For example, if the correlation between changes in exchange rates and price level is 0.8, one would expect that when the exchange rate rises, the price level will rise 80% of the time. The

[12] Meese and Rogoff (1983) showed in their seminal paper that main empirical exchange rate models have inferior out-of-sample forecasting ability compared to models such as a random walk. These results have been confirmed by a large number of studies.

focus here is to create random scenarios that would properly capture this relationship. Since there are more than two risk factors in this simulation framework, the correlation is created by decomposing the covariance matrix using Cholesky decomposition.

In the Cholesky decomposition method, a new upper triangular matrix, A, is found such that the transpose of A times A equals the covariance matrix of financial variables. Then, random numbers are drawn from a chosen distribution to create a random vector, Z. If A is multiplied by Z, a vector of random risk factors that are correlated according to the original covariance matrix is reached (Marrison, 2002). This vector is subsequently used to create simulation paths that are consistent with historical correlations between financial variables. In TDSM, the decomposition of the covariance matrix and the process of random number generation are handled using built-in functions in the modeling platform.

TDSM provides flexibility to produce scenario paths based on different approaches: normal distribution scenarios, fat tail scenarios, asymmetric shocks, yield curve shifting scenarios and bootstrapping. Normal distribution scenarios are the default case, in other words normally distributed errors form the benchmark stochastic environment. In addition to scenarios based on normal distribution, the stochastic environment is enriched to cover extreme cases with regard to the realizations of financial variables. In these alternative scenario generation methods, only the error term distributions are changed in the process of random number generation, while underlying stochastic model parameters remain as is.

The main elements of scenario generation options can be detailed as follows:

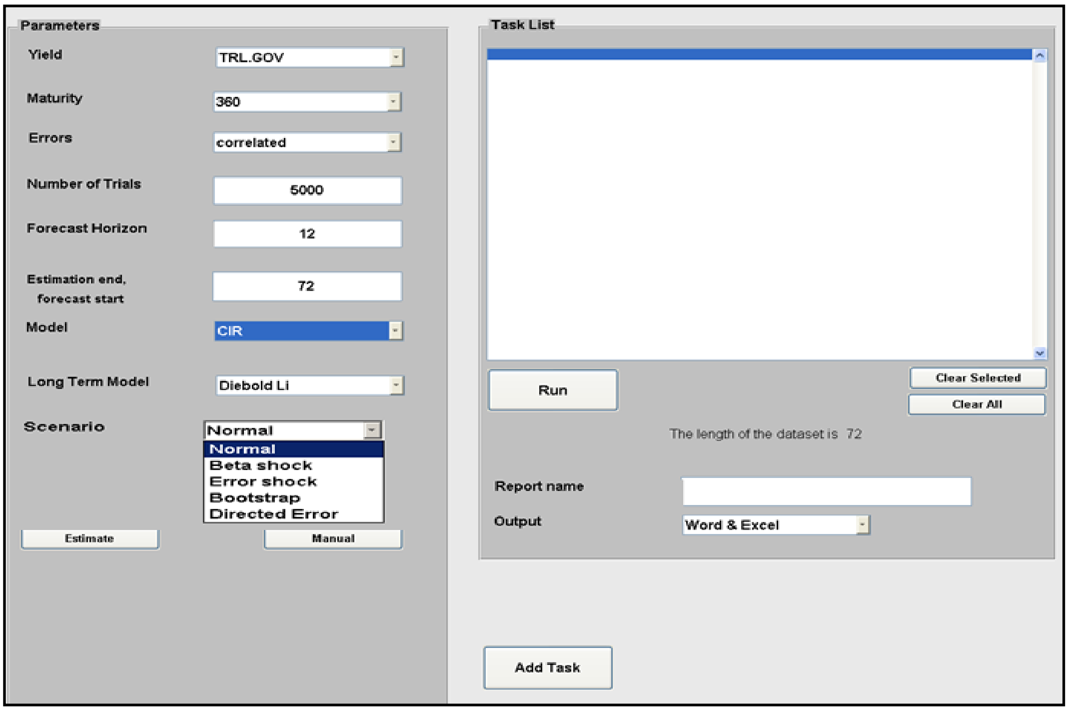
- Normal distribution scenarios: Produces simulation paths by using random shocks drawn from standard normal distribution (N (0,1)).
- Fat tail scenarios (error shock): Fat tails are represented via t-distribution. This uses a combination of random numbers from t-distribution with selected degrees of freedom and standard normal distribution and provides the user with the flexibility to choose an error shock based on future views. For example, if one chooses 10% as the shock scenario ratio and 100 as the number of simulation paths, then 10 of the scenarios are formed via random numbers from the t-distribution and the remaining 90 scenarios from standard normal distribution.
- Asymmetric Shocks (Directed Error): This scenario option can be considered as a subset of fat tail scenarios with one significant exception. In this case, the user can

select whether the random numbers from the t-distribution are generated from the right tail or left tail.

- Yield curve shifting scenarios (beta shock): As discussed in the previous section, this option is designed to cover potential slope changes of the yield curves. Selecting yield curve shifting scenario gives shocks to the slope of the term structure based on the historical minimum and maximum calculated in the formation period.
- Bootstrapping: This option follows a classical bootstrapping methodology for scenario generation in which historical changes are used.

The financial scenario generator is run through a graphical user interface (GUI) created on the Matlab platform. The GUI enables the user to select the variables to be simulated, the number of scenario paths, forecast horizon (in months) together with methods to be used for generating the random shocks. Relevant parameters, such as drift and volatility, are then produced for the selected variables, based on historical data. A print screen of this interface is presented below for illustrative purposes (Figure 2).

Figure 2: The Graphical User Interface for the Simulation Framework



An important feature of the scenario generation module is that it allows decision maker (DM) interference to the process. A DM may believe that historical data will not fully reflect the improvement (or deterioration) in the fundamentals of the country or the world economy over time. In this case, after the model parameters of the financial variables are computed internally, the DM can use the manual estimation option to produce financial scenarios based on his/her future views or current market based data, considering that estimations based on historical data are not “desired” reflectors of the future state of the economy. Forward looking scenarios can be generated based on medium-term program targets of the government, central bank inflation targets, or market expectations such as forward rates and implied option volatilities. For example, in exchange rate scenario generation, one can adjust the drift term in line with rate of increase implied by forward rates. Once the “estimated” or “user calibrated” parameters are determined for all financial variables, these are used to create final simulation paths.

The simulation algorithms then produce the summary statistics (mean, 70th percentile, etc.) regarding the distributions of financial variables. The DM can also interfere at this stage if the summary statistics of these simulated paths reveal that the estimations based on historical data deviates significantly from his/her market expectations. The DM could then calibrate the model parameters by “trial-error” method until he/she reaches the desired distributions.

Even though, the “manual” option provides additional flexibility, in the implementation of TDSM, scenario generation based on automatic model estimation is considered as the default case. In practice, no user calibration is performed without taking into account the scenario trees generated by using the selected financial models described in section 4.2.1.

Once the simulation paths are generated, they are exported to MS Excel platform[13] automatically, where monthly forecasts are converted to quarterly[14]—granularity of the model—data and then these quarterly paths are fed into the cash-flow engine module of TDSM in order to estimate cost and risk profiles of alternative debt strategies.

4.3 Cash-Flow Engine

The cash-flow engine is where computations are carried out with regard to the borrowing requirement, the bonds issued, the payments and finally the output metrics, given the scenario paths. The central feature of the engine is the debt-stock matrix. This is a matrix where each row represents a different financing instrument (i.e., Treasury bill, coupon bond, floating-rate note, etc.) and each column represents a different attribute of the corresponding financing

[13] Using MS Excel platform to transfer simulation paths to Matlab enables users to create their scenarios manually (deterministic scenarios) and feed them easily to the Cash Flow Engine.

[14] Although, TDSM performs mainly on quarterly granularity, financial scenario generator is based on monthly historical data and subsequently provides monthly forecasts. Since historical financial data was restricted to the beginning of 2002, this approach enables to extend the length of data set.

instrument. These attributes include the coupon rate, payment frequency, currency, issue date, and maturity date among others.

At the inception of each simulation and at the beginning of the simulation horizon, the debt-stock matrix provides a description of the initial debt stock. The first task performed by the model involves, in fact, the initialization of this debt-stock matrix using the actual debt stock from the debt stock database. The majority of subsequent operations in the model are performed by either updating the debt stock matrix or by using the information embedded in the matrix to perform an ancillary calculation. In summary, the debt stock matrix, by virtue of the fact that it represents a description of the entire debt stock at any given point of time, is the central element of the TDSM implementation.

The principal application of any debt strategy model is the computation of the cost-risk distributions for a number of alternative financing strategies over a pre-defined time horizon. In the model, this is accomplished by a nested loop. The loop has three layers. The first layer loops over the number of financing strategies. The second layer loops over the number of stochastically generated interest and exchange rate outcomes. The third and final layer loops over each of the time steps across the simulation horizon.

The bulk of the computation occurs at each quarterly time step. At each of these quarterly time steps, a sequence of stages is performed.

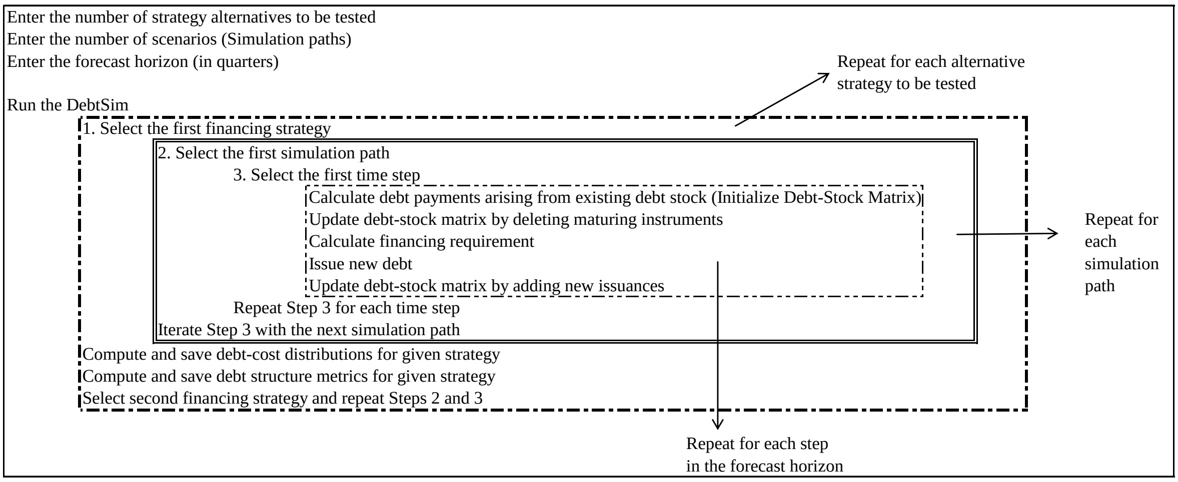
- The first stage in this sequence is the computation of cash flows, performed by screening the debt stock matrix using the attributes corresponding to the instruments, kept in each row.
- Consequently, the matrix is updated by deletion of maturing bonds/bills.
- The next stage is the computation of the gross funding requirement, which equals the net cash flow at that time step. The inflows are determined by the assumptions regarding the primary surplus, privatization revenues, etc. for each quarter. These quantities are fed into the model at inception and assumed to remain fixed across all scenarios and financing strategies. The outflows are given by redemptions calculated in the first stage plus some pre-fixed payments of the government, also fed during initialization.
- The fourth stage involves applying the financing strategy, represented as a vector of weights, to compute the amount of issuance in the current time step. TDSM uses a flow-based rule to determine the issuance and meet the gross funding requirement.

Each individual element of the vector corresponds to a bond or bill and denotes the proportion to be issued in that security to meet the funding requirement. The user chooses from a pre-defined set of bonds and decides on the ratio of each instrument in the incumbent strategy. The borrowing composition is fixed for each quarter of a given year, while it may change from year to year in the decision horizon.

- In the fifth stage, the debt-stock matrix is updated based on instruments issued.

The above quarterly approach is repeated for all time steps, scenarios and strategies. Finally, the expected cost and risk indicators of alternative strategies are generated as outputs. The analysis of the outputs is discussed in the following section. The general framework of the stochastic simulation algorithm and the summary of the previously discussed sequences are presented below (Figure 3)[15].

Figure 3: Stochastic Simulation Algorithm of TDSM



5. Experimental Framework: Runs and Analysis

Simulation models are developed to experiment with alternative policy designs. Once the coding process, i.e., representing the behavior of the system in a computer language is over, the analysts' next task is to develop an experimental framework that includes several steps from checking the accuracy of the system representation to designing policy inputs, running the model and analyzing the outputs.

Similar to other simulation analyses, experimenting with TDSM involves the following steps:

[15] Figure 3 is adopted from "Stochastic Simulation Algorithm" in Bolder and Deeley (2011, p.7) due to similarities of simulations frameworks.

5.1 Verification and Validation

After completing the coding, the first stage is to ensure that the computer program performs properly. This “verification” process includes debugging and ensuring that the code produces correct computations. The verification of TDSM involves checking the cash-flow calculations, i.e., interest and principal payments for different types of bonds and ensuring that the borrowing requirement and the issuance amounts are correctly computed in the model given a certain scenario and a borrowing strategy. This process is crucial after the construction of the model and should be repeated following modifications such as adding new instruments to the model.

The “Validation” stage is about testing the consistency of the model and determining that the model is a correct representation of the real system as far as the modellers’ purposes are concerned. The TDSM is validated by analyzing the financial scenarios generated to ensure that they are justifiable given the current macroeconomic outlook and future possibilities. At this stage, one can rely on expert judgement regarding economic fundamentals and/or market based forecasts with regard to the span of the scenario tree. For example, one should not expect to have negative inflation rates in an environment of depreciating local currency if the price level is heavily influenced by prices of imported goods. In such a case, the model specification should be re-visited.

The cost and risk metrics are also analyzed, depending on the decision makers’ choices and characteristics of the instruments added to the model. These two steps together make sure that the right model is built rightly.

5.2 Experimental Design

Experimental design involves determining the policy alternatives to be tested in the model. Risk managers in charge of model building are responsible for determining which strategy alternatives are worth considering and whether the alternatives are applicable given the market constraints. At this stage, coordination with the front-office may also be required to be able to incorporate their views about market conditions.

In designing strategy alternatives, maximum issuance limits are set for some particular borrowing instruments based on historical and expected market demand, i.e., constraints are imposed. An issuance floor is set for some securities across the yield curve in line with the objective of developing and maintaining an efficient and liquid market for government securities. In addition, if the market appetite for a type of security is expected to increase in the future, this is reflected through the increasing issuance amounts over time, within the decision horizon.

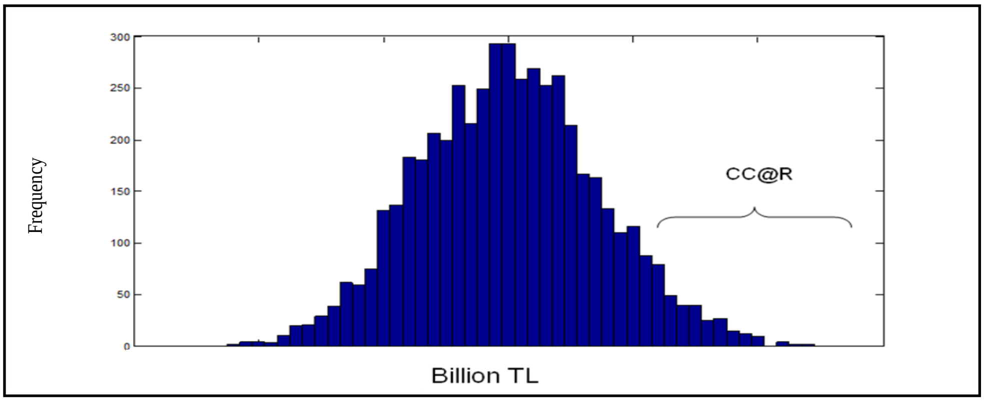
The strategy alternatives are chosen to ensure that each alternative represents a specific policy direction (with a story behind it) that can be pursued by DMs. Some of the alternatives tested previously include strategies with regard to “cost minimization,” “moderate risk reduction,” “currency risk reduction,” “ambitious risk reduction,” and “maintain current profile.” Alternatives based solely on a single borrowing instrument are also tested to identify the different cost and risk characteristics of the securities (for example, 100% CPI-indexed bonds vs. 100% floating-rate notes). The strategy alternatives are designed by plugging in the appropriate weights in the financing strategy vector, described in section 4.3. For example, weights of local currency instruments are increased in the vector to develop a “currency risk reduction” strategy.

5.3 Runs and Analysis of Results

Following the creation of simulation paths and selection of alternative strategies to be tested, TDSM then calculates the cash flows associated with each strategy. This module also compiles summary statistics (mean, conditional-at-risk values for different percentiles, etc.) of the distributions for these strategies with respect to the measures discussed in Section 3.1. The future levels of debt structure metrics discussed in Section 3.3 is calculated for each strategy tested. As the model is capable of producing interest and principal payments and future debt stock levels for each strategy under each simulation path, other cost and risk indicators can easily be calculated.

Each alternative strategy following a different policy direction is then numerically presented in terms of the debt structure metrics. For example, the share of local debt will be higher for a “currency risk reduction” strategy compared to a “maintain current profile” strategy. Debt structure metrics for alternative strategies is then reported to senior management together with the expected cost and risk levels of these strategies for each year in the decision horizon. An illustrative result for the distribution of cash-based interest costs under 5,000 simulation paths for a strategy tested is presented in Graph 2.

Graph 2: Distribution of Interest Costs



Since the first cost metric is based on cash-based interest payments and the other two metrics are based on debt stock levels, different measures may give different rankings of the strategies. In particular, alternative strategies assuming a higher share of foreign currency-denominated and/or inflation indexed debt in the future may provide the best rankings in terms of cash-based interest cost, while they may rank differently with respect to debt stock related measures. Therefore, a comprehensive analysis on rankings of different strategies is provided for strategy formulation process. In many multiple criteria cases, there is not a single choice that is best for all criteria. Usually, one needs to give up in some criteria in order to improve in others. In general, if we have more than one decision criteria, a solution (a strategy in our case) is said to be "inefficient" or "dominated" if there exists another solution that is equally good in all objectives, and better in at least one measure. Some of the strategies tested in the simulation model may be dominated by other strategies, and these are then excluded from the analysis. The DMs then evaluate the efficient solutions and decide on one strategy depending on their preferences. The risk management team also presents its strategy recommendation with underlying rationales to DMs in order to assist the decision-making process.

As a complement to the primary outputs of TDSM, analyses with regard to market demand for government auctions and different securities, volatility and covariance profile of exchange rates and redemption profile for upcoming years are also reported to DMs in order to enhance the decision-making process in strategy formulation. This can be supplemented with a stress-testing component that aims at quantifying and comparing the exposure of public debt dynamics to different types of shocks such as a depreciation of the exchange rate.

These outputs and analyses are not the sole criteria in making the final decision about the debt management strategy. Assessments about the macroeconomic environment, the market outlook, the considerations of front offices and other restrictions or objectives arising from the economic policy framework are also key factors in strategy selection.

6. Concluding Remarks

The paper addresses two main objectives. The first is to share Turkey's experiences on the process of developing a model to support debt management strategy decisions. Model development for debt managers is especially challenging since the academic literature on this field and material publicly shared by debt management offices is limited. That said, we learned much from the examples of countries that were kind enough to disclose the details of their models, and we hope this paper will provide some insight for debt offices that are considering, or in the process of building, their own analytical tools. For this reason, we have described not just the technicalities, but also the conceptual stages of the model's development, which we consider to be crucial in model building. In this sense, this paper also provides an opportunity to compare the methodology used in Turkey to the approaches followed by other debt management offices (mainstream applications). The Turkish Treasury's model can be classified in the class of debt management strategy models in which only financial variables are stochastically simulated and macroeconomic variables are treated as exogenous. The approach used for modeling the relationship between the financial variables is based on the statistical analysis of their behavior and correlations.

The second objective in introducing the model is to invite comments on its quantitative structure to gain insights for further progress. As debt management offices operate in a dynamic environment, their models also need to cope with altering environmental factors. Therefore, no model can be used indefinitely with a static form. The TDSM has been in use in its current structure since 2007, with some modifications required by the debut of new instruments, such as inflation-linkers. The introduction of new instruments not only necessitates an adjustment of cash-flow calculations in the engine, but may also necessitate a reassessment of the cost and risk indicators, as in the case of Turkey's cost metric.

The TDSM is actively used in debt management strategy formulation and decision-making process. The model proved to be quite robust even during the global financial crises. The scenarios generated were able to cover extreme events, and hence, the tail risk. Nevertheless, we believe there is room for modifications and improvement, particularly in scenario generation and future work should focus in this area.

References

Anderson P, 2011, “Optimal Uses of Risk Management Techniques”, Presentation at the Eighth UNCTAD Debt Management Conference, <http://r0.unctad.org/dmfas/docs/DMconf2011/Anderson_WB.pdf> (retrieved on December 17, 2011).

Balibek E and Köksalan M, 2010, “A Multi-Objective Multi-Period Stochastic Programming Model for Public Debt Management”, European Journal of Operational Research 205: 205-217.

Banks J and Carson SC, 1984, Discrete Event System Simulation, Prentice Hall: Englewood Cliffs, New Jersey.

Bergström PB, Holmlund A and Lindberg S, 2002, “The SNDO’s Simulation Model for Government Debt Analysis”, Swedish National Debt Office (SNDO), <http://www.riksgalden.se> (retrieved on March 08, 2011).

Bolder DJ, 2002, “Towards a More Complete Debt Strategy Simulation Framework”, Bank of Canada, Working Paper 2002-13.

Bolder DJ, 2003, “A Stochastic Simulation Framework for the Government of Canada’s Debt Strategy”, Bank of Canada, Working Paper 2003-10.

Bolder DJ and Rubin T, 2007, “Optimization in a Simulation Setting: Use of Function Approximation in Debt Strategy Analysis”, Bank of Canada, Working Paper 2007-13.

Bolder DJ and Deeley S, 2011, “The Canadian Debt-Strategy Model: An Overview of the Principal Elements”, Bank of Canada, Working Paper 2011-3.

Cox JC, Ingersoll JE and Ross SA, 1985, “A Theory of Term Structure of Interest Rate”, Econometrica 53: 385-407.

Chan KC, Karolyi GA, Longstaff FA and Sanders AB, 1992, “An Empirical Comparison of Alternative Models of the Short-Term Interest Rate”, Journal of Finance 47: 1209-1227.

Consiglio A and Stanio A, 2010, “A Stochastic Programming Model for the Optimal Issuance of Government Bonds”, Annals of Operations Research,doi:10.1007/s10479-010-0755-5

Danmarks Nationalbank, 2005, Danish Government Borrowing and Debt, <http://www.nationalbanken.dk> (retrieved on January 29, 2011).

Diebold FX and Li C, 2006, "Forecasting the Term Structure of Government Bond Yields", Journal of Econometrics 130: 337-364.

Granger R, 2005, "Risk Management of Government Debt in Portugal", In: Advances in Risk Management of Public Debt, Vol.6. OECD Publishing: Paris.

Hansen LP, 1982, "Large Sample Properties of Generalized Method of Moments Estimators", Econometrica 50: 1029-1054.

International Monetary Fund and The World Bank, 2001, Guidelines for Public Debt Management, International Monetary Fund Publication Services: Washington, D.C.

International Monetary Fund and The World Bank, 2003, Guidelines for Public Debt Management-Accompanying Document and Selected Case Studies, International Monetary Fund Publication Services: Washington, D.C.

Jensen OS and Risbjerg L, 2005, "Recent Developments in the Management of Market Risk", In: Advances in Risk Management of Public Debt, Vol.6. OECD Publishing: Paris.

Kocher PA and Nebenführ G, 2005, "Risk Management of Government Debt in Austria", In: Advances in Risk Management of Public Debt, Vol.6. OECD Publishing: Paris.

Marrison C, 2002, The Fundamentals of Risk Measurement, McGraw-Hill.

Meese RA and Rogoff K, 1983, "Empirical Exchange Rate Models of Seventies: Do They Fit Out of Sample?", Journal of International Economics 14: 3-24.

Mitchell G, 1993, The Practice of Operational Research, John Wiley and Sons: Chichester, West Sussex.

Nelson CR and Siegel AF, 1987, "Parsimonious Modelling of Yield Curves", Journal of Business 60: 473-489.

Organization for Economic Cooperation and Development (OECD), 2010, Debt Structure and Risk Metrics – OECD Questionnaire on Debt Portfolio Management.

Organization for Economic Cooperation and Development (OECD), 2005, Advances in Risk Management of Government Debt, Vol.6. OECD Publishing: Paris.

Pick A and Anthony M, 2006, "A Simulation Model for the Analysis of the UK's Sovereign Debt Strategy", UK Debt Management Office Research Paper.

Risbjerg L and Holmlund A, 2005, "Analytical Framework for Debt and Risk Management", In: Advances in Risk Management of Public Debt, Vol.6. OECD Publishing: Paris.

Silva AC, Carvalho LO and Medeiros OL, 2010, Public Debt: The Brazilian Experience, The World Bank and National Treasury of Brazil, Brasilia.

The World Bank and International Monetary Fund, 2009, Developing a Medium-Term Debt Management Strategy (MTDS)-Guidance Note for Country Authorities, Washington, D.C.

Turkish Treasury, 2004, Public Debt Management Report, February 2004, <www.treasury.gov.tr> (retrieved on February 14, 2011).

Turkish Treasury, 2008, Annual Public Debt Management Report 2008, (In Turkish), <www.treasury.gov.tr> (retrieved on February 17, 2011).

Velandia A, 2002, "A Risk Quantification Model for Public Debt Management", The World Bank Working Paper 2002-45233.