Substitution and Technological Change under Carbon Cap and Trade

Lessons from Europe

Timothy J. Considine
Donald F. Larson

The World Bank
Development Research Group
Sustainable Rural and Urban Development Team
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Abstract

The use of carbon-intense fuels by the power sector contributes significantly to the greenhouse gas emissions of most countries. For this reason, the sector is often key to initial efforts to regulate emissions. But how long does it take before new regulatory incentives result in a switch to less carbon intense fuels? This study examines fuel switching in electricity production following the introduction of the European Union’s Emissions Trading System, a cap-and-trade regulatory framework for greenhouse gas emissions. The empirical analysis examines the demand for carbon permits, carbon based fuels, and carbon-free energy for 12 European countries using monthly data on fuel use, prices, and electricity generation. A short-run restricted cost function is estimated in which carbon permits, high-carbon fuels, and low-carbon fuels are variable inputs, conditional on quasi-fixed carbon-free energy production from nuclear, hydro, and renewable energy capacity. The results indicate that prices for permits and fuels affect the composition of inputs in a statistically significant way. Even so, the analysis suggests that the industry’s fuel-switching capabilities are limited in the short run as is the scope for introducing new technologies. This is because of the dominant role that past irreversible investments play in determining power-generating capacity. Moreover, the results suggest that, because the capacity for fuel substitution is limited, the impact of carbon emission limits on electricity prices can be significant if fuel prices increase together with carbon permit prices. The estimates suggest that for every 10 percent rise in carbon and fuel prices, the marginal cost of electric power generation increases by 8 percent in the short run. The European experience points to the importance of starting early down a low-carbon path and of policies that introduce flexibility in how emission reductions are achieved.

This paper—a product of the Sustainable Rural and Urban Development Team, Development Research Group—is part of a larger effort in the department to understand how climate change policies affect energy markets. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at DLarson@worldbank.org.
SUBSTITUTION AND TECHNOLOGICAL CHANGE UNDER CARBON CAP AND TRADE: LESSONS FROM EUROPE

TIMOTHY J. CONSIDINE AND DONALD F. LARSON

Timothy Considine is an SER Professor of Economics in the Department of Economics and Finance, University of Wyoming. Donald F. Larson is a Senior Economist in the World Bank’s Research Group. The author’s gratefully acknowledge support from the World Bank’s Knowledge for Change Trust Fund and the School of Energy Resources at the University of Wyoming and research assistance from Muhammed Hassan. Comments and suggestions from brown bag participants from the Economics and Finance Department at the University of Wyoming are appreciated. Any errors or omissions are the responsibility of the authors.
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1. Introduction

The European Union (EU) has pioneered the development of a carbon dioxide (CO2) emissions trading program, known as the Emissions Trading System (ETS). Operating since early 2005, the program mandates an overall limit or cap on carbon emissions that originate from large industrial facilities and electric power generating plants and allows the trading of emission permits under the cap. By allocating a supply of permits and creating a regulatory demand for CO2, the EU ETS creates a market for disposing carbon dioxide emissions in the atmosphere. As a consequence, markets have developed that price CO2 emissions.

Under the legal and regulatory framework established by the EU ETS, producers of carbon intensive goods and services covered by the program must consider emissions in their production decisions, weighing the costs of purchasing permits with the benefits of selling excess permits that are created by using less carbon-intensive inputs or by investing in less carbon-intensive technologies. The objective of this study is to understand this process and the nature of short-run relationships among permit use, input substitution and technological change under carbon cap and trade.

In pursuit of this goal, the study specifies and estimates an econometric model of fuel substitution in electric power production in Europe. According to Ellerman and Buchner (2007), the electric power sector accounts for 60 percent of carbon emissions in the EU and constitutes 90 percent of the potential demand and 50 percent of total supply of carbon allowances.

Electric power producers have a variety of options to reduce their carbon emissions. In the short-run, they can shift their mix of generation, raising their utilization of carbon-free capacity, such
as nuclear power, shifting to lower carbon sources, such as natural gas fired generation, and reducing their use of high carbon sources, such as coal and oil-fired generation. This fuel switching is limited by installed capacity. Longer term, electricity producers can invest in new capacity, such as advanced nuclear plants, coal with carbon capture and sequestration systems, or renewable sources, including wind, solar, biomass, and geothermal capacity.

Dynamically, decisions to invest in new capacity will be influenced in part by the ability of the existing fleet of generating plants to adapt to carbon emission constraints. As carbon emission limits become more stringent, the ability of electric power producers to adjust becomes more difficult and the marginal cost of electricity rises, inducing new investment in carbon-free sources of electricity. How readily electricity producers respond to price signals remains a key question in estimating the costs of carbon emission controls.

The model presented below is designed to estimate this short-run adaptability arising from input substitution and technological change in the electric power sector. The framework uses a restricted cost function in which electricity producers minimize the variable costs of production including inputs of coal, natural gas, petroleum, and carbon allowances subject to inputs of carbon-free energy resources, including nuclear and renewable resources. These last two energy resources are treated as quasi-fixed inputs because data on prices for carbon-free energy resources are unavailable. This is the same problem faced by Halvorsen and Smith (1986) in their analysis of substitution possibilities for internally produced and un-priced ore inputs in metal mining and has implications for how we interpret our results. We return to this topic later in the paper.

Unlike the Halversen and Smith study, which used a translog (TL) function, the model presented below is based upon the Generalized Leontief (GL) restricted cost function developed by Morrison (1988). Caves and Christenson (1980) show that the GL outperforms the TL when technology has limited substitution, which is likely in electric power generation. Morrison also shows
that the GL allows closed form solutions for equilibrium levels of quasi-fixed inputs, which facilitates computation of substitution elasticities and their standard errors.

The model is estimated using a panel of monthly time series observations from January 2005 through March of 2008 for a cross section of twelve European countries. The relatively large number of observations and considerable variation in the data allows the estimation of variable returns to scale and input biases from technological change. For our purposes, an important advantage is that the model can be used to test whether the carbon cap-and-trade regime induced carbon saving technological innovation. Moreover, the study provides explicit measures of the degree of carbon abatement, such as carbon emissions per unit of electricity output, under the EU ETS and, most importantly, explains how this abatement was achieved.

Specifically, the degree and nature of fuel switching induced by carbon pricing and relative fuel prices is estimated. This empirical assessment of carbon substitution possibilities sheds light on whether carbon pricing significantly increases the demand for less carbon intensive fuels, such as natural gas, at the expense of carbon intensive fuels, such as coal. These substitution possibilities ultimately determine whether the demand for carbon is price inelastic, which would imply significant adjustment costs to a low carbon society, or whether carbon demand is elastic, facilitating a less costly path to achieving significant reductions in carbon emissions.

The next section provides some additional background on the European program. Section 3 presents the economic framework, discussing the theoretical underpinnings for the empirical model. The parametric specification of the econometric model is then presented in the fourth section along with a discussion of the estimation techniques. Section 5 provides an overview of the data sample, including descriptive statistics by country on electricity generation by type, net imports, total generation, and the composition of so-called combustible fuels, including natural gas, coal, and petroleum. Trends in the carbon intensity of electricity and in fuel shares in the sample are also identified and discussed. The sixth section of the paper discusses the econometric results and the
implications for assessing substitution and technological innovation under carbon cap and trade. The final section summarizes the major findings and discusses the policy implications of the results.

2. EU Emissions Trading Scheme

In 2003, the European Council and Parliament approved legislation that eventually launched the EU ETS in 2005. The ETS is a cap-and-trade program that limits carbon dioxide emissions from more than 10,000 installations located in the thirty member states of the European Economic Area.¹ Currently, the sectors covered by the program include energy activities (e.g. electric-power generation greater than 20 megawatts), ferrous metals industries (iron and steel), mineral industries (cement, glass, ceramics, oil refineries, etc.), and pulp and paper industries.² The program is considered a key element in the European Union’s plan to meet its commitment under the Kyoto Protocol to reduce greenhouse gas emissions by 6 percent compared to 1990 levels by the end of 2012.

Our study period, January 2005 to March 2008, covers two phases of the program. Practical implementation of the program meant establishing an extensive system of procedures for allocating allowances, for monitoring how they are used, and for matching allowances with measured emissions. For this reason, Phase 1 (2005-2007) of the cap-and-trade program was intended in part as an opportunity to work out operational difficulties in advance of Phase 2 of the program, which corresponds to the first round of commitments (2008-2012) under the Kyoto Protocol.

As discussed, under the program regulated installations are issued permits, called EU allowances (EUAs), equivalent to one ton of emitted carbon dioxide. The allocations are made in accordance with National Allocation Plan (NAP), drawn-up by individual Member States. At the end of each

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¹ The area includes the EU’s 27 member states, plus Iceland, Liechtenstein and. Norway
² For more information on the EU ETS see Watanabe and Robinson (2005), Convery and Redmond (2007) and Europa (2007).
year, regulated installations must surrender allowances equivalent to their emissions. Surplus and short-falls can be matched through sales and purchases. The allowances are tracked in national registries that were linked to form a system-wide registry during the program’s second phase.

Though restrictions apply, the system is open to other tradable units established under the Kyoto Protocol, including Certified Emission Reductions (CERs) from developing countries. This is significant, since it potentially links the two types of carbon offsets into a large and liquid market, making the findings of this study relevant for developing countries. Legislation known as the “Linking Directive” lays out the relationship between EUAs and the Kyoto-system tradable units.3

3. The Economic Model

The output of electricity depends upon inputs of labor and maintenance, capital service flows from generating equipment and structures, and primary fuels. In addition, under the EU ETS producers of electricity are required to obtain pollution permit allowances to offset their emissions of carbon dioxide. Hence, the disposal of the carbon dioxide by-products of electricity generation now becomes a factor of production. These observations imply the following production function for electricity:

\[ Y_t = f(K_t, L_t, E_t, C_t) \]  

where \( Y_t \) is output of electricity in period \( t \), \( K_t \) is capital service flows, \( L_t \) is salaried and hourly worker services, maintenance, and non-fuel supplies, \( E_t \) is an aggregate of energy inputs, and \( C_t \) is carbon emissions.

Assuming capital and labor are fixed in the short run, under duality theory the following long-run cost function exists:

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3 For an early assessment of the ETS, see the volume edited by Ellerman, Buchner and Carraro (2007). For more on carbon markets in general, see Larson et al. (2009).
\[ TC_i = C(w_{et}, w_{ct}, K_t, L_t, Y_t) + \mu_{kt} K_t + \mu_{lt} L_t \]  \hspace{1cm} (2)

where \( w_{et} \) and \( w_{ct} \) are prices for energy and carbon respectively and \( \mu_{kt} \) and \( \mu_{lt} \) are the user costs corresponding with stocks of labor and capital stocks. Prices for carbon emission allowances represent the societal valuation of the impacts of carbon emissions on common property atmospheric resources implicit in the target level of allowable emissions and the corresponding allocation of permits. This specification is similar to the study conducted by Considine and Larson (2006) of sulfur dioxide pollution allowances.

For the empirical analysis below, \( K_t \) and \( L_t \) are unobservable. To specify an empirical model, therefore, requires assuming the existence of a weakly separable sub-aggregate of energy and carbon emissions within the variable cost function. In particular, the weakly separable model implies that substitution possibilities between fuels and carbon emissions are independent of substitution possibilities between labor and capital, which is likely a reasonable assumption within a short-run context. The cost minimization problem, therefore, is to minimize energy and carbon emission allowance costs subject to output levels.

In the context of the model, there are three types of energy aggregates: i) primary fossil fuels, including coal, petroleum, and natural gas; ii) nuclear fuel and hydroelectric energy; and iii) renewable energy resources, including wind, solar, and geothermal energy. As discussed, while market prices for primary fuels are observable, those for nuclear, hydroelectric, and renewable energy are not. To accommodate this, the study assumes the existence of a weakly separable sub-aggregate for primary energy and carbon emissions contingent upon levels of nuclear, hydroelectric, and renewable energy generation, levels of output and the state of technology. More specifically, this implies the following short-run restricted energy and emission allowance cost function:

\[ G_i = G(w_{et}, w_{ct}, w_{st} | N_i, R_i, Y_i, Z_i) \]  \hspace{1cm} (3)
where $Z_t$ is an index of technological change, $w_{1t}$ is the price for carbon emission allowances; $w_{2t}$ is the price for solid and liquids fuels, such as coal and fuel oil with relatively high carbon content; $w_{3t}$ is price of natural gas with relatively low carbon content; $N_t$ is the consumption of nuclear and hydroelectric energy, which is carbon-free with low operating costs; and $R_t$ is renewable energy resource use, which are also carbon-free but associated with relatively higher operating costs.

Two sets of substitution possibilities are recovered from this model. The first set provides estimates of first-order substitution possibilities among primary fuels and carbon allowances when levels of carbon-free energy are held fixed. The second set of substitution possibilities are recovered from the convexity conditions, $\frac{\partial G}{\partial N_t}, \frac{\partial G}{\partial R_t}$, and estimate rates of substitution among carbon-intensive primary fuels, carbon emissions, and carbon-free energy sources.

An engineering perspective on this model can be attained by noting that the consumption of a primary fuel at a specific plant is equal to the heat rate, which is defined as the amount of fuel consumed per unit of electricity, multiplied by the level of power generation from that facility. So from this perspective, the short-run restricted variable cost function specified above in equation (3) can be viewed as a model that selects the least cost mix of plant capacity operating in any time period. This model is consistent with least cost scheduling algorithms commonly employed by electricity companies and system operators.

4. Econometric model

The Generalized Leontief (GL) function developed by Morrison (1988) is best suited for this particular problem because Caves and Christensen (1980) found it more likely to maintain cost minimizing curvature conditions under limited input substitution possibilities, which is a reasonable prior assumption for electric power generation. Another important reason is that the GL provides a closed-form solution for stocks of quasi-fixed factors, which facilitates computation of long-run elasticities. For this study, the GL takes the following form:
where the $\alpha's$, $\delta's$, and $\gamma's$ are unknown parameters, the $\varepsilon's$, $\nu's$, and $\mu's$ are stochastic errors. The empirical model includes three variable inputs: high carbon fuel, $x_u$, low-carbon fuel, $x_v$, and carbon emissions, $x_y$. The restricted cost function is symmetric and homogeneous of degree zero in prices. The asymmetric way that output and technological change enter the cost function facilitates parametric testing of long-run constant returns to scale.

The input demand functions for high carbon fossil fuels (petroleum and coal), low carbon fuels (natural gas), and carbon are equal to the derivative of (4) with respect to factor prices. These expressions are as follows:

\begin{equation}
X_i = \left[ \sum_{c=1}^{12} \delta_c D_{ct} + Y_t \left( \sum_{j=1}^{3} \alpha_{ij} \left( \frac{w_j}{w_i} \right)^{1/2} + \sum_{i=1}^{3} \delta_{ij} Y_{i}^{1/2} + \gamma_{yi} Y_{i}^{1/2} + \gamma_{yy} Y_{i}^{1/2} + \gamma_{yz} Y_{i}^{1/2} + \gamma_{zz} Z_{i}^{1/2} \right) \right] + \delta_{ni} Y_{i}^{1/2} + \gamma_{ny} Y_{i}^{1/2} + \gamma_{nz} Y_{i}^{1/2} + \gamma_{nz} Z_{i}^{1/2} + \gamma_{rz} Z_{i}^{1/2} + \gamma_{rr} R_{i}^{1/2} + \forall i \end{equation}

4 The countries in order from one to twelve are Austria, Denmark, Finland, France, Germany, Greece, Netherlands, Portugal, Spain, Poland, Sweden, and United Kingdom.
The variable input-output ratios are a function of relative input prices conditional upon electricity production and the availability of hydroelectric, nuclear, and renewable energy resources. The full model includes the restricted variable cost function (4) and the three variable input demand equations (5).

While the model can be estimated with full information maximum likelihood, a more robust procedure is estimation with Generalized Methods of Moments with country and monthly dummy variables and lagged values of the right-hand side variables, including those involving input prices, quasi-fixed factor levels, and output.

5. An Overview of the Data Sample

The above model is estimated with a pooled, monthly data sample across twelve countries in the EU from January 2002 to March 2008. The International Energy Agency (IEA) reports monthly electric power generation from nuclear, hydroelectric, geothermal and renewable resources and from the combustion of fossil fuels. The IEA does not report the types of combustible fuels but the EuroStat database does report the consumption of petroleum, coal, and natural gas in electric power generation. The EuroStat database, however, does not report data on generation from geothermal and renewable electricity generation. Given the rising importance of renewable energy in the generation portfolio, the more inclusive IEA data on generation is adopted in this study while the EuroStat data on fossil fuel use is utilized. A comparison of the generation data reported by the two agencies reveals the average differences are 3.2 percent.

An overview of the generation and net imports data appears in Table 1, which reports the sample means. The largest producers of electricity are Germany, France, United Kingdom and Spain. The mid tier includes Sweden and Poland and the other six countries have total indigenous production between 3,000 and 8,000 gigawatt hours. France and Germany are the largest producers at 47,624 and 45,666 gigawatt hours respectively. All twelve countries in the sample produce fossil-
fuel-fired electricity. Five countries do not produce nuclear electricity, including Austria, Denmark, Greece, Portugal, and Poland. Denmark and the Netherlands produce negligible amounts of hydroelectric power while France and Sweden are the largest producers of hydroelectricity. Germany and Spain produce rather substantial amounts of renewable electricity (see Table 1).

An overview of coal, petroleum, and natural gas consumption in electric power generation appears in Table 2. The largest coal consumers include Germany, United Kingdom, Poland, and Spain. The United Kingdom is the largest consumer of natural gas, Germany is second, and the Netherlands and Spain are significant consumers as well. Spain consumes significant amounts of petroleum to generate power along with Greece, the United Kingdom, and Germany.

Trends in the carbon intensity of indigenous electricity production are displayed in Figure 1. Carbon emissions are computed by multiplying fuel use by its respective carbon emission factor. The denominator is indigenous electricity production to reflect the shifts between combustible fuels and carbon-free generation such as nuclear, hydroelectric, and renewable energy resources. For the aggregate of the twelve countries, carbon intensity decreased from 2004 to 2005 but then increased very slightly from 2005 to 2007. This aggregate reflects a great deal of variability in carbon intensity trends among countries. Poland has the highest carbon intensity among the twelve countries, which actually increased between 2005 and 2007. With the exception of Denmark, Germany, the United Kingdom, and Greece, five countries reduced the carbon intensity of their electricity production – Portugal, Netherlands, Spain, Austria, and Finland. Electricity generation from renewable energy increased in each of these countries. Were it not for expanded use of renewable energy, Germany would have experienced even greater growth in the carbon intensity of their electricity production. France and Sweden have very low levels of carbon intensity due to their extensive use of nuclear and hydroelectric resources to generate electric power (see Figure 1).

The shares of natural gas in total fossil fuel consumption by country for the four full calendar years 2004 to 2007 are displayed in Figure 2. The share of natural gas for the aggregate of the twelve
countries increased from 33 to 36 percent from 2004 to 2007. Expanding use of gas in Greece, the Netherlands, the United Kingdom, Portugal, Spain, and Sweden offset declines in Finland and Denmark and the flat trends in the other countries. The empirical model below may shed some light on the role of relative fuel and carbon permit prices in these fuel share adjustments. Nevertheless, the shift to less carbon intensive natural gas, the increasing generation of renewable energy (see Figure 3), and the declining carbon intensity in several countries suggests that carbon dioxide emission abatement may have occurred even during the trial period for the EU ETS.

During the first phase of the EU ETS, exchanges emerged to trade spot and futures contracts derived from Phase I and Phase II. Because the Phase I allowances could not be carried over into Phase II, future contracts based either on Phase I or Phase II allowances were independently priced. For a variety of reasons, evidence suggests that allowances were over-issued during Phase I. As a result, toward the end of the trial period in 2007, emission allowance prices fell to zero (see Figure 4). However, when allocations for the second phase were determined, additional oversight was given to the European Commission and this appears to have resulted in a binding Phase II cap.

In the short-run, fuel substitution decisions are likely linked to the relative costs of obtaining or selling marginal allowances and for this reason we use spot prices for carbon permits as reported by PointCarbon in our model. This is a reasonable proposition and can be tested in the analysis below by determining whether carbon prices are statistically significant in the input demand functions.

In interpreting our results, we should point out that the pricing incentives for short-run substitution, which our model measures, may be different that the incentives for new investment in capacity. Said somewhat differently, fuel-switching and related actions are short-term tactics meant to minimize costs, while investments are made with an expected flow of profits in mind. Ellerman (2008), for one, argues that the investment decisions were guided by market valuations of traded Phase II allowances, as given by prices for the December 2008 futures contract also shown in Figure 4. And while there is no clear reason to expect that such investments were brought forward into
Phase I when spot prices were low, investment studies will need to reconcile the conflicting incentives given by the pricing of Phase I and Phase II allowances.

Prices for natural gas, petroleum, and coal paid by electricity generators in the United Kingdom are directly observable on a monthly basis from the United Kingdom (2009). For the other countries, this study must estimate monthly prices based upon regional monthly wholesale prices published by Platts (2009) for various market hubs in the EU and quarterly prices published by the International Energy Agency (2009) that measure prices paid by end users including taxes. Quarterly averages are computed from the monthly data from Platts. Next, we compute the ratio of these quarterly averages to the quarterly data reported by the IEA. These ratios represent the spreads between prices in each country and the market hub. Monthly estimates for prices in the remaining eleven countries result from multiplying these ratios by the monthly data from Platts.

6. Model Estimation Results

The above econometric model of the restricted variable cost function (4) and the three variables input demand functions given by (5) are estimated as a system of equations. Given that fuel and carbon permit prices and output could be endogenous, an instrumental variables estimator is needed. The Generalized Method of Moments (GMM) estimator provides for consistent parameter estimates and allows correction of the standard errors for heteroscedasticity and autocorrelation in the error terms. The instruments include lagged values of input prices, generation levels for nuclear, hydroelectric, and renewable generation, total power generation, and country and monthly dummy variables. The lagged instruments vary for each equation and correspond with the specific specification of the right-hand side variables in equations (4) and (5). So, for example, the instruments for the input demand functions include square roots of lagged price ratios. This approach is intended to ensure that the instruments are correlated with the explanatory variables but remain independent of the error terms. Country and monthly dummy variables are included as instruments in all four equations.
The test of the over-identifying restrictions for the unrestricted model is 93.2 with a probability value of 0.51, which suggests that the model given above cannot be rejected. Only one technological change coefficient was significant at the 5 percent level or less and that was for natural gas, which indicated gas-saving technological change, most likely reflecting the steady improvements in the thermal efficiency of combined cycle gas turbine technology. As a result, the hypothesis of non-neutral technological change is tested by computing a test statistic equal to the difference between the test statistics of the over-identifying restrictions for the unrestricted model and the restricted model with neutral technological change imposed via the following parameter restrictions:
\[ \delta_{zi} = 0 \quad \forall \, i, \gamma_{yz} = \gamma_{zn} = \gamma_{zr} = 0 . \]

The value of this test statistic is 9.16 with a probability value of 24%, indicating that the null hypothesis of neutral technological change cannot be rejected. This finding suggests that at least for the early stages of the EU ETS, exogenous technological change has not induced pervasive changes in the relative factor intensities of power generation.\(^5\)

As a result, the following presentation of results will focus on the estimates for the model assuming neutral technological change. In this case, the instruments involving the trend proxy for technological change are dropped and the model is re-estimated. The test of the over-identifying restrictions is 92.3 with 85 degrees of freedom and a probability value of 27.8%. As expected, the model with neutral technological change cannot be rejected. The parameter estimates appear in Table 3. Of the 36 country dummy variables, 20 have probability values that suggest less than a 5% chance of being zero while three have probability values less than 10%. For the 21 coefficients on the relative price, output, and quasi-fixed factors, 15 have probability values less than 5 percent and two less than 10 percent. The coefficients involving output, however, have relatively high probability

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\(^5\) The parameter estimates for the neutral technological change model are relatively close to those for the non-neutral model. There are no sign changes between the two sets of estimates.
values, although as we shall see below the output elasticities contain the other parameters and often are highly significant.

The goodness of fit statistics are reported in Table 4 indicate an excellent fit of the data with R-squared coefficients ranging from 0.97 to 0.99. The Durbin-Watson statistics indicate first-order autocorrelation, which is why we allow a first order moving average correction in the GMM estimation. An explicit structural correction for autocorrelation is not pursued because it could introduce specification error and would violate the conditions that allow the input demand functions to be integrated back to the cost function. Given the relatively large sample size used in this study, the theoretical result that the GMM estimates are asymptotically efficient seems reasonable.

Two sets of elasticities can be computed from the restricted cost function and the input demand functions. The elasticities of demand holding levels of the quasi-fixed factors are equivalent to short-run elasticities often defined in the literature. Given that we are estimating a separable cost sub-function holding capital and labor fixed, this study uses the term partial adjustment for these elasticities, which are reported in Table 5. The demands for carbon permits and fuels are essentially perfectly inelastic assuming levels of nuclear, hydroelectric, and renewable resources are fixed. In some sense, these elasticities are an artifact of this extremely restrictive ceteris paribus measurement and would explain the violation of the concavity conditions for these partial adjustment elasticities. Nonetheless, the other elasticities reported in Table 5 show that greater levels of nuclear and renewable resources reduce the demands for fuels and carbon, as one would expect. Indeed, the convexity conditions are satisfied for all observations. Likewise, predicted marginal cost is positive

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6 Using annual data on installed capacity and monthly estimates of replacement costs for new capacity, we generated monthly time series for capital stocks. Likewise, using quarterly data for employment levels in the electric generating and transmission sector and quarterly data on wages, we generated price and quantities for labor. We then estimated a model using (4) and (5) with carbon, labor, and energy as variable inputs and non-carbon energy and capital as quasi-fixed inputs. The test of the overidentifying restrictions was decisively rejected, suggesting that the extrapolated data could be introducing measurement errors. This finding verifies our approach to base our analysis on reported data, which admittedly can only allow measurement of the short-run flexibility of the power grid to switch generation sources in response to relative prices and output.
for all observations. Moreover, the marginal cost function shifts upward with rising carbon and fuel prices and downward with more nuclear generation.

The full adjustment elasticities allow the levels of quasi-fixed factors to change. These elasticities result from solving the envelope conditions for the quasi-fixed factors and differentiating these functions to obtain the elasticities. These derivations appear in Appendix A. The estimated full adjustment elasticities appear in Table 6. Overall, they reflect very inelastic factor demands. The demand for carbon permits is very inelastic with an own price elasticity of -0.068 indicating that ceteris paribus a 10 percent increase in carbon permit prices results in less than a one percent reduction in carbon use. This inelasticity reflects significant complementarity between carbon emissions and fuels. While nuclear and renewable energy are substitutes with carbon permits, the cross price elasticities indicated very limited substitution. For example, a 10 percent reduction in the price of renewable energy induces slightly less than a 2 percent drop in carbon emissions.

The output elasticities are all positive as expected with the natural gas output elasticity at more than 3, reflecting the well-known role of gas in leveling peaks and troughs in seasonal demand. The output elasticity of carbon permits is also significant at more than 2, suggesting that demand side reductions, if they can be achieved, would substantially reduce the demand for carbon emissions.

The marginal cost elasticities are also all significant. The estimated carbon price elasticity of marginal cost is 0.211 (see Table 6), indicating that for every 10 percent increase in carbon prices, the marginal generation cost of electricity increases by 2 percent. If fuel prices increase with carbon prices, the sum of the carbon and fuel price marginal cost elasticities (see Table 6) suggest that for every 10 percent increase in carbon prices, the marginal cost of electricity could increase 8 percent.7

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7 A markup pricing model was also estimated in which two additional estimating equations were added to equations (4) and (5), a demand for electricity and a price markup over marginal cost equation derived by assuming electric utilities are engage in monopoly pricing. The estimated price elasticities of demand holding output fixed are very similar to the results presented above. The elasticities in this context, which allows for endogenous output and
The last set of elasticities is the Morishima elasticities of substitution, which are a unit-less measure of substitution. The analysis by Blackorby and Russell (1989) proves that the Morishima elasticity is a superior measure of substitution for this study because it provides a clear distinction between substitutions induced by carbon permit prices versus other input price changes. Morishima elasticities are defined as follows:

\[ M_{ij} = \frac{\partial \ln x_j}{\partial \ln w_i} - \frac{\partial \ln x_i}{\partial \ln w_i} = \frac{-\partial \ln \left( \frac{x_i}{x_j} \right)}{\partial \ln w_i}. \]  

These elasticities measure the curvature of an isoquant, or the percentage change in a factor input ratio for a given percentage change in price, holding all other factor prices constant. As the above equation illustrates, the effect of varying \( w_i \) on the factor input ratio, \( \frac{x_i}{x_j} \), is composed of two parts – the effect of \( w_i \) on \( x_i \) and the effect of \( w_i \) on \( x_j \). Blackorby and Russell [7] show that these elasticities are inherently asymmetric.

The estimated Morishima elasticities of substitution appear in Table 7. All but four of these 20 substitution elasticities are significant at the five percent level. On the other hand, all of them are less than one. For instance, the ratio of nuclear resources to carbon emissions rises 0.852 percent for every percent increase in carbon emission prices. In contrast, the ratio of renewable generation to carbon emissions increases only 0.345 percent for each percent change in carbon emission prices. This suggests that nuclear energy serves as an important swing fuel in meeting carbon emission constraints.

Finally, there is significant complementarity between carbon emissions and high carbon fuels when the prices for the latter increase but a small and insignificant response of carbon emission prices, are very complicated and at this juncture of this research would obscure our focus on technological change and substitution. Nevertheless, this approach may merit future investigation.
relative to high carbon fuel consumption as carbon emission permits change. This reflects the very limited reductions in high carbon fuel consumption in response to carbon permit prices during the first phase of the EU ETS. Overall, the Morishma elasticities reflect very limited price-induced substitution between alternative generation fuels in the production of electricity in the short-run.

7. Conclusions

This study provides an analysis of the underlying economic forces inducing adjustments in electricity production factor intensities during the first phase of the European Union’s Emissions Trading System regulating emissions of greenhouse gas emissions. Our empirical analysis examines the demand for carbon permits, carbon based fuels, and carbon-free energy for 12 European countries using monthly data on fuel use, prices, and electricity generation. Our empirical model is unique because it considers all possible sources of generation within one model. Heretofore, empirical models of factor substitution in electric power generation were confined to studies of steam power generation using combustible fuels apart from nuclear of hydroelectric generation because prices for the latter fuels are not observable. Our approach uses a restricted variable cost function treating these factors as quasi-fixed to estimate the shadow value of these resources.

Our results suggest several conclusions. Perhaps the most important finding is that very limited substitution possibilities in combination with low carbon permit prices may explain the limited success of the EU ETS in achieving carbon emission reductions in the electric power generation sector. While our empirical results demonstrate that switching to nuclear and renewable energy is induced by higher carbon permit prices, the extent of this substitution is limited. Other substitution possibilities are also very limited. These results suggest that the current configuration of electricity generating assets is inflexible and that to achieve substantial reductions in carbon emissions more flexibility must be introduced, most likely from significant investments in new generation capacity.
A second implication of the model is that the effects of the cap on electricity prices can be significant, if fuel prices increase together with carbon permit prices as is likely. In this case, our estimates suggest that for every 10 percent rise in carbon and fuel prices, the marginal cost of electric power generation increases by 8 percent in the short-run. Consequently, if EUA allocations are fixed and fuel prices exogenous, the degree to which the costs of a carbon cap are passed on to consumers in the short run will be determined by how open the system is to alternative carbon offset, such as CERs, and the relative price of those allowances.

The European experience points to the importance of starting early down a low-carbon path. Because fixed investments in power generation are long-lived and irreversible, inflexibilities resulting from past investments will be long-lived as well. Consequently, it is important for countries that do not currently cap greenhouse gas emissions but hope to promote growth that is less carbon intensive to find alternative policies that consider the costs of future adjustments.

References


20
Table 1: Average annual electric power generation by type and net imports in gigawatt hours, January 2002 to March 2008

<table>
<thead>
<tr>
<th>Country</th>
<th>Fossil Fuel</th>
<th>Nuclear</th>
<th>Hydroelectric</th>
<th>Renewables</th>
<th>Indigenous Production</th>
<th>Net Imports</th>
<th>Total</th>
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## Table 2: Average Annual Fossil fuel consumption in terajoules, January 2002 to March 2008

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Figure 1: Carbon intensity of indigenous electricity production by country, 2004 to 2007
Figure 2: Shares of natural gas in fossil fuel use in power generation by country, 2004 to 2007
Figure 3: Renewable electricity generation, 2002-2007
Figure 4: EU ETS carbon emission allowance prices
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### Table 4: Summary Fit Statistics

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<tr>
<th>Equation</th>
<th>Mean of Dependent Variable</th>
<th>Standard Error of Regression</th>
<th>R-Squared</th>
<th>Durbin-Watson</th>
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### Table 5: Partial Adjustment Elasticities of Demand and Marginal Cost (asymptotic t-ratios in parentheses)

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<tr>
<th></th>
<th>Carbon Price</th>
<th>Coal &amp; Oil Prices</th>
<th>Natural Gas Prices</th>
<th>Nuclear Generation</th>
<th>Renewable Generation</th>
<th>Total Output</th>
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</thead>
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### Table 6: Full Adjustment Elasticities of Demand and Marginal Cost (asymptotic t-ratios in parentheses)

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<th>Coal &amp; Oil Price</th>
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<th>Nuclear Prices</th>
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### Table 7: Full Adjustment Morishima Elasticities of Substitution (asymptotic t-ratios in parentheses)

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<td>(1.9)</td>
<td>(6.2)</td>
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Appendix A
Derivation of the Elasticities

This appendix provides the derivations for the restricted and unrestricted elasticities of demand and substitution. The derivations all follow from differentiating the following short-run restricted cost function:

\[ G_t = Y_t \left\{ \sum_{i=1}^{3} \alpha_{ij} (w_i w_j)^{1/2} + \sum_{i=1}^{3} \delta_{ji} w_i Y_t^{1/2} + \sum_{i=1}^{3} \delta_{ji} w_i Z_t^{1/2} \right\} + \sum_{i=1}^{3} w_i \left( \gamma_{ii} Y_t + 2 \gamma_{ij} Y_{i,t}^{1/2} Z_j^{1/2} + \gamma_{i} Z_t \right) \]

\[ + \sum_{i=1}^{3} \delta_{mi} w_i N_t^{1/2} + \sum_{i=1}^{3} \delta_{ni} w_i R_t^{1/2} \]

\[ + Y_t^{1/2} \left\{ \sum_{i=1}^{3} w_i \left( \gamma_{ii} N_t + 2 \gamma_{im} N_i^{1/2} R_t^{1/2} + \gamma_{m} R_t \right) + \sum_{i=1}^{12} w_i \delta_{ici} D_{ci} \right\} \]

(A1)

The variables are defined in the paper above. The three input demand functions are the partial derivatives of the restricted cost function (A1) with respect to input prices:

\[ \frac{\partial G_t}{\partial w_{it}} = X_{it} + \sum_{c=1}^{12} \delta_{ci} D_{ci} + Y_t \left\{ \sum_{j=1}^{3} \alpha_{ij} \left( \frac{w_j}{w_i} \right)^{1/2} + \delta_{ji} Y_t^{1/2} + \delta_{ji} Z_t^{1/2} + \gamma_{ij} Y + 2 \gamma_{ij} Y^{1/2} Z_j^{1/2} + \gamma_{i} Z_t \right\} \]

\[ + \delta_{mi} \left( Y_t N_t \right)^{1/2} + \delta_{ri} \left( Y_t R_t \right)^{1/2} + \gamma_{mn} Y_t N_t^{1/2} + \gamma_{mr} Y_t R_t^{1/2} + \gamma_{mn} \left( Y_t Z_t N_t \right)^{1/2} + \gamma_{mr} \left( Y_t Z_t R_t \right)^{1/2} \]

(A2)

The marginal cost function is as follows:

The marginal cost function is as follows:
\[ \frac{\partial G_t}{\partial Y_t} = \sum_{i=1}^{3} \sum_{j=1}^{3} \alpha_{ij} \left( \frac{w_i}{w_j} \right)^{1/2} + \frac{3}{2} \sum_{i=1}^{3} \delta_{x} Y_t^{1/2} + \sum_{i=1}^{3} \delta_{z} Z_t^{1/2} \]

\[ + \sum_{i=1}^{3} w_i \left( 2 \gamma_{x} Y_t + 3 \gamma_{y} Y_t^{1/2} Z_t^{1/2} + \gamma_{z} Z_t \right) + \frac{1}{2} \sum_{i=1}^{3} \delta_{x} \left( \frac{N_t}{Y_t} \right)^{1/2} + \frac{1}{2} \sum_{i=1}^{3} \delta_{z} \left( \frac{R_t}{Y_t} \right)^{1/2} \]

\[ + \sum_{i=1}^{3} w_i \left[ \gamma_{yn} N_t^{1/2} + \gamma_{yr} R_t^{1/2} + \frac{1}{2} \gamma_{zn} \left( \frac{Z_t}{N_t} \right)^{1/2} + \frac{1}{2} \gamma_{zr} \left( \frac{Z_t}{R_t} \right)^{1/2} \right] \]  

\[ \text{(A3)} \]

The conditional own-price elasticities of demand are defined as follows:

\[ \frac{\partial \ln X_{it}}{\partial \ln w_{it}} = -\frac{1}{2} \left( \frac{Y_t}{X_{it}} \right) \left[ \sum_{j=1}^{3} \alpha_{ij} \left( \frac{w_{jt}}{w_{it}} \right)^{1/2} \right] \quad \forall \ i \]

\[ \text{(A4)} \]

While the cross-price elasticities of demand are:

\[ \frac{\partial \ln X_{it}}{\partial \ln w_{jt}} = -\frac{1}{2} \left( \frac{Y_t}{X_{it}} \right) \left[ \alpha_{ij} \left( \frac{w_{jt}}{w_{it}} \right)^{1/2} \right] \quad \forall \ i \neq j \]

\[ \text{(A5)} \]

The conditional input demand elasticities with respect to output are:

\[ \frac{\partial \ln X_{it}}{\partial \ln Y_t} = \left( \frac{Y_t}{X_{it}} \right) \left[ \sum_{j=1}^{3} \alpha_{ij} \left( \frac{w_{jt}}{w_{it}} \right)^{1/2} + \frac{3}{2} \delta_{x} Y_t^{1/2} + \delta_{z} Z_t^{1/2} + 2 \gamma_{y} Y_t + 3 \gamma_{y} Y_t^{1/2} Z_t^{1/2} + \gamma_{z} Z_t \right] \]

\[ + \frac{1}{2} \left\{ \delta_{x} \left( \frac{N_t}{Y_t} \right)^{1/2} + \delta_{z} \left( \frac{R_t}{Y_t} \right)^{1/2} + \gamma_{zn} \left( \frac{Z_t}{N_t} \right)^{1/2} + \gamma_{zr} \left( \frac{Z_t}{R_t} \right)^{1/2} \right\} \quad \forall \ i \]

\[ \text{(A6)} \]

The conditional input demand elasticity with respect to levels of hydroelectric and nuclear generation is as follows:

\[ \frac{\partial \ln X_{it}}{\partial \ln N_t} = \frac{N_t}{X_{it}} \left[ \frac{1}{2} \left( \frac{Y_t}{N_t} \right)^{1/2} \left[ \delta_{x} + \gamma_{yn} Y_t^{1/2} + \gamma_{zn} Z_t^{1/2} \right] + \gamma_{yn} + \gamma_{zn} \left( \frac{R_t}{N_t} \right)^{1/2} \right] \]

\[ \text{(A7)} \]

Likewise, the conditional input demand elasticity with respect to renewable generation is:
Finally, the conditional input demand elasticity with respect to technological change is:

\[
\frac{\partial \ln X_{it}}{\partial \ln R_t} = \frac{R_t}{X_t} \left\{ \frac{1}{2} \left( \frac{Y_t}{R_t} \right)^{\frac{1}{2}} \left[ \delta_n + \gamma_{xy} Y_t^{\frac{1}{2}} + \gamma_{xZ} Z_t^{\frac{1}{2}} \right] + \gamma_{rr} + \gamma_{nr} \left( \frac{N_t}{R_t} \right)^{\frac{1}{2}} \right\} 
\]

(A8)

The concavity conditions are determined by calculating the Eigen values of the three-by-three matrix formed from the partial derivatives of (A1) with respect to input prices.

The elasticity of marginal cost with respect to output is as follows:

\[
\frac{\partial \ln MC_t}{\partial \ln Y_t} = \frac{1}{MC_t} \left\{ \frac{3}{4} \sum_{i=1}^{3} \delta_{yi} w_{it} Y_t^{\frac{1}{2}} + \sum_{i=1}^{3} w_{it} \left( 2 \gamma_{xy} Y_t + \frac{3}{2} \gamma_{xZ} Z_t^{\frac{1}{2}} \right) - \frac{1}{4} \sum_{i=1}^{3} \delta_{ni} w_{it} N_t^{\frac{1}{2}} Y_t^{\frac{1}{2}} Y_t^{\frac{1}{2}} 
- \frac{1}{4} \sum_{i=1}^{3} \delta_{ni} w_{it} R_t^{\frac{1}{2}} Y_t^{\frac{1}{2}} Y_t^{\frac{1}{2}} \right\} 
\]

(A10)

The technical change elasticity of marginal cost is given by:

\[
\frac{\partial \ln MC_t}{\partial Z_t} = \frac{1}{MC_t} \left\{ \frac{3}{2} \sum_{i=1}^{3} \delta_{zi} w_{it} Z_t^{\frac{1}{2}} + \sum_{i=1}^{3} w_{it} \left( \frac{3}{2} \gamma_{xy} Y_t^{\frac{1}{2}} Z_t^{\frac{1}{2}} + \gamma_{z} \right) + \frac{1}{4} \sum_{i=1}^{3} w_{it} \left( \gamma_{zn} \left( \frac{N_t}{Z_t} \right)^{\frac{1}{2}} + \gamma_{zr} \left( \frac{R_t}{Y_t Z_t} \right)^{\frac{1}{2}} \right) \right\} 
\]

(A11)

The partial derivatives of marginal cost with respect to observed variable inputs are as follows:
The partial derivatives of marginal cost with respect to the quasi-fixed inputs are as follows:

\[
\frac{\partial \ln MC_t}{\partial \ln w_{it}} = \left( \frac{w_{it}}{MC_t} \right) \left\{ \sum_{j=1}^{3} \alpha_{ij} \left( \frac{w_{jt}}{w_{it}} \right)^{1/2} + \frac{3}{2} \delta_{yi} Y_t^{1/2} + \delta_{zi} Z_t^{1/2} \right. \\
+ \left. \left( 2\gamma_{yy} Y_t + 3\gamma_{yz} Y_t^{1/2} Z_t^{1/2} + \gamma_{zz} Z_t \right) + \frac{1}{2} \delta_{yi} \left( \frac{N_t}{Y_t} \right)^{1/2} + \frac{1}{2} \delta_{zi} \left( \frac{R_t}{Y_t} \right)^{1/2} \right. \\
+ \gamma_{yn} N_t^{1/2} + \gamma_{yr} R_t^{1/2} + \frac{1}{2} \gamma_{yz} \left( \frac{Z_t N_t}{Y_t} \right)^{1/2} + \frac{1}{2} \gamma_{zn} \left( \frac{Z_t R_t}{Y_t} \right)^{1/2} \right\} 
\]  
(A12)

The partial derivatives of marginal cost with respect to the quasi-fixed inputs are as follows:

\[
\frac{\partial \ln MC_t}{\partial \ln N_t} = \frac{1}{MC_t} \left\{ \frac{1}{4} \sum_{i=1}^{3} \delta_{ni} w_{it} N_t^{1/2} Y_t^{-1/2} + \frac{3}{2} \sum_{i=1}^{3} w_{it} \left[ \gamma_{yn} N_t^{1/2} + \frac{1}{2} \gamma_{zn} \left( \frac{N_t Z_t}{Y_t} \right)^{1/2} \right] \right\} 
\]  
(A13)

The convexity conditions for the quasi-fixed level of hydroelectric and nuclear generation resources is as follows:

\[
\frac{\partial G_t}{\partial N_t} = \mu^*_{nt} = \frac{1}{2} \sum_{i=1}^{3} \delta_{ni} w_{it} \left( \frac{Y_t}{N_t} \right)^{1/2} + \frac{1}{2} Y_t^{1/2} \sum_{i=1}^{3} w_{it} \left[ \gamma_{yn} \left( \frac{Y_t}{N_t} \right)^{1/2} + \gamma_{zn} \left( \frac{Z_t}{N_t} \right)^{1/2} \right] \\
+ \frac{1}{4} \sum_{i=1}^{3} w_{it} \left[ \gamma_{na} + \gamma_{ar} \left( \frac{R_t}{N_t} \right)^{1/2} \right] \right\} 
\]  
(A14)

This derivative must be negative. The other convexity condition for renewable energy is:

\[
\frac{\partial G_t}{\partial R_t} = \mu^*_{nt} = \frac{1}{2} \sum_{i=1}^{3} \delta_{ni} w_{it} \left( \frac{Y_t}{R_t} \right)^{1/2} + \frac{1}{2} Y_t^{1/2} \sum_{i=1}^{3} w_{it} \left[ \gamma_{yr} \left( \frac{Y_t}{R_t} \right)^{1/2} + \gamma_{zn} \left( \frac{Z_t}{R_t} \right)^{1/2} \right] \\
+ \frac{1}{4} \sum_{i=1}^{3} w_{it} \left[ \gamma_{nr} + \gamma_{ar} \left( \frac{N_t}{R_t} \right)^{1/2} \right] \right\} 
\]  
(A15)
As Morrison (1988) shows, the convexity conditions can be solved, in this case simultaneously, for the equilibrium levels of the quasi-fixed inputs. First, consider the solution for equilibrium levels of nuclear and hydroelectric generation:

\[
\mu^*_{nt} - \sum_{i=1}^{3} w_{it} \gamma_{ii} = \left( \frac{1}{2} \sum_{i=1}^{3} \delta_{nt} w_{it} Y_{i}^{1/2} + \frac{1}{2} Y_{i}^{1/2} \sum_{i=1}^{3} w_{it} \left( \gamma_{yn} Y_{n}^{1/2} + \gamma_{zn} Z_{n}^{1/2} \right) + \sum_{i=1}^{3} w_{it} \gamma_{nm} R_{i}^{1/2} \right)
\]

\[
N_{t} = \left( \frac{1}{2} \sum_{i=1}^{3} \delta_{nt} w_{it} Y_{i}^{1/2} + \frac{1}{2} Y_{i}^{1/2} \sum_{i=1}^{3} w_{it} \left( \gamma_{yn} Y_{n}^{1/2} + \gamma_{zn} Z_{n}^{1/2} \right) + \sum_{i=1}^{3} w_{it} \gamma_{nm} R_{i}^{1/2} \right)^{-1/2}
\]

Similarly, solving (A11) for the level of renewable generation is as follows:

\[
R_{t} = \left( \frac{1}{2} \sum_{i=1}^{3} \delta_{nt} w_{it} Y_{i}^{1/2} + \frac{1}{2} Y_{i}^{1/2} \sum_{i=1}^{3} w_{it} \left( \gamma_{yr} Y_{r}^{1/2} + \gamma_{zr} Z_{r}^{1/2} \right) + \sum_{i=1}^{3} w_{it} \gamma_{rm} N_{i}^{1/2} \right)^{-1/2}
\]

Substituting (A10) into (A11) and solving for the equilibrium level of renewable generation yields:

\[
N_{t}^* = \left( \frac{GM_{t} RM_{t} + \gamma_{nr} \sum_{i=1}^{3} w_{it} HM_{t}}{QM_{t} RM_{t} - \left[ \sum_{i=1}^{3} w_{it} \gamma_{nr} \right]^{2}} \right)^{2} = \left[ \frac{NN_{t}}{ND_{t}} \right]^{2},
\]

\[
R_{t}^* = \left( \frac{HM_{t} QM_{t} + \gamma_{nr} \sum_{i=1}^{3} w_{it} GM_{t}}{QM_{t} RM_{t} - \left[ \sum_{i=1}^{3} w_{it} \gamma_{nr} \right]^{2}} \right)^{2} = \left[ \frac{RN_{t}}{RD_{t}} \right]^{2}
\]

where
\[ QM_i = \mu_m^* + \gamma_m \sum_{i=1}^{3} w_{it} \]
\[ RM_i = \mu_r^* + \gamma_r \sum_{i=1}^{3} w_{it} \]
\[ GM_i = \frac{1}{2} Y_{it}^{1/2} \left[ \sum_{i=1}^{3} \delta_m w_{it} + \sum_{i=1}^{3} w_{it} \left( \gamma_{yn} Y_{it}^{1/2} + \gamma_{zn} Z_{it}^{1/2} \right) \right] \]
\[ HM_i = \frac{1}{2} Y_{it}^{1/2} \left[ \sum_{i=1}^{3} \delta_r w_{it} + \sum_{i=1}^{3} w_{it} \left( \gamma_{yr} Y_{it}^{1/2} + \gamma_{zr} Z_{it}^{1/2} \right) \right] \]

The elasticities of demand for these quasi-fixed inputs with respect to carbon and fossil energy prices take the following form:

\[
\frac{w_a}{N_t} \frac{\partial N_t}{\partial w_{it}} = 2 \frac{w_a}{N_t} \frac{NN_t}{N_t^3} \left[ \gamma_{rr} GM_i + RM_i GP_i + \gamma_{nr} \left( HM_i + HP_i \sum_{i=1}^{3} w_{it} \right) \right] - \frac{NN_t}{N_t^3} \left[ \gamma_{rr} QM_i + \gamma_{nr} RM_i - 2 \gamma_{nr}^2 \sum_{i=1}^{3} w_{it} \right] \quad \forall \ i
\]

\[
\frac{w_a}{R_t} \frac{\partial R_t}{\partial w_{it}} = 2 \frac{w_a}{R_t} \frac{RN_t}{R_t^3} \left[ \gamma_{nn} HM_i + QM_i HP_i + \gamma_{nr} \left( GM_i + GP_i \sum_{i=1}^{3} w_{it} \right) \right] - \frac{RN_t}{R_t^3} \left[ \gamma_{nn} QM_i + \gamma_{nr} RM_i - 2 \gamma_{nr}^2 \sum_{i=1}^{3} w_{it} \right] \quad \forall \ i
\]

where

\[ GP_i = \frac{1}{2} Y_{it}^{1/2} \left( \delta_{at} + \gamma_{yn} Y_{it}^{1/2} + \gamma_{zn} Z_{it}^{1/2} \right) \]
\[ HP_i = \frac{1}{2} Y_{it}^{1/2} \left( \delta_{at} + \gamma_{yn} Y_{it}^{1/2} + \gamma_{zn} Z_{it}^{1/2} \right) \]

The own-price elasticities of demand for the two quasi-fixed inputs are as follows:

\[
\frac{\mu_m^*}{N_t} \frac{\partial N_t}{\partial \mu_m} = -2 \left[ \frac{NN_t^2}{N_t^3} \right] RM_i \frac{\mu_m^*}{N_t}
\]
\[
\frac{\mu_r^*}{R_t} \frac{\partial R_t}{\partial \mu_r} = -2 \left[ \frac{RN_t^2}{R_t^3} \right] QM_i \frac{\mu_r^*}{R_t}
\]
While the cross-price elasticities are:

\[
\frac{\mu_{nt}^* \partial N_t^*}{N_t^* \partial \mu_{nt}^*} = 2 \left( \frac{NN_t}{ND_t^2} \right) \left[ GM_t - \frac{NN_t}{ND_t} QM_t \right] \frac{\mu_{nt}^*}{N_t^*}
\]

\[
\frac{\mu_{nt}^* \partial R_t^*}{R_t^* \partial \mu_{nt}^*} = 2 \left( \frac{RN_t}{RD_t^2} \right) \left[ HM_t - \frac{RN_t}{RD_t} RM_t \right] \frac{\mu_{nt}^*}{R_t^*}
\]

The output elasticities are respectively:

\[
\frac{Y_t}{N_t^*} \frac{\partial N_t}{\partial Y_t} = 2 \left( \frac{NN_t}{ND_t^2} \right) \left[ Y_t \frac{RM_t GY_t + \gamma_{ny} \sum_{i=1}^{3} w_{it} HY_t}{Y_t N_t} \right] \frac{1}{N_t^*}
\]

\[
\frac{Y_t}{R_t^*} \frac{\partial R_t}{\partial Y_t} = 2 \left( \frac{RN_t}{RD_t^2} \right) \left[ Y_t \frac{QM_t HY_t + \gamma_{nr} \sum_{i=1}^{3} w_{it} GY_t}{Y_t R_t} \right] \frac{1}{R_t^*}
\]

where

\[
GY_t = \frac{1}{4Y_t^{1/2}} \left[ \sum_{i=1}^{3} \delta_{in} w_{it} + \gamma_{zn} \sum_{i=1}^{3} w_{it} Z_t^{1/2} \right] + \frac{1}{2} \gamma_{zn} \sum_{i=1}^{3} w_{it}
\]

\[
HY_t = \frac{1}{4Y_t^{1/2}} \left[ \sum_{i=1}^{3} \delta_{ir} w_{it} + \gamma_{zn} \sum_{i=1}^{3} w_{it} Z_t^{1/2} \right] + \frac{1}{2} \gamma_{zn} \sum_{i=1}^{3} w_{it}
\]

Finally, the technological change elasticities are as follows:

\[
\frac{1}{N_t^*} \frac{\partial N_t^*}{\partial Z_t} = \frac{1}{2N_t^*} \left( \frac{NN_t}{ND_t^2} \right) \left[ Y_t \frac{1}{Z_t} \right]^{1/2} \left[ \gamma_{zn} R M_t + \gamma_{nr} \gamma_{zn} \sum_{i=1}^{3} w_{it} \right] \sum_{i=1}^{3} w_{it}
\]

\[
\frac{1}{R_t^*} \frac{\partial R_t^*}{\partial Z_t} = \frac{1}{2R_t^*} \left( \frac{RN_t}{RD_t^2} \right) \left[ Y_t \frac{1}{Z_t} \right]^{1/2} \left[ \gamma_{zn} Q M_t + \gamma_{nr} \gamma_{zn} \sum_{i=1}^{3} w_{it} \right] \sum_{i=1}^{3} w_{it}
\]

The partial adjustment elasticities for carbon and energy inputs allow the quasi-fixed factors to adjust, at this stage assuming output and prices fixed. The general expressions for these full adjustment elasticities of demand are as follows:

\[
\frac{\partial \ln X_{it}^{FA}}{\partial \ln w_{jt}} = \frac{\partial \ln X_{it}^{FA}}{\partial \ln w_{jt}} \left|_{\mu_{nt}^*, \mu_{rt}^*} \right. + \frac{\partial \ln X_{it}^{FA}}{\partial \ln N_t^*} \frac{\partial \ln N_t^*}{\partial \ln w_{jt}} + \frac{\partial \ln X_{it}^{FA}}{\partial \ln R_t^*} \frac{\partial \ln R_t^*}{\partial \ln w_{jt}} \forall i, j
\]
where

\[
\frac{N^*}{X^*} \frac{\partial X_{it}}{\partial N^*_t} = \left( \frac{N^*}{X^*} \right) \left\{ \frac{1}{2} \left( \frac{Y_t}{N^*_t} \right)^{1/2} \left[ \delta_{nt} + \gamma_{yn} Y_t^{1/2} + \gamma_{zn} Z_t^{1/2} \right] + \gamma_{nn} + \gamma_{nr} \left( \frac{R^*_t}{N^*_t} \right)^{1/2} \right\}
\]

\[
\frac{R^*}{X^*} \frac{\partial X_{it}}{\partial R^*_t} = \left( \frac{R^*}{X^*} \right) \left\{ \frac{1}{2} \left( \frac{Y_t}{R^*_t} \right)^{1/2} \left[ \delta_{rt} + \gamma_{yr} Y_t^{1/2} + \gamma_{zr} Z_t^{1/2} \right] + \gamma_{nr} + \gamma_{nn} \left( \frac{R^*_t}{R^*_t} \right)^{1/2} \right\} \quad \forall \ i
\]

and where \( X^*_t \) are the levels of variable inputs at equilibrium levels of the quasi-fixed factors.

The elasticities of variable input demands with respect to prices for quasi-fixed factors are as follows:

\[
\frac{\partial \ln X_{it}^{FA}}{\partial \ln \mu_{nt}} = \frac{\partial \ln X_{it}}{\partial \ln N^*_t} \frac{\partial \ln N^*_t}{\partial \ln \mu_{nt}} + \frac{\partial \ln X_{it}}{\partial \ln R^*_t} \frac{\partial \ln R^*_t}{\partial \ln \mu_{nt}}
\]

\[
\frac{\partial \ln X_{it}^{FA}}{\partial \ln \mu_{rt}} = \frac{\partial \ln X_{it}}{\partial \ln N^*_t} \frac{\partial \ln N^*_t}{\partial \ln \mu_{rt}} + \frac{\partial \ln X_{it}}{\partial \ln R^*_t} \frac{\partial \ln R^*_t}{\partial \ln \mu_{rt}} \quad \forall \ i
\]

Similarly, the partial adjustment output and technological elasticities are defined as follows:

\[
\frac{\partial \ln X_{it}^{FA}}{\partial \ln Y_t} = \left. \frac{\partial \ln X_{it}}{\partial \ln Y_t} \right|_{R, R} + \frac{\partial \ln X_{it}}{\partial \ln N^*_t} \frac{\partial \ln N^*_t}{\partial \ln Y_t} + \frac{\partial \ln X_{it}}{\partial \ln R^*_t} \frac{\partial \ln R^*_t}{\partial \ln Y_t}
\]

\[
\frac{\partial \ln X_{it}^{FA}}{\partial \ln Z_t} = \left. \frac{\partial \ln X_{it}}{\partial \ln Z_t} \right|_{R, R} + \frac{\partial \ln X_{it}}{\partial \ln N^*_t} \frac{\partial \ln N^*_t}{\partial \ln Z_t} + \frac{\partial \ln X_{it}}{\partial \ln R^*_t} \frac{\partial \ln R^*_t}{\partial \ln Z_t}
\]

The full adjustment marginal cost elasticities are defined in a similar fashion.
\[
\frac{\partial \ln MC_t^{FA}}{\partial \ln w_{it}} = \frac{\partial \ln MC_t^{FA}}{\partial \ln w_{it}} \bigg|_{R^*, N^*} + \frac{\partial \ln MC_t}{\partial \ln N^*_{it}} \frac{\partial \ln N^*_{it}}{\partial \ln w_{it}} + \frac{\partial \ln MC_t}{\partial \ln R^*_{it}} \frac{\partial \ln R^*_{it}}{\partial \ln w_{it}} \quad \forall \; i
\]

\[
\frac{\partial \ln MC_t^{FA}}{\partial \ln \mu_{it}} = \frac{\partial \ln MC_t^{FA}}{\partial \ln \mu_{it}} \bigg|_{R^*, N^*} + \frac{\partial \ln MC_t}{\partial \ln N^*_{it}} \frac{\partial \ln N^*_{it}}{\partial \ln \mu_{it}} + \frac{\partial \ln MC_t}{\partial \ln R^*_{it}} \frac{\partial \ln R^*_{it}}{\partial \ln \mu_{it}} \quad i = N, R
\]

\[
\frac{\partial \ln MC_t^{FA}}{\partial \ln Y_t} = \frac{\partial \ln MC_t^{FA}}{\partial \ln Y_t} \bigg|_{R^*, N^*} + \frac{\partial \ln MC_t}{\partial \ln N^*_{it}} \frac{\partial \ln N^*_{it}}{\partial \ln Y_t} + \frac{\partial \ln MC_t}{\partial \ln R^*_{it}} \frac{\partial \ln R^*_{it}}{\partial \ln Y_t}
\]

\[
\frac{\partial \ln MC_t^{FA}}{\partial Z} = \frac{\partial \ln MC_t^{FA}}{\partial Z} \bigg|_{R^*, N^*} + \frac{\partial \ln MC_t}{\partial \ln N^*_{it}} \frac{\partial \ln N^*_{it}}{\partial Z} + \frac{\partial \ln MC_t}{\partial \ln R^*_{it}} \frac{\partial \ln R^*_{it}}{\partial Z}
\]

(A32)

All the full adjustment elasticities are evaluated at the grand mean of the observations.

References