A Flaw in the Model… That Defines How the World Works

Volker Bieta
Hellmuth Milde
Nadine Weber

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Abstract

The authors of this paper claim that modeling financial markets based on probability theory is a severe systematic mistake that led to the global financial crisis. They argue that the crisis was not just the result of risk managers using outdated financial data, but that the employed efficiency model—also referred to as the stochastic model—is basically flawed. In an exemplary way, the analysis proves that this model is unable to account for interactions between market participants, neglects strategic interdependences, and hence leads to erroneous solutions. The central message is that the existing efficiency model should be replaced by an approach using agent-based scenario analysis.

This paper—a product of the IDA Resource Mobilization—is part of a larger effort in the department to better understand the recent financial crisis. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at nweber@worldbank.org.
“A FLAW IN THE MODEL … THAT DEFINES HOW THE WORLD WORKS”*

Volker Bieta, Hellmuth Milde and Nadine Weber**

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** Dr. Volker Bieta is lecturer of game theory at the Technical University of Dresden, Germany; Dr. Hellmuth Milde is professor of Business Administration at the Centre Universitaire de Luxembourg; Dr. Nadine Weber is Financial Officer in the IDA Resource Mobilization Department at the World Bank in Washington, D.C., USA.

I. Introduction

When considering the theory of financial markets, two different views emerge: the old view and the new view. The old view regards financial assets as claims on cash flows whose magnitude and variability are exogenously given. The prices of these assets are based on current information about future cash flows. A market is called “informationally efficient” if all information available at a given point of time is indeed used by market participants. As a result, according to the efficiency model, all information available is reflected in existing market prices.

However, this view is misleadingly simple and seriously incomplete. For example, according to the old view, information is available at zero cost. This is of course nonsense because information is never a free good. As a consequence, we observe information insiders and information outsiders in the world around us. Informational asymmetry is an empirical fact. In addition, the old view ignores the existence of conflicts of interest, incentive schemes, and agency problems. In his paper “Information and the Change in the Paradigm in Economics,” Professor Joseph Stiglitz (2003)\(^1\) challenged Adam Smith’s idea that the invisible hand can lead free markets to efficient outcomes.

In a Congressional hearing on October 23, 2008, Alan Greenspan pointed out that the “Mickey Mouse world”\(^2\) of the efficiency model is basically responsible for the recent financial crisis. Greenspan is not alone in his critical position. Many prominent voices blame the old view for the financial market disaster, including Justin Fox (2009), Nassim Taleb (2007), and Pablo Triana (2009a),\(^3\) to name a few. Our contribution keeps company with these critics. We will argue that the old view should be replaced by the new view, which strives to overcome the asymmetries of information. How to apply the new concepts and the new tools is nicely discussed in the textbook by Professor Jean Tirole (2006).\(^4\)

In our paper, we will use an example to demonstrate how the new view meets reality. Starting with an agency scenario in the banking industry the tools of incentive contracting and strategic decision making are used to solve the problem. Cash flows are no longer exogenously given but endogenously derived.

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\(^2\) Authors’ expression.


II. How Irrelevance and Force Majeure Disregard Behavioral Uncertainty and Strategic Interdependences

Consider a banker who climbs a tree and starts sawing the branch on which he is sitting. Wouldn’t you think that this banker is bird-brained? Be assured he is not as in his fantasy the branch does not break and he does not fall down. The banker’s fantasy is based on the assumption of irrelevance—the backbone of modern financial theory: Actions of market participants are irrelevant for the state of the market. This means in our analogy that the banker’s action of sawing does not change the state of the branch.

However, there is more to this story. Bankers do not only dream that the branch, which symbolizes the price of a financial asset, does not crash; they also observe that it sways somewhat. To explain this movement, which is not at all related to the sawing, bankers invented a coin tossing force majeure to decide whether the branch moves to the right or to the left depending on the random outcome of the tossed coin. The assumption of a random walk is explained by the efficient market hypothesis and the model of perfect competition that create a situation in which all parties are price takers and only decide about quantities. Yet in such a situation, someone has to decide about the price; this is where the force majeure—also called the auctioneer—enters the picture, setting the price based on presumably randomly incoming supply and demand quotes.

The described situation, where (1) a market participant’s actions do not influence the state of the market and (2) the state of the market is determined by tossing a coin, is the basis of the efficiency model and stochastic risk theory, which is oriented on natural sciences. Prototypes for financial processes (Black, Scholes, Merton) are the heat equation, molecular movements, and particle fluctuations (Brown, Einstein, Langevin). There are no means of human influence in natural sciences; hence, only event risk is addressed. This situation resembles the game of roulette, a game independent of human behavior in which a passive player, the croupier, starts the wheel, which is analogous to tossing a coin. The passive player is like nature and can be represented by an exogenous stochastic density function. The active player seeks maximal output under a passive state of nature. This becomes evident in the case of Markowitz: Without exogenous data for means, variances, and covariances, no solution can be computed.

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However: “There are essentially two sources of uncertainty: the possibility of uncontrollable events and the unpredictability of human behavior.” The former refers to event uncertainty while the latter refers to behavioral uncertainty. As soon as two active players are involved, we can no longer ignore behavioral uncertainty. Examples include poker and chess. There is no room for an exogenous density function since the player who is able to checkmate the other will not toss a coin to determine her move. Players will take the move leading to the highest payoff, which is decided by a set of rules that determine each player’s compensation under different actions. These rules create incentives for players to act in certain ways dependent on the opposite player’s actions, which are generally unknown. Hence, with multiple active players, we face information asymmetry; we are no longer situated in perfect markets and are confronted with the principal-agent-model, where the principal is the information outsider and the agent is the information insider. The insider (agent) can and should use her advantage. Every decision taken by the insider (agent) has consequences for both players. However, the outsider (principal) determines the incentive scheme by setting the compensation rules. If so desired, the outsider (principal) can provide disadvantageous rules to the insider (agent), but the insider (agent) will then use her information to get payback and line her own pockets. If this is realized by the outsider (principal), she might fire the insider (agent). These interactions are called strategic interdependences.

The concept of solving strategic interdependences is game theory. Players can choose from among several strategies, allowing for a multitude of strategic combinations. Each combination leads to one expected payoff per player. In game theory, each player chooses the strategy of greatest benefit assuming the opposite player continues in position. The situation in which no player can gain by making a unilateral deviation from the original combination is called the Nash equilibrium.

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8 We want to highlight the existing confusion between the two terms “risk” and “uncertainty”. Whereas risk is associated with a situation of unknown outcomes but a known probability distribution, uncertainty refers to unknown outcomes under an unknown probability distribution (see Frank Knight (1921), “Risk, Uncertainty, and Profit”, New York 1921). Nevertheless, both expressions are used interchangeably in financial theory. Chess and poker describe situations of uncertainty; meanwhile, roulette can be associated with the term risk, since a-priori probabilities are available. In financial theory no a-priori probabilities exist, and this drawback is bypassed by using the past to estimate them, adding an additional, non-negligible source of error, as many authors have pointed out (see, for example, René Stulz (2009), “Six Ways Companies Mismanage Risk”, Harvard Business Review, March 2009, pp. 86-97). Modern financial theory thinks of financial markets as being risky (roulette-like) when they are actually uncertain.

9 Named after John Forbes Nash, American mathematician, laureate of the 1994 Nobel Prize in Economic Sciences for his work in game theory, known through the Hollywood movie “A Beautiful Mind”.
III. Example

This example shows how the efficiency model leads to an erroneous solution. We will assume two projects; the first is chosen following the efficiency approach while the second is chosen following the strategic approach. Changing the incentive scheme by altering the compensation rules is like changing constraints and generally should lead to a new solution. However, this is the case only in the strategic approach, while the efficiency model is incapable of accounting for the new situation. Such a model cannot produce useful results.

We will first discuss the case of “pure strategies,” where exactly one solution can be determined, although reality is not that straightforward all the time, as is evident when we alter the compensation rules and move on to “mixed strategies,” where a concrete solution no longer exists.

III.A Pure Strategies

Consider a businessman who can choose between two mutually exclusive projects of identical volume but differing risks. Further, consider that a bank will finance the project 100 percent, but is incapable of monitoring the businessman’s choice. Yet, the bank offers two types of credit contracts with different interest rates and collaterals.

In this example, we have two active players: the businessman (=beneficiary=debtor=client) and the bank (=creditor). Both players have two mutually exclusive choices of action: The businessman can choose between two projects, which form his strategy set, and the bank can choose between two credit contracts, which form its strategy set. The businessman is the information insider (agent) who has insider knowledge about the risk of the two projects. The bank is the information outsider (principal) who determines the distribution rules and hence the incentive scheme by selecting the credit contract.

Let us formalize the example based on the following assumptions:

- Both bank and businessman are risk neutral to allow discounting at the risk-free interest rate.
- The risk-free interest rate is 5 percent and we assume simple interest.
- The maturity for both projects and credit contracts is one year.
- The volume of both projects and credit contracts is 100 units.
We determine that one project is high risk (H) and the other is low risk (L). Assume that each project has two possible cash flows depending on two possible states of the world, which occur with certain probabilities. Table 1 provides numeric assumptions to be used in this example. Figures in Table 1 would typically result from scenario analysis.

### Table 1

<table>
<thead>
<tr>
<th>Project</th>
<th>State of the world 1</th>
<th>State of the world 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cash flow volume</td>
<td>Probability of state</td>
</tr>
<tr>
<td></td>
<td>175</td>
<td>90%</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
<td>10%</td>
</tr>
<tr>
<td>H</td>
<td>250</td>
<td>60%</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>40%</td>
</tr>
</tbody>
</table>

As can be observed, projects L and H differ in that they provide a small cash flow with high probability if the risk is low and a high cash flow with a lower probability if the risk is high.

Based on the information provided, we can calculate the present values (PV) and net present values (NPV) for the two projects:

\[
PV(L) = \frac{175 \times 0.9 + 0 \times 0.1}{1.05} = 150.00 \\
NPV(L) = -100 + PV(L) = 50.00 \\
PV(H) = \frac{250 \times 0.6 + 0 \times 0.4}{1.05} = 142.86 \\
NPV(H) = -100 + PV(H) = 42.86
\]

Using the “NPV rule,” the businessman chooses project L since its NPV of 50 is higher than the NPV of project H, which is only 42.86. To calculate the NPVs solely the exogenously given probabilities for the two states of the world are needed; hence, this method falls under the efficiency or stochastic approach. The decision is taken without considering that the bank is an active player and offers two types of credit contracts. Is this realistic?

Table 2 provides numeric assumptions regarding the credit contracts offered by the bank. One contract requires collateral (C) but comes at a low interest rate; a second contract does not require collateral (N) but comes at a high interest rate. Note that interest rates in this example were purposefully chosen to be unrealistically high to be able to demonstrate clear differences in size.
Table 2

<table>
<thead>
<tr>
<th>Credit Contract</th>
<th>Interest rate</th>
<th>Collateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>50%</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>30%</td>
<td>99</td>
</tr>
</tbody>
</table>

Having two active players, we need to use strategic considerations to determine the choices for both. Since the businessman can choose between projects L and H and the bank can choose between contracts N and C, exactly four strategic combinations are possible: L/N, L/C, H/N, H/C. Every combination leads to exactly one expected cash flow to the businessman and one to the bank, as calculated in Table 3.

Table 3

<table>
<thead>
<tr>
<th>Cash Flows to Businessman</th>
<th>Cash Flows to Bank</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/N</td>
<td>[(175-100*1.5)<em>0.9 -0</em>0.1]/1.05 = 21.43</td>
<td>+[100<em>1.5</em>0.9+0*0.1]/1.05 = 28.57</td>
</tr>
<tr>
<td>L/C</td>
<td>[(175-100*1.3)<em>0.9 -99</em>0.1]/1.05 = 29.14</td>
<td>+[100<em>1.3</em>0.9+99*0.1]/1.05 = 20.86</td>
</tr>
<tr>
<td>H/N</td>
<td>[(250-100*1.5)<em>0.6 -0</em>0.4]/1.05 = 57.14</td>
<td>+[100<em>1.5</em>0.6+0*0.4]/1.05 = -14.29</td>
</tr>
<tr>
<td>H/C</td>
<td>[(250-100*1.3)<em>0.6 -99</em>0.4]/1.05 = 30.86</td>
<td>+[100<em>1.3</em>0.6+99*0.4]/1.05 = 12.00</td>
</tr>
</tbody>
</table>

For each project, the cash flows to businessman and bank total the projects’ NPVs. It now becomes evident how the two types of credit contracts offered by the bank provide rules that determine the distribution of each project’s cash flow (NPV) between the two players depending on the strategic combination of project and contract chosen. These distribution rules create clear incentives for both players to choose one of their two feasible actions. Table 4 summarizes the payoffs in a game matrix,
with the businessman as the “line player” to whom the first entry in each cell corresponds and the bank as the “column player” to whom the second entry in each cell corresponds.

<table>
<thead>
<tr>
<th>Businessman</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>21.43 / 28.57</td>
</tr>
<tr>
<td>H</td>
<td>57.14 / -14.29</td>
</tr>
</tbody>
</table>

The Nash equilibrium is given with the combination H/C, where the businessman chooses project H at an expected cash flow of 30.86 and the bank offers contract C at an expected cash flow of 12.00. Neither the businessman nor the bank will deviate from this choice as a deviation for either would lead to lower expected cash flows (29.14 and -14.29, respectively). Taking strategic considerations into account, the businessman thus chooses project H and not, as was calculated under the efficiency approach, project L.

This erroneous solution under the efficiency approach clearly results from negligence of the credit contracts, which determine the distribution of payoff between the two players. Only in cases when nothing is to be distributed does this approach make sense, such as in the case of 100 percent equity financing. However, in the case of debt financing, as in our example, the bank bears part of the costs and therefore the businessman has incentives to choose the high risk project (H). That is exactly what happened, ultimately leading to the financial crisis.

Note that statistical data are needed in both models to calculate the expected cash flows per player in each action combination. However, whereas in the efficiency approach the mere existence of statistical data is sufficient, the strategic approach goes much further by allowing each player to actively make a decision. This decision considers the distribution of cash flows between the players as determined by the distribution rules. Distribution rules create incentives that can only be captured by applying strategic considerations. However, it is also important to note that the strategic approach no longer leads to an optimum from a social point of view since the project with the lower NPV is selected.
III.B Mixed Strategies

With strategic analysis, we can go one step further and test new incentives. How do we need to alter the conditions for the businessman to choose contract L? Certainly, the new distribution rules must allow the businessman to earn more with project L by conserving the bank’s expected cash flows. Without going into the details of the calculations, assume the slightly changed credit contracts as displayed in Table 5. Note that only contract C was changed and is now called C’.

Table 5

<table>
<thead>
<tr>
<th>Credit Contract</th>
<th>Interest rate</th>
<th>Collateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>50%</td>
<td>0</td>
</tr>
<tr>
<td>C’</td>
<td>28%</td>
<td>105</td>
</tr>
</tbody>
</table>

We can again calculate four strategic combinations: L/N, L/C’, H/N, H/C’. Since we only changed the conditions for contract C, only the second column of the Table 4 changes. The new game matrix is displayed in Table 6.

Table 6: Game matrix

<table>
<thead>
<tr>
<th>Bank</th>
<th>N</th>
<th>C’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L: 21.43 / 28.57</td>
<td>C’: 30.29 / 19.71</td>
</tr>
<tr>
<td>H: 57.14 / -14.29</td>
<td>29.71 / 13.14</td>
<td></td>
</tr>
</tbody>
</table>

If we look at the combination H/C’, we see that the businessman now has an incentive to deviate from project H and choose project L as this would increase his NPV from 29.71 to 30.29. However, the combination L/C’ is not an equilibrium since the bank would not offer contract C’ given that contract N increases its cash flow from 19.71 to 28.57. No Nash equilibrium in pure strategies can be determined as each player keeps changing the strategy as soon as the other player’s action is known.
In such a situation, both do best by keeping the other guessing. One way of doing this is by randomly selecting a strategy. This random selection is what game theory refers to as “mixed strategy,”\(^\text{10}\) which is a probability distribution over pure strategies. The player calculates probabilities for all pure strategies, and the final output is driven by a probability mechanism. A Nash equilibrium in mixed strategies is given where the players choose the probability distribution that maximizes their expected cash flows. The beauty comes with the general existence of a Nash equilibrium in mixed strategies, whereas oftentimes no equilibrium in pure strategies can be found.

To determine the equilibrium in mixed strategies, we need to calculate endogenous probabilities for the businessman to choose between projects L and H and for the bank to choose between contracts N and C’. Assume that the businessman takes project L with probability Prob(L) and the bank selects credit contract N with probability Prob(N). It follows that the probabilities for H and C’ are Prob(H) = 1-Prob(L) and Prob(C’) = 1-Prob(N), respectively. We can now calculate the expected cash flows for the businessman (E(businessman)) and the bank (E(bank)) and determine the optimum by setting the first derivative equal zero:

\[
\begin{align*}
E(\text{businessman}) &= \text{Prob}(L) \times \text{Prob}(N) \times 21.43 \\
& \quad + \text{Prob}(L) \times (1 - \text{Prob}(N)) \times 30.29 \\
& \quad + (1 - \text{Prob}(L)) \times \text{Prob}(N) \times 57.14 \\
& \quad + (1 - \text{Prob}(L)) \times (1 - \text{Prob}(N)) \times 29.71 \\
\frac{\partial E(\text{businessman})}{\partial \text{Prob}(L)} &= \text{Prob}(N) \times 21.43 + (1 - \text{Prob}(N)) \times 30.29 \\
& \quad - \text{Prob}(N) \times 57.14 - (1 - \text{Prob}(N)) \times 29.71 = 0 \\
& \quad \leftrightarrow \quad \text{Prob}(N) = 1.6% \\
\end{align*}
\]

\[
\begin{align*}
E(\text{bank}) &= \text{Prob}(L) \times \text{Prob}(N) \times 28.57 \\
& \quad + \text{Prob}(L) \times (1 - \text{Prob}(N)) \times 19.71 \\
& \quad + (1 - \text{Prob}(L)) \times \text{Prob}(N) \times (-14.29) \\
& \quad + (1 - \text{Prob}(L)) \times (1 - \text{Prob}(N)) \times 13.14 \\
\frac{\partial E(\text{bank})}{\partial \text{Prob}(N)} &= \text{Prob}(L) \times 28.57 - \text{Prob}(L) \times 19.71 \\
& \quad - (1 - \text{Prob}(L)) \times (-14.29) - (1 - \text{Prob}(L)) \times 13.14 = 0 \\
& \quad \leftrightarrow \quad \text{Prob}(L) = 75.6% \\
\end{align*}
\]

\(^{10}\) See, for example, Charalambos Aliprantis and Subir Chakrabarty (2000), “Games and Decision Making”, Oxford University Press 2000, p. 68.
Each player chooses the strategy with the highest endogenous probability. The businessman chooses project L with a probability of 75.6% while the bank chooses credit contract C’ with a probability of 98.4%. It follows that the combination of L/C’ will be chosen in 74.4% of all cases. It is important to note that this is no longer a 100 percent secure combination as we have seen under pure strategies. No unique solution exists, and neither player is able to determine a clear course of action. This shows the disadvantage of the strategic approach.

**IV. Reasons for Maintaining the Old Practices**

The arguments against the efficiency model are telling and substantial. However, even after the crisis, bankers, risk managers and other financial professionals are not willing to abandon the old practices. Why are clients, taxpayers, and politicians accepting this? We see the following possible explanations:

(1) The efficiency model survived for quite some time, which many take as a rationale for its validity. Yet even a constructionally defective model can persist if it is not incriminated. This is exactly what happened: Before the American real estate market broke down, the model never had to withstand a stress test.

(2) The Basel regulations determine the minimum amount of capital, which is calculated using the Value at Risk (VaR) concept. This concept is based on the efficiency approach and therefore forces bankers to stick to it.

(3) Governments of many countries stabilized the banking sector with subventions to avoid its breakdown. Bankers and financial professionals know that politicians would and will always act in this way to avoid adverse social consequences. Yet this in turn takes away the risk and hence does not incentivize bankers and others to abandon their old practices.

(4) Whereas the efficiency approach provides a scapegoat for bad results (i.e., any loss is the model’s fault), financial professionals are taken into account under the strategic approach since they have to determine the distribution rules.

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11 It should be pointed out that the endogenous probabilities relate to the actions of the two players. They must not be confused with the exogenous probabilities for the two states of the world in the efficiency approach.

12 Pablo Triana has requested abandoning Value at Risk; see Pablo Triana (2009b), “President Obama, Please Kill VaR”, The Huffington Post, June 25, 2009.
(5) The assumption of an exogenous density function allows for the calculation of concrete prices for complicated financial instruments like derivatives. Without concrete prices, these instruments could not be sold, and many finance professionals would lose a lucrative business segment.

**V. Concluding Remarks**

As mentioned herein, the prospective exorbitant future gains incentivized bankers to apply the efficiency model. We also explained that the efficiency model views incentives as being irrelevant. This is an inconsistency in the model design: If incentives are irrelevant, the financial crisis should not have occurred. The fact that it unfortunately was a real and not just a virtual phenomenon proves that incentives play a non-negligible role. The only rational conclusion is that the existing model should be abandoned and replaced by a strategic approach.

Finally, we would like to briefly address the question of guilt. Thus far, we have mostly held bankers (and other financial professionals) accountable for the dilemma. Yet bankers got their education from universities and institutes offering international professional certifications, whose curricula are dominated by the efficiency approach without introducing the dangers of blindly applying the taught models. Therefore, the first step in addressing the chaos of today’s financial markets has to be an enhancement of university and institute curricula to include lessons on strategic thinking for future financial professionals.
References


