Marshallian Externality, Industrial Upgrading, and Industrial Policies

Jiandong Ju
Justin Yifu Lin
Yong Wang

The World Bank
Development Economics Vice Presidency
September 2011
Abstract

A growth model with multiple industries is developed to study how industries evolve as capital accumulates endogenously when each industry exhibits Marshallian externality (increasing returns to scale) and to explain why industrial policies sometimes succeed but sometimes fail. The authors show that, in the long run, the laissez-faire market equilibrium is Pareto optimal when the time discount rate is sufficiently small or sufficiently large. When the time discount rate is moderate, there exist multiple dynamic market equilibria with diverse patterns of industrial development. To achieve Pareto efficiency, it would require the government to identify the industry target consistent with the comparative advantage and to coordinate in a timely manner, possibly for multiple times. However, industrial policies may make people worse off than in the market equilibrium if the government picks an industry that deviates from the comparative advantage of the economy.

This paper is a product of the Development Economics Vice Presidency. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The authors may be contacted at jdju@ou.edu, justinlin@worldbank.org, and yongwang@ust.hk.
Abstract
A growth model with multiple industries is developed to study how industries evolve as capital accumulates endogenously when each industry exhibits Marshallian externality (increasing returns to scale) and to explain why industrial policies sometimes succeed but sometimes fail. The authors show that, in the long run, the laissez-faire market equilibrium is Pareto optimal when the time discount rate is sufficiently small or sufficiently large. When the time discount rate is moderate, there exist multiple dynamic market equilibria with diverse patterns of industrial development. To achieve Pareto efficiency, it would require the government to identify the industry target consistent with the comparative advantage and to coordinate in a timely manner, possibly for multiple times. However, industrial policies may make people worse off than in the market equilibrium if the government picks an industry that deviates from the comparative advantage of the economy.

Key Words: Marshallian Externality, Industrial Policies, Structural Transformation, Industrial Upgrading, Economic Growth

JEL Codes: L50, O14, O25, O41

This paper benefits from the discussions with Gene Grossman, Norman Loayza, and Aart Kraay. Roula Yazigi provides excellent editorial support. Wang gratefully acknowledges the hospitality and financial support from the World Bank during his stay in the 2010-2011 academic year.

1University of Oklahoma and Tsinghua University. Email Address: jdju@ou.edu
2World Bank. Email address: justinlin@worldbank.org
3Hong Kong University of Science and Technology. Email address: yongwang@ust.hk
1 Introduction

This paper studies the role of industrial policies in a growth model with multiple industries. In the real world, for better or worse, almost all the economies have adopted and are still adopting various industrial policies. Yet the effect of industrial policies is always highly controversial. On the one hand, many empirical and case studies suggest that industrial policies have mostly failed and the overall performance of industrial policies is very mixed and insignificant; but on the other hand, ample research, especially case studies, contends that industrial policies and government facilitation are crucially important for most, if not all, of the cases of successful industrial development.

The main purpose of this paper is to help reconcile these two different empirical facts and views, which tend to convey opposite implications for the desirability of industrial policies. More specifically, we try to shed some new light on why "similar" industrial policies work in some cases but fail in others. Any useful discussion on industrial policies must involve the existence of certain market imperfections. In this paper we will revisit and focus on the issue of Marshallian externality, the importance of which is widely recognized in the existing literature. For example, one influential study on coordination failure with the presence of externality is the big-push theory proposed by Rosenstein-Roden and beautifully formalized by Murphy, Shleifer and Vishny (1989). The key policy implication derived from this literature is that government intervention (support) is justified if an industry exhibits Marshallian externality.

However, if the existence of Marshallian externality is a sufficient condition for government intervention, how to explain why many "big-push" industrial policies failed? For example, from the 1960s to the 1980s, the former Soviet Union set the capital-intensive and technology-intensive aerospace industry as its target industry, in which Marshallian externality indeed existed. The government tried hard to coordinate and push the industry to be established. But the social welfare loss actually exceeded the gain from such intervention because the resources for consumption goods production were tremendously reduced and resource allocation created huge distortions when developing such an excessively capital-intensive industry. Similar stories of industrial policies could also be told about China, India and many other developing countries after World War II. The dominant social thought at that time was interventionism based on the rationale of "big push". These countries tried to emulate the rich countries by establishing as soon as possible the same industries that prevailed in the most developed countries. Those target industries were typically capital-intensive and also had significant Marshallian externality. Coordination was indeed implemented but

---

1 See, for example, Wade (1990) and Chang (2003), Lin (2009) and Lin and Monge (2010).
2 For example, Harrison and Rodriguez-Clare (2009) provide a nice literature review. Also see Pack and Saggi (2006) and the papers cited there.
3 For example, Canda (2006) provides fourteen detailed cases studies for successful industrial upgrading; Rodrik (1996, 2006) discussed, among others, the positive role of industrial policies in several East Asia economies; Ohashi (2005) studies the steel industry in Japan. Also see Lin and Monge (2010) and the papers cited in all the aforementioned research.
the industrial development largely failed because the target industries simply went against the comparative advantage of those capital-scarce economies (see Lin, 2009). Meanwhile, by contrast, industrial policies are widely believed successful in Japan, South Korea, Singapore, and Taiwan for the same period. These economies followed comparative advantage and upgraded their industries step by step toward more capital-intensive ones as they accumulated capital on the growth path.

These observations suggest that we should bear in mind the factor endowment when examining industrial policies. Unfortunately, almost the whole existing literature of Marshallian externality and industrial policies assumes, presumably for analytical simplicity, that labor is the only input in the production function. As a result, there is no explicit role for capital and the structure of the factor endowments in the discussion of industrialization and policies. A novel feature of this paper is, therefore, to reexamine the industrial policies by explicitly introducing capital (and its accumulation) into our theoretical analysis. Adding this extra layer of complexity not only makes the model more realistic, but more importantly, there are at least three potential gains that warrant such an investigation.

First, it points to the importance of identifying the right industry target that needs government intervention. The existing literature on coordination failure largely ignores this important issue by typically assuming that there exist two industries with only one industry exhibiting Marshallian externality, thus it is common knowledge which industry needs government support. However, as we argued, the existence of Marshallian externality (or pecuniary externality) alone is not sufficient to warrant government intervention. Factor endowment also matters. We have to take the capital intensity of each different industry and the capital abundance into account when identifying the right industry targets. Moreover, the target industry that needs coordination is also changing endogenously as the structure of factor endowment changes over time. Consequently, in our model we assume that there are three industries with different capital intensities and more than two industries exhibit Marshallian externality. It is necessary to have at least three industries to discuss the effect of industrial policies that aim to facilitate "leap forward" in the industrial upgrading, as occurred in the former Soviet Union and many developing countries.

Second, an important technical question in the literature of industrial policies is how to select an equilibrium because there typically exist multiple equilibria in a coordination game. Discussions have focused on what determines the relative importance of history versus expectation (see, Krugman (1991), Matsuyama (1991), for example). Once capital is introduced into the model, however, agglomeration is no longer the only economic force that determines the production cost (hence the competitiveness) of an industry. The relative price of capital and labor also matters in a general equilibrium fashion. As our model will show, sometimes the factor price effect dominates the agglomeration effect so that the equilibrium may restore uniqueness, making equilibrium selection less controversial.

Third, in the existing literature of coordination failures and industrial poli-
cies, dynamic analyses mainly focus on the stability of an equilibrium by assuming some costly adjustment process in a heuristic and ad hoc way (see Mussa (1978), Panagary (1986), Krugman (1991), etc.) or when there exists demographic change (see Matuyama, 1991). Besides, none of these papers studies the industrial upgrading in the economic growth framework. However, once capital is introduced, it becomes very natural to examine the industrial policy and industrial upgrading issues in the standard Ramsey growth framework with endogenous saving, which allows us to analyze industrial policies along with GDP growth and industrialization.

In light of these arguments, we build a simple growth model with three industries, each of which exhibits increasing returns to scale (Marshallian externality) and differs in capital intensity. The key result is that industrial policies may succeed or fail, depending on whether the target industry for coordination is correctly identified in a timely manner. We show that in some cases industries may eventually upgrade successfully in an intervention-free market economy despite the existence of Marshallian externality and coordination problems. However, industrial upgrading under laissez-faire policies may be seriously delayed and thus can be Pareto improved if the right industry target is identified in time and "pushed" by the government appropriately. But if the government identifies the wrong industry and "pushes" it, the economy would end up in an equilibrium Pareto inferior to the laissez-faire market equilibrium. The "right" industry target in our model endogenously changes over time, depending on the capital endowment of the economy. We hope that our model will help deepen our understanding on the important question why industrial policies have succeeded in some circumstances but failed in others, although Marshallian externality always exists in all these cases.

The rest of the paper is organized as follows. Section 2 studies a static economy with only two industries and the basic economic forces are explained. It is extended to a static economy with three industries in Section 3, which allows for the discussion on "leap-forward" industrial upgrading and policies. In the subsequent sections, the industrial upgrading and policies are analyzed in a Ramsey growth framework. In particular, Section 4 studies the dynamic model in which endowment structure changes endogenously. Section 5 examines the robustness of our theoretical results to the variations of the equilibrium concepts and relevant functional forms. The last section concludes.

2 Two Industries

Consider a static and small open economy populated by \( L \) identical agents, each of whom is endowed with one unit of labor and \( E \) units of capital. There are two different industries, each producing a distinct consumption good. Call them industry 1 and industry 2. The utility function is

\[
U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma},
\]
where the final aggregate consumption \( C \) is a composite of both consumption of good 1 (denoted by \( c_1 \)) and consumption of good 2 (denoted by \( c_2 \)). Function \( C(c_1, c_2) \) can take any form as long as \( C_1(c_1, c_2) > 0 \) and \( C_2(c_1, c_2) > 0 \). Let \( p_i \) denote the world price of consumption good \( i \). Normalize \( p_1 = 1 \) and assume \( p_2 = \lambda \), where \( \lambda > 1 \). Consumption goods can be traded freely at the world market but the production factors (labor and capital) cannot move across borders.

To operate, each firm requires one unit of labor. Therefore, the total number of firms is \( L \) in the economy. The technology to produce good \( i \in \{ 1, 2 \} \) in each firm is:

\[
F_i(k_i) = A(n_i)k_i^{\alpha_i}, \quad \alpha_i \in (0, 1) \tag{1}
\]

where \( n_i \) is the total number of firms in industry \( i \) and \( k_i \) is capital input in each representative firm in industry \( i \). There exists Marshallian externality so \( A(n_i) \) is a strictly increasing function of \( n_i \). To sharpen the analytical results, let us assume \( A(n_i) \equiv A_0 e^{\xi n_i} \), where \( \xi \) and \( A_0 \) are both positive parameters. Without loss of generality assume \( \alpha_2 > \alpha_1 \) so good 2 is more capital-intensive. \( L \) is an integer, so is \( n_i \).

Both labor and capital can freely move across the two industries. All the markets are perfectly competitive and each firm earns zero profit in the equilibrium after paying the capital and labor cost. Let \( w \) and \( r \) denote the equilibrium wage and rental price of capital.

**Lemma 1** If both industries operate, then the capital of each firm in an industry will strictly increase with the number of firms in the same industry.

**Proof.** The equalization of capital return across the two industries implies

\[
\alpha_1 A(n_1) \left( \frac{E - (L - n_1)k_2}{n_1} \right)^{\alpha_1 - 1} = \alpha_2 \lambda A(L - n_1)k_2^{\alpha_2 - 1}, \tag{2}
\]

which immediately implies that \( k_2(n_1) \) is a strictly decreasing function of \( n_1 \) and hence a strictly increasing function of \( n_2 \). (2) can be also rewritten as

\[
\alpha_1 A(n_1)k_4^{\alpha_1 - 1} = \alpha_2 \lambda A(L - n_1) \left( \frac{E - Lk_1}{L - n_1} + k_1 \right)^{\alpha_2 - 1},
\]

which implies that \( k_1(n_1) \) must be a strictly increasing function. \( \blacksquare \)

The economic intuition is straightforward. When an agent moves, say, from industry 1 to industry 2, the marginal productivity of capital of each firm in industry 2 strictly increases and thus becomes larger than that in industry 1, so capital is attracted to industry 2.

**Proposition 2** In any equilibrium, at most one industry exists.
Proof. By contradiction, suppose in some equilibrium both industries coexist, that is, \( n_i \geq 1 \) for both \( i = 1, 2 \). The free mobility of capital implies:

\[
r = \alpha_1 A(n_1)k_1^{\alpha_1 - 1} = \alpha_2 \lambda A(n_2)k_2^{\alpha_2 - 1}
\]

(3) Zero profit condition, equalization of wage across the two industries and (3) jointly imply

\[
w = (1 - \alpha_1)A(n_1)k_1^{\alpha_1} = \lambda(1 - \alpha_2)A(n_2)k_2^{\alpha_2}.
\]

(4) Labor market clearing condition is

\[
n_1 + n_2 = L.
\]

(5) Capital market clearing condition is

\[
n_1 k_1 + n_2 k_2 = E.
\]

(6) So we have four unknowns: \( n_1, n_2, k_1, k_2 \) and four equations (3) to (6). Then \( r \) and \( w \) can be pinned down by (3) and (4) respectively. Consider an individual agent (firm) in industry 1. If it unilaterally moves to industry 2, then

\[
n_1' = n_1 - 1, n_2' = n_2 + 1,
\]

this deviating agent rationally expects that the market-clearing rental price of capital would change from \( r \) to \( r' \), correspondingly, \( k_1 \) and \( k_2 \) also change to \( k_1' \) and \( k_2' \), which must still satisfy (3) and (6), it remains to check whether the (entrepreneurial) wage becomes better.

\[
(1 - \alpha_1)A(n_1)k_1^{\alpha_1} < \lambda(1 - \alpha_2)A(n_2 + 1)k_2^{\alpha_2},
\]

which, according to (4), is equivalent to

\[
\lambda(1 - \alpha_2)A(n_2)k_2^{\alpha_2} < \lambda(1 - \alpha_2)A(n_2 + 1)k_2^{\alpha_2},
\]

or

\[
k_2^{\alpha_2} < \left[ \frac{A(n_2 + 1)}{A(n_2)} \right]^{\frac{1}{\alpha_2}} = e^{\xi/\alpha_2},
\]

which must hold because the previous Lemma implies \( k_2 < k_2' \). It contradicts that the economy was in an equilibrium. ■

This proposition states that the two industries cannot coexist in any equilibrium. Next we explore the necessary and sufficient conditions under which only one industry exists. First suppose it is an equilibrium that all the firms choose to stay in industry 1, then

\[
r = \alpha_1 A(L)\left( \frac{E}{L} \right)^{\alpha_1 - 1}; \quad w = (1 - \alpha_1)A(L)\left( \frac{E}{L} \right)^{\alpha_1}.
\]
Now consider an individual agent (firm) in industry 1. Suppose it unilaterally shifts to industry 2, then
\[ n_1' = L - 1, n_2' = 1, \]
this deviating agent rationally expects that market clearing rental price of capital may change from \( r \) to \( r' \), correspondingly, \( k_1 \) and \( k_2 \) also change to \( k_1' \) and \( k_2' \) to equalize the marginal return to capital in both industries after this deviation. That is,
\[ r' = \alpha_1 A(L - 1)k_1'^{(\alpha_1 - 1)} = \alpha_2 \lambda A(1)k_2'^{(\alpha_2 - 1)} \]
and the capital market must remain clear
\[ (L - 1)k_1' + k_2' = E. \]
These two equations imply
\[ \alpha_1 A(L - 1) \left[ \frac{E - k_2'}{L - 1} \right]^{\alpha_1 - 1} = \alpha_2 \lambda A(1)k_2'^{(\alpha_2 - 1)} \quad (7) \]
which uniquely determines \( k_2' \). It remains to check whether the (entrepreneurial) wage of this agent becomes worse after the deviation:
\[ (1 - \alpha_1)A(L) \left( \frac{E}{L} \right)^{\alpha_1} > \lambda(1 - \alpha_2)A(1)k_2'^{\alpha_2}, \]
which can be rewritten as
\[ \left[ \frac{(1 - \alpha_1)A(L) \left( \frac{E}{L} \right)^{\alpha_1}}{\lambda(1 - \alpha_2)A(1)} \right]^{1/\alpha_2} > k_2'. \quad (8) \]
(8) holds if and only if \( E < E^* \), where \( E^* \) can be uniquely determined by
\[ \alpha_1 A(L - 1) \left[ \frac{E^* - \left( \frac{(1 - \alpha_1)A(L)(E^*)^{\alpha_1}}{\lambda(1 - \alpha_2)A(1)} \right)^{1/\alpha_2}}{L - 1} \right]^{\alpha_1 - 1} = \alpha_2 \lambda A(1) \left[ \frac{(1 - \alpha_1)A(L) \left( \frac{E^*}{L} \right)^{\alpha_1}}{\lambda(1 - \alpha_2)A(1)} \right]^{(\alpha_2 - 1)/\alpha_2}. \quad (9) \]
In other words, "all firms stay in industry 1" is a Nash equilibrium if and only if \( E \leq E^* \) \((\lambda, L, \alpha_1, \alpha_2, \xi)\). We can show that
\[ \frac{\partial E^*}{\partial \lambda} (\lambda, L, \alpha_1, \alpha_2, \xi) < 0; \quad \lim_{\lambda \to \infty} E^* (\lambda, L, \alpha_1, \alpha_2, \xi) = 0 \quad (10) \]
\[ \frac{\partial E^*}{\partial L} (\lambda, L, \alpha_1, \alpha_2, \xi) < 0, \text{ suppose } L > \frac{\alpha_1}{\xi} \quad (11) \]
\[ \frac{\partial E^*}{\partial \xi} (\lambda, L, \alpha_1, \alpha_2, \xi) > 0, \text{ suppose } L > 1 + \alpha_2 \quad (12) \]
The intuition for (10) is that as the price of good 2 increases, it would strengthen the incentive of a marginal firm to deviate to industry 2, so capital endowment has to be smaller to keep the marginal firm from deviating. (11) is due to the fact that the marginal impact of each firm on the strength of Marshallian externality becomes larger as the number of firms increases (due to the exponential functional form), therefore once a firm deviates from industry 1 to industry 2, the marginal decrease in the productivity of the firms in industry 1 becomes larger if the total number of firms increases, therefore, it makes unilateral deviation more attractive. (12) is because the stronger the Marshallian externality in the current industry, the weaker the incentive to deviate away from the less capital-intensive industry.

Conjecture

\[ \frac{\partial E^* (\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial \alpha_2} < 0; \quad \frac{\partial E^* (\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial \alpha_1} > 0. \]

Similarly, if it is an equilibrium that all the firms choose to stay in industry 2, then

\[ r = \alpha_2 A(L) \left( \frac{E}{L} \right)^{\alpha_2 - 1}; \quad w = (1 - \alpha_2) A(L) \left( \frac{E}{L} \right)^{\alpha_2}. \]

Now consider an individual agent (firm) in industry 2, if it unilaterally shifts to industry 1, then

\[ n'_1 = 1, n'_2 = L - 1, \]

this deviating agent rationally expects that market clearing rental price of capital may change from \( r \) to \( r' \), correspondingly, \( k_1 \) and \( k_2 \) also change to \( k'_1 \) and \( k'_2 \) to equalize the marginal return to capital in both industries after this deviation. That is,

\[ r' = \alpha_1 A(1) k'^{\alpha_2 - 1}_1 = \alpha_2 \lambda A(L - 1) k'^{\alpha_2 - 1}_2 \]

and the capital market must remain clear

\[ (L - 1) k'_2 + k'_1 = E. \]

These two equations imply

\[ \alpha_1 A(1) k'^{\alpha_1 - 1}_1 = \alpha_2 \lambda A(1) \left( \frac{E - k'_1}{L - 1} \right)^{\alpha_2 - 1}, \]

which uniquely determines \( k'_1 \). It remains to check whether the (entrepreneurial) wage of the deviating agent becomes worse.

\[ \left[ \frac{(1 - \alpha_2) A(L) \left( \frac{E}{L} \right)^{\alpha_2}}{\lambda (1 - \alpha_1) A(1)} \right]^{\frac{1}{\alpha_1}} > k'_1. \]  

(13) holds if and only if \( E > E^{**} \), where \( E^{**} \) is uniquely determined by
In other words, "all firms stay in industry 2" is a Nash equilibrium if and only if \( E > E^{**} \). \( E^{**} = E^{**}(\lambda, L, \alpha_1, \alpha_2) \). We can show that

\[
\frac{\partial E^{**}(\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial \lambda} > 0, \text{ when } \alpha_1 < \frac{1}{2}; \\
\frac{\partial E^{**}(\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial L} < 0, \text{ suppose } L > \frac{\alpha_1}{\xi}; \\
\frac{\partial E^{**}(\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial \xi} < 0, \text{ suppose } L > 1 + \alpha_2.
\]

The intuition for (15) is that, since the price of good 2 is multiplied to the productivity of the firms in industry 2, an increase in the price of good 2 will amplify the marginal loss in the attractiveness of industry 2 when a firm unilaterally deviates from industry 2 to industry 1, therefore making such a deviation more likely to happen. The intuition for (16) is that, as the total number of firms increases in industry 2, the Marshallian externality becomes stronger; therefore the capital stock must be sufficiently scarce (hence expensive) in order to induce one firm to unilaterally deviate from industry 2 to industry 1, which employs less capital. (17) is due to the fact that industry 2 becomes more attractive as the Marshallian externality is strengthened in that industry, so it requires the capital stock to be more scarce to induce a firm to deviate from industry 2 to industry 1.

**Conjecture**

\[
\frac{\partial E^{**}(\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial \alpha_2} > 0; \quad \frac{\partial E^{**}(\lambda, L, \alpha_1, \alpha_2, \xi)}{\partial \alpha_1} < 0
\]

**Remark 3** When \( E^{**} \leq E^* \), we have the following result: When \( E \in (0, E^{**}) \), there is a unique equilibrium, in which all the firms are in industry 1; when \( E \in [E^{**}, E^*) \), there are two equilibria: "all in industry 1" and "all in industry 2"; when \( E \in [E^*, \infty) \), there is a unique equilibrium, in which all the firms are in industry 2.

**Remark 4** When \( E^* < E^{**} \), we have the following result: When \( E \in (0, E^*) \), there is a unique equilibrium, in which all the firms are in industry 1; When \( E \in (E^{**}, E^*) \), there is no pure strategy equilibrium; When \( E \in [E^{**}, \infty) \), there is a unique equilibrium, in which all the firms are in industry 2.

**Remark 5** \( E^{**} \leq E^* \) when \( \lambda \) is sufficiently small.
3 Three Industries

Now suppose there is another industry called industry 3 with production function given by (1) and \( \lambda_3 > \alpha_2 \). Assume \( p_3 = \lambda_3 > p_2 = \lambda_2 > p_1 = 1 \).

Define

\[
E_1 \equiv E^*(\lambda_2, L, \alpha_1, \alpha_2) \\
E_2 \equiv E^*(\lambda_3, L, \alpha_1, \alpha_3) \\
E_3 \equiv E^{**}(\lambda_2, L, \alpha_1, \alpha_2) \\
E_4 \equiv E^*(\frac{\lambda_3}{\lambda_2}, L, \alpha_2, \alpha_3) \\
E_5 \equiv E^{**}(\frac{\lambda_3}{\lambda_2}, L, \alpha_2, \alpha_3) \\
E_6 \equiv E^{**}(\lambda_3, L, \alpha_1, \alpha_3)
\]

where functions \( E^*(\ldots, \ldots) \) and \( E^{**}(\ldots, \ldots) \) are given by (9) and (14), respectively. Obviously, \( 0 < E_i < \infty \) for any \( i \in \{1, 2, 3, 4, 5, 6\} \).

**Proposition 6** There must exist at most one industry in any equilibrium. "All firms are in industry 1" is an equilibrium if and only if \( E \leq \min\{E_1, E_2\} \); "all firms are in industry 2" is an equilibrium if and only if \( E \in [E_3, E_4] \); and "all firms are in industry 3" is an equilibrium if and only if \( E \geq \max\{E_5, E_6\} \).

**Proof.** It is straightforward to show that whenever \( E \leq E_1 \), no firms have incentive to unilaterally deviate to industry 2 if initially all firms are in industry 1. Whenever \( E \leq E_2 \), no firms have incentive to unilaterally deviate to industry 3 if initially all firms are in industry 1; Therefore, if and only if \( E \leq \min\{E_1, E_2\} \), it is an equilibrium that "all firms are in industry 1". Similarly, whenever \( E \geq E_3 \), no firms have incentive to unilaterally deviate to industry 1 if initially all firms are in industry 2. Whenever \( E \leq E_4 \), no firms have incentive to unilaterally deviate to industry 3 if initially all firms are in industry 2. Therefore, whenever \( E_3 \leq E \leq E_4 \), it is an equilibrium that "all firms are in industry 2". Whenever \( E \geq E_5 \), no firms have incentive to unilaterally deviate to industry 2 if initially all firms are in industry 3; whenever \( E \geq E_6 \), no firms have incentive to unilaterally deviate to industry 1 if initially all firms are in industry 3; so whenever \( E \geq \max\{E_5, E_6\} \), it is an equilibrium that "all firms are in industry 3".

This proposition implies that "all firms are in industry 1" is a unique equilibrium so long as \( E \) is sufficiently small. Likewise, "all firms are in industry 3" is a unique equilibrium so long as \( E \) is sufficiently large. However, nothing ensures the non-emptiness of the interval \([E_3, E_4]\), so it is possible that "all firms are in industry 2" can never be an equilibrium for any \( E \).

In addition, there could be multiple equilibria for any given \( E \). For example, whenever \( E \in \{\max\{E_5, E_6\}, \min\{E_1, E_2\}\} \cap [E_3, E_4] \), there exist three equilibria: "all in industry 1", "all in industry 2" and "all in industry 3". Whenever there exist multiple equilibria, those equilibria can always be Pareto ranked.
Lemma 7 \[ \max\{E_3, E_5\} < E_6 \text{ and } E_2 < \min\{E_1, E_4\}. \]

**Proof.** Because of the properties of functions \( E^*(\cdot, \cdot, \cdot) \) and \( E^{**}(\cdot, \cdot, \cdot) \) together with the facts that \( \alpha_1 < \alpha_2 < \alpha_3 \) and \( \lambda_1 < \lambda_2 < \lambda_3 \). □

**Corollary 8** Suppose \( \lambda_3 \) is sufficiently small. When \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_6) \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in [E_6, E_2] \), there are three equilibria: all in 1, all in 2, and all in 3; when \( E \in (E_2, E_4] \), there are two equilibria: all in industry 2 and all in industry 3; when \( E \in (E_4, \infty) \), there is a unique equilibrium, in which all stay in industry 3.

**Proof.** \( E_6 \leq E_2 \) when \( \lambda_3 \) is sufficiently small, so the previous lemma implies
\[
\max\{E_3, E_5\} < E_6 \leq E_2 < \min\{E_1, E_4\}. \tag{18}
\]

The rest of the argument comes from Proposition 6. □

Appendix 1 discusses what happens if (18) is not satisfied. It would be useful to know when which industry is Pareto optimal. For any \( i < j \), the total output when "all firms are in industry \( i \)" is \( \lambda_i A(L) L^{\frac{1}{\alpha_i}} \), which dominates the output when "all firms are in industry \( j \)" if and only if \( E < \left( \frac{\lambda_j}{\lambda_i} \right)^{\frac{1}{\alpha_j - \alpha_i}} L \).

Define
\[
\theta_1 \equiv \left( \frac{\lambda_2}{\lambda_1} \right)^{\frac{1}{\alpha_1 - \alpha_2}} L; \quad \theta_2 \equiv \left( \frac{\lambda_3}{\lambda_2} \right)^{\frac{1}{\alpha_2 - \alpha_3}} L; \quad \theta_3 \equiv \left( \frac{\lambda_3}{\lambda_1} \right)^{\frac{1}{\alpha_1 - \alpha_3}} L.
\]

We have the following useful lemma:

**Lemma 9** Suppose \( \theta_1 < \theta_3 < \theta_2 \). Industry 1 (that is, all firms are in industry 1) Pareto dominates when \( E < \theta_1 \), industry 2 Pareto dominates when \( E \in (\theta_1, \theta_2) \), industry 3 Pareto dominates when \( E \in (\theta_2, \infty) \).

For example, \( \theta_1 < \theta_3 < \theta_2 \) hold when
\[
\frac{\lambda_2}{\lambda_1} = \frac{\lambda_3}{\lambda_2} > 1 \text{ and } \alpha_1 - \alpha_2 > \alpha_2 - \alpha_3 > \alpha_1 - \alpha_3.
\]

This lemma tells us which is the Pareto optimal industry, but nothing so far ensures that these Pareto optimal outcomes be automatically achieved by the free market.

**Conjecture 10** The social optimal industry is always a Nash equilibrium. That is, \( \theta_1 \in (0, E_2], \theta_2 \in (E_3, E_4], \theta_3 \in [E_6, \infty) \).
4 Dynamic Model

Consider a dynamic model

$$\max_{C(t),i(t)} \int_0^\infty \frac{C(t)^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt$$

subject to

$$\dot{K}(t) = F[K(t), i(t)] - \delta K(t) - C(t)$$
$$K(0) = K_0 \text{ is given and sufficiently small}$$

where $i(t) \in \{1, 2, 3\}$ indicates the industry choice at time $t$ and the corresponding production function is

$$F[K(t), i] = \lambda_i A(L)L^{1-\alpha_i}K^{\alpha_i}, \text{ for } i = 1, 2, 3.$$  

Obviously, the existence of Marshallian externality makes the welfare theorems no longer applicable here. Therefore, we will proceed by first characterizing the Pareto optimal allocation that a benevolent social planner would choose. Then we will analyze the market equilibrium without government intervention, followed by a comparative analysis of the two scenarios, based on which policies will be discussed. For analytical simplicity, attention will be mainly focused on the case without productivity growth within an industry.

4.1 Pareto Optimal Allocation

Obviously, the social planner would choose the Pareto optimal industry based on Lemma 9.

$$i^{PO}(t) = \begin{cases} 1, & \text{when } K(t) \leq \theta_1 \\ 2, & \text{when } \theta_1 < K(t) \leq \theta_2 \\ 3, & \text{when } K(t) > \theta_2 \end{cases}$$ (19)

Establish the discounted-value Hamiltonian when $K(t) \leq \theta_1$,

$$H = \frac{C(t)^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} + \eta \left[ \lambda_1 A(L) L^{1-\alpha_1} K^{\alpha_1} - \delta K(t) - C(t) \right] + \psi_1 [\theta_1 - K(t)],$$

where $\eta$ is the co-state variable and $\psi_1$ is the Lagrangian multiplier. First order conditions and K-T condition are

$$e^{-\rho t} C(t)^{-\sigma} = \eta$$
$$\dot{\eta} = -\frac{\partial H}{\partial K} = -\alpha_1 \eta \lambda_1 A(L) L^{1-\alpha_1} K^{\alpha_1 - 1} + \psi_1$$
$$\psi_1 [\theta_1 - K(t)] = 0, \psi_1 \geq 0, K(t) \leq \theta_1$$
When $K(t) < \theta_1$, we have

$$-\rho - \frac{C(t)}{C(t)} = -\alpha_1 \lambda_1 A(L) L^{1-\alpha_1} K^{\alpha_1-1}$$

$$\dot{K}(t) = \lambda_1 A(L) L^{1-\alpha_1} K^{\alpha_1} - \delta K(t) - C(t).$$

so the steady state is

$$K_{ss}^1 = \left[ \frac{\alpha_1 \lambda_1 A(L)}{\rho} \right]^{\frac{1}{1-\alpha_1}} L$$

$$C_{ss}^1 = \lambda_1 A(L) L^{1-\alpha_1} (K_{ss}^1)^{\alpha_1} - \delta K_{ss}^1$$

$$= [\rho - \delta \alpha_1] \alpha_1 \frac{\lambda_1 A(L)}{\rho} \left[ \frac{\lambda_1 A(L)}{\rho} \right]^{\frac{1}{1-\alpha_1}} L.$$

Similarly, we obtain when $K(t) \in (\theta_1, \theta_2)$, the steady state is

$$K_{ss}^2 = \left[ \frac{\alpha_2 \lambda_2 A(L)}{\rho} \right]^{\frac{1}{1-\alpha_2}} L$$

$$C_{ss}^2 = [\rho - \delta \alpha_2] \alpha_2 \frac{\lambda_2 A(L)}{\rho} \left[ \frac{\lambda_2 A(L)}{\rho} \right]^{\frac{1}{1-\alpha_2}} L.$$

and when $K(t) \in (\theta_2, \infty)$, the steady state is

$$K_{ss}^3 = \left[ \frac{\alpha_3 \lambda_3 A(L)}{\rho} \right]^{\frac{1}{1-\alpha_3}} L$$

$$C_{ss}^3 = [\rho - \delta \alpha_3] \alpha_3 \frac{\lambda_3 A(L)}{\rho} \left[ \frac{\lambda_3 A(L)}{\rho} \right]^{\frac{1}{1-\alpha_3}} L.$$

We restrict the parameters such that

$$K_{ss}^1 < K_{ss}^2 < K_{ss}^3, C_{ss}^1 < C_{ss}^2 < C_{ss}^3.$$

Suppose $K_{ss}^i > \theta_i$ for both $i = 1, 2$. There is a unique Pareto optimal dynamic allocation as illustrated by curve $BB$ in the following phase diagram (see Figure 1). The economy starts with a sufficiently small initial capital stock

4For example, it holds when

$$\alpha_1 = 0.1; \alpha_2 = 0.2, \alpha_3 = 0.4, \lambda_2 = 1.2, \lambda_3 = 1.44, \xi = 0.03, L = 100.$$ and

$$\delta = 0.05, \rho = 0.05,$$

because then we have

$$E_3 < E_6 < \theta_1 < \theta_2 < K_{ss}^1 < K_{ss}^2 < E_2 < K_{ss}^3 < E_4.$$ (20)
$K_0$, and all the firms stay in industry 1, so the economy will move northeast along the curve $BB$ until capital reaches $\theta_1$, at which point all the firms move to industry 2, so the economy continues to move northeast until the capital reaches $\theta_2$, at which point all the firms simultaneously shift to industry 3 and stay there afterwards. Therefore, the economy will follow the saddle path and eventually converge to the steady state $SS_3$. In other words, it requires sequential multiple coordination to achieve the Pareto optimality.

[Figure 1]

Suppose $K_1^{**} > \theta_1$ but $K_2^{**} < \theta_2$. There are still several different possibilities. When $\theta_2$ is sufficiently close to $K_2^{**}$, the Pareto optimal allocation may follow a non-monotonic path as illustrated by curve $BHB$ in Figure 2. The economy starts from industry 1 and then shifts to industry 2 when capital reaches $\theta_1$, after which the economy stays in industry 2 until capital reaches $\theta_2$ at point $H$. The economy stays in industry 3 afterwards and eventually converges to steady state $SS_3$.

[Figure 2]

However, if $\theta_2$ is sufficiently larger than $K_2^{**}$ but not too large (close to $K_3^{**}$), then in order to approach steady state 3 by following the rule (19), the economy would have to follow a discontinuous path as illustrated in Figure 3. The economy initially follows the lotus $BH$ and consumption jumps from $H$ to $D$ precisely when capital reaches $\theta_2$, the economy shifts to industry 3 afterwards and eventually converges to steady state $SS_3$.

[Figure 3]

Discontinuous consumption is certainly not desirable for consumers and hence cannot be Pareto optimal. When $\theta_2$ is sufficiently larger than $K_2^{**}$, it may be strictly better off to give up industry 3 and instead converge to steady state $SS_2$ with industry 2 although it is feasible to upgrade to industry 3. See Figure 4.

[Figure 4]

Similar reasoning also applies when $K_1^{**} < \theta_1$, in which case the Pareto optimal dynamic allocation may eventually converge to steady state $SS_1$ with industry 1 when $\theta_1$ exceeds $K_1^{**}$ significantly.

To summarize,

**Proposition 11** Suppose $K(0)$ is sufficiently small. When the time discount rate $\rho$ is sufficiently small ( $K_i^{**} > \theta_i$ for both $i = 1, 2$), the Pareto optimal allocation is such that industries will upgrade step by step from industry 1 to industry 2 and then to industry 3. When $\rho$ is sufficiently large ( $K_1^{**} \ll \theta_1$), the Pareto optimal allocation is that the industries will remain in industry 1. When $\rho$ is in some middle range, the Pareto optimal allocation is such that the industries will first stay in industry 1 and then upgrade to industry 2 and stay there forever.
4.2 Laissez-faire Market Equilibrium

Next let us examine the market equilibrium (or equilibria). At each time point, given the inherited capital stock, the equilibrium industrial choices are the same as in the static model shown in Section 3. That is, all the firms simultaneously and non-cooperatively make their industrial choices (production decision) at each time point. The consumption and saving decisions are made after output is produced. For simplicity, we will focus on the symmetric equilibria in which all the firms behave identically. To simplify the analysis, we assume \( \lambda_3 \) is sufficiently small (so that Corollary 8 applies) and \( \theta_1 < \theta_3 < \theta_2 \) (so that Lemma 9 applies).

Suppose the initial capital stock is sufficiently small \((K(0) < E_3)\), so the market equilibrium must start with industry 1. By revoking Corollary 8, we have the following lemma:

**Lemma 12** Industry 1 can be the industry in the long-run steady state only if \( K_{ss}^1 \in (0, E_2) \), industry 2 can be in the steady state only if \( K_{ss}^2 \in [E_3, E_4] \), industry 3 can be in the steady state only if \( K_{ss}^3 \in [E_4, \infty) \).

This lemma is useful in determining the steady state of the economy. For example, industry 1 cannot be the steady state industry if \( \rho \) is sufficiently small such that \( K_{ss}^1 > E_2 \). Using this logic, we can easily obtain the following proposition after studying the associated Hamiltonian system.

**Proposition 13** When \( \rho \) is sufficiently large such that \( K_{ss}^3 < E_3 \), then the only dynamic equilibrium is that industry 1 will persist forever, which is also Pareto optimal. By contrast, when \( \rho \) is sufficiently small such that \( K_{ss}^1 > E_4 \), the economy will eventually approach steady state 3, although there may exist multiple equilibrium transitional dynamic paths, some of which are strictly Pareto dominated.

**Proof.** According to Corollary 8, the market equilibrium industry must be industry 3 whenever \( K(t) > E_4 \) and must be industry 1 whenever \( K(t) \leq E_3 \).

This proposition implies that the market equilibrium itself is Pareto optimal when people are sufficiently impatient, even though each industry exhibits Marshallian externality. When people are sufficiently patient, the market itself will successfully upgrade the industry and end up with the Pareto optimal industry in the long run. The most complicated case is when \( \rho \) falls in the middle range, to which we now turn.

Now we have \( E_3 \leq K_{ss}^3 \) and \( K_{ss}^1 \leq E_4 \). In general, there could be a continuum of different dynamic market equilibria. For example, consider the following case: \( K_{ss}^1 < E_3 \). One possible market equilibrium is that the economy follows from the beginning the unique saddle path that converges to steady state 1 (denoted by point \( SS_1 \)). Another possible equilibrium is that the economy stays in industry 1 and capital exceeds \( E_3 \) at some time point, then the economy may immediately shift to industry 2 or shift to industry 2 after a while so long
as capital exceeds $E_3$, and the economy eventually converges to steady state 2 (denoted by point $SS_2$). One possible equilibrium path is depicted by curve $BHSS_2$ in Figure 5. Notice that the economy shifts to industry 2 at point $H$, which is indeterminate depending on the initial consumption (that is, the y-axis of point B). In other words, there could be infinite such type of equilibria as long as $K_2^{**} \in [E_3, E_4]$.

[FIGURE 5]

A third possible equilibrium is illustrated in Figure 6. The economy starts with industry 1 at point B and then switches to industry 2 at point $H$ and stays in industry 2 until point $A$, at which the economy shifts to industry 3 and stays there forever by following the unique saddle path leading to steady state 3 (denoted by $SS_3$). Again, there can be an infinite number of such equilibrium as long as $K_3^{**} \in [E_6, \infty)$.

[FIGURE 6]

There may exist other types of equilibria as well. To illustrate the multiplicity of the market equilibria even further, consider the different case, in which $E_3 \leq K_1^{**}$. It is a possible equilibrium that the economy stays in industry 1 until after the capital exceeds $K_1^{**}$ at some time $t$. What happens after time $t$ is as if we have an economy with initial capital $K_0 = K(t)$. What happens when $K_0 \geq E_3$? Suppose $E_3 \leq K_1^{**} \leq K_0 \leq K_2^{**} \leq E_6$.

At time 0, the economy could embark on either industry 1 or industry 2. This can be illustrated in the following phase diagram (Figure 7).

[FIGURE 7]

If the economy chooses industry 1 at time 0 and stays in that industry afterwards, the consumption will be denoted by point $C_0^{AA}$, and the equilibrium capital stock and consumption will follow the unique saddle path $AA$ and gradually decrease and eventually the economy approaches steady state 1. Call this equilibrium 1. If industry 2 is chosen at time 0 and the economy stays in that industry, the initial consumption will be point $C_0^{BB}$ and the equilibrium capital stock and consumption will gradually increase and eventually the economy approaches steady state 2. Call this equilibrium 2.

It is not obvious which equilibrium path is Pareto superior: although the consumption at steady state 2 is strictly higher than in steady state 1, the consumption on the transitional path is strictly larger at the first equilibrium path and this gap exists at least for a while. We know, however, when $\theta_1 \geq K_2^{**}$, the first equilibrium path must Pareto dominate the second equilibrium path. Likewise, when $\theta_1 \leq K_1^{**}$, the second equilibrium path Pareto dominates the first one.

The following phase diagram (Figure 8) demonstrates two other possible types of equilibria, called equilibrium 3 and equilibrium 4, respectively.
In equilibrium 3, the economy starts with consumption at point $C^0$ and the economy stays in industry 1, therefore the equilibrium consumption and capital stock will move southeast until it hits the saddle path $AA$ at point $H$. From that point on, all the firms shift to industry 2, so the equilibrium path will move northeast along the saddle path $BB$, and the economy eventually approaches steady state 2. Since it is a perfect foresight equilibrium in which all the firms know when the industry shift will occur, $C^0$ is optimally chosen such that the economy hits the saddle path $BB$ precisely at the time of industry switch. Likewise, if people commonly expect that the industry switch will occur at some capital level different from $H$, then the initial consumption will also be adjusted accordingly. On this type of equilibrium path, the consumption flow is smooth but non-monotonic. The initial relatively low consumption is to ensure that capital increases enough to reach the saddle path $BB$. Conceptually, we can Pareto rank these equilibria because we can compute the welfare for each equilibrium numerically or analytically. Notice that equilibrium 3 can never be the Pareto optimal equilibrium unless the capital stock at point $H$ is exactly equal to $1$, in which case equilibrium 3 must be Pareto superior to equilibrium 1 (shown in Figure 7).

In equilibrium 4, all the firms first stay in industry 2 with initial consumption denoted by point $C_0^B$ in Figure 8, the equilibrium path moves northwest until it arrives at the saddle path $AA$ at point $G$, after which all the firms switch to industry 1, so the economy moves southwest along $AA$ and eventually approaches steady state $SS_1$.

There may also exist other equilibria which involve an infinite number of switches across industries. For example, in an equilibrium perfectly coordinated firms could be continuously jumping (almost vertically) between the $AA$ path and $BB$ path back and forth, although it is obviously not Pareto optimal because consumption changes discontinuously. The consumption stream could also be smooth in an equilibrium with an infinite number of switches across industries. In particular, there may exist a limit cycle, as illustrated in the next graph (Figure 9). The equilibrium allocation moves along the closed orbit counterclockwise. At point $G$, all the firms are in industry 1, consumption decreases while capital stock remains unchanged. All the firms stay in industry 1 until point $M$ is reached, at which all the firms simultaneously shift to industry 2 and stay there so the equilibrium path starts moving northeast. When point $B$ is reached, all the firms switch back to industry 1 so the economy moves southwest and meets point $G$ again. Such a limit cycle, if existing, cannot be Pareto optimal unless the capital stock associated with the two switching points $B$ and $M$ are both equal to $\theta_1$.

The diversity and multiplicity of the market equilibria may help explain why the patterns of industrial development are so different across different countries or regions in the real world.
4.3 Industrial Policies

Since there may exist multiple market equilibria of industrial development and some of them are not Pareto optimal, it creates potential room for welfare-enhancing government intervention through various forms of policies, which we generally refer to as industrial policies.

For example, consider the case in Figure 1. If the government is fully benevolent and capable enough, it would coordinate the industries to follow the Pareto optimal equilibrium path as described in Section 4.1. That is, it requires the government to identify the right target industry and coordinate the industrial upgrading for multiple times because the Pareto optimal industry shifts over time as the capital accumulates. Imagine that the government intervention is restricted to selecting a market equilibrium when multiple equilibria exist. As discussed earlier, there can be multiple dynamic market equilibria including those similar to Figures 1, 2, 4, 5, 6, etc., but nothing ensures that the Pareto optimal equilibrium be realized.

If the government ignores the role of the endowment structure and prematurely sets industry 3 as its target industry, which is indeed the case observed in many developing countries that pursued "comparative-advantage-defying" development strategies, then it may want to upgrade the industry as soon as possible. More specifically, it would choose industry 2 immediately when the capital exceeds $E_3$ and choose industry 3 immediately when the capital exceeds $E_6$. By doing so, the economy could eventually approach industry 3, but the cost is that the consumption is suppressed too much on the transitional path and the welfare may be even lower than some laissez-faire market equilibrium.

If the government identifies the right industry target but lacks the capacity to coordinate in time, the outcome is also inefficient. For example, if there exists coordination failure, the economy may not be able to escape the initial equilibrium industry soon enough or even fail to upgrade its industry at all. Or perhaps the government may manage to coordinate to industry 2 after the capital exceeds $E_3$ but for some reason fail to coordinate to industry 3 before the capital closely approaches $K_{ss}^*$, then the economy eventually will converge to steady state 2. In other words, the economy only achieves partial upgrading of its industry.

All the above analysis on industrial policies is essentially the selection of a certain market equilibrium out of the multiple market equilibria. Actually the government typically needs to monitor the whole development process to ensure that firms are not deviating from the desired path at any point. In other words, the role of government is not only to coordinate the firms to change to the right industries at the right time, but also to maintain the status quo for the rest of the time.

If the set of industrial policies is expanded such that the government can implement any resource allocation only subject to the resource feasibility constraint, then a government which pursues "comparative-advantage-defying" development strategies would choose to establish industry 3 from the beginning.
and it can also eventually achieve industry 3, which can never be achieved by the market itself. Of course, it is not Pareto optimal because the total consumption is depressed too much.

In the pertinent literature, to tackle the multiplicity of market equilibria, it is often assumed that there exists an ad hoc frictional adjustment process, so that whether the industrial upgrading occurs depends on whether "expectation" can dominate "history", which in turn depends on the magnitude of the discount rate and the parameters of the adjustment process (see, for example, Krugman, 1991). If the discount rate is sufficiently large and/or the adjustment process is sufficiently difficult, then "history" dominates and therefore there is a unique equilibrium, in which industrial upgrading (or industrialization) cannot occur, otherwise "expectation" dominates and there are multiple equilibria, some of which have industrial upgrading. Our previous proposition already partly characterizes how the market equilibrium is affected by the time discount rate. Now, to simplify the analysis, suppose that economy starts with initial capital smaller than \( E_3 \) and has the strongest path dependence (by revoking Corollary 8) in the sense that the equilibrium remains to be that "all the industries are in industry 1" whenever \( K(t) \in (0, E_2] \). When \( K(t) \) exceeds \( E_2 \), all the firms could either suddenly shift to industry 2 or all move to industry 3, depending on people's expectation; and the industries will stay in industry 3 when \( K(t) > E_4 \).

In particular, if we resort to the argument that the first mover would unilaterally deviate from industry 1 to industry 3 when \( K(t) \) just crosses \( E_2 \), which can be verified, then it implies

\[
i^*(t) = \begin{cases} 
1, & \text{when } K(t) \leq E_2 \\
3, & \text{when } E_2 < K(t)
\end{cases}
\]

If, however, we assume that there exists some sufficiently small cost of industrial upgrading and the cost is larger when directly shifting from 1 to 3 than that when shifting from 1 to 2, holding other things unchanged, then the market equilibrium industry is

\[
i^*(t) = \begin{cases} 
1, & \text{when } K(t) \leq E_2 \\
2, & \text{when } E_2 < K(t) \leq E_4 \\
3, & \text{when } K(t) > E_4
\end{cases}
\]

To make the analysis concrete, consider the following parametric example:

\( \alpha_1 = 0.1; \alpha_2 = 0.2; \alpha_3 = 0.4; \lambda_2 = 1.2; \lambda_3 = 1.44; \xi = 0.03; L = 100; \delta = 0.05; \rho = 0.05 \),

then we have

\[
E_3 < E_6 < \theta_1 < \theta_2 < K_1^{ss} < K_2^{ss} < \left[ \frac{\lambda_1 A(L)}{\delta} \right]^{1/\rho_1} L < E_2 < K_3^{ss} < E_4,
\]

then under rule (21) or rule (22), there exists a unique market equilibrium, in which the firms always stay in industry 1 and the economy eventually converges
to steady state 1. By contrast, the Pareto optimal equilibrium for this numerical example is exactly given by Figure 1 and the economy will eventually converge to steady state 3. This comparison implies that this "constrained" market equilibrium is not Pareto optimal, therefore, the government may improve the welfare by appropriately "relaxing" the "path dependence" constraint, and alleviating all the constraints once and for all is not sufficient to ensure that the Pareto optimal equilibrium be realized because there are many possible market equilibria.

So far we have not been explicit about the concrete policy tools that government can employ to implement its industrial policy. Traditional policy variables include the provision of various forms of subsidies, such as tax rebate/holidays, investment credit, export subsidy, etc., for those investors/firms that follow the recommendation of the government. Policy variables also include imposing punishment, through tax for example, on those investors/firms that do not follow the recommendation of the government. All these policy tools are applicable here. It is worth mentioning that, in our model economy, it is often the case that the Pareto optimal allocation can be achieved by the market itself, although not ensured because there are multiple equilibria. Under that circumstance, the best the government can do is just to help reduce the coordination cost among the firms because each firm/investor has sufficiently strong incentive to achieve the Pareto optimal equilibrium and there is no conflict of interest among those identical investors/firms. When the Pareto optimal allocation can never be implemented by the market itself because, for example, there exists coordination failure or sufficiently large cost associated with the efficiency-enhancing collective action as we have just discussed, then the government should rectify the relevant market failure with carrot and stick.

Most importantly, we want to emphasize that the crucial prerequisite for successful industrial policy is that the government identifies the "right" industry target in time, that is, the industry which is most consistent with the capital endowment of the economy. The existence of Marshallian externality itself is insufficient to justify government support for that industry. Moreover, the "right" industry target may endogenously shift over time as the economy develops, therefore successful industrial policy may require sequential and timely "push" instead of "once-and-for-all" intervention, as typically examined in the existing literature. To illustrate the importance of "right identification", we show that when the government identifies the wrong industrial target and "push" it accordingly, the economy may be even worse off than the laissez-faire market

\[ E_2 < \frac{\lambda_1 A(L_i)}{\delta} \rightarrow_{E_1} L \]

is the largest possible value for capital when the initial capital is sufficiently small. If \[ E_2 < \frac{\lambda_1 A(L_i)}{\delta} \rightarrow_{E_1} L \], there may exist a second equilibrium, in which the economy stays in industry 1 until the capital reaches \( E_2 \), after which the economy switches to industry 3 and moves along the saddle path leading to steady state 3. If rule (22) is adopted, no matter whether \[ E_2 > \frac{\lambda_1 A(L_i)}{\delta} \rightarrow_{E_1} L \] holds or not, there will be a unique equilibrium, in which the firms always stay in industry 1 and the economy eventually converges to steady state 1.
equilibrium even though the target industry does exhibit Marshallian externality.

5 Further Discussion

In this section, we show that our theoretical results are robust when certain assumptions in the model setting are changed.

5.1 Sequential Entrance

The market equilibrium at each time point in our previous analysis is the static Nash equilibrium. What would be the subgame Nash equilibrium if firms are allowed to move sequentially? Suppose \( E^{**} \leq E^* < E \). The first firm wants to move from industry 1 to industry 2 holding other firms staying in industry 1. Now given that the first mover has moved, would there be a second mover at the same level of \( E \)? If he stays, he earns

\[
(1 - \alpha_1)A(L - 1) \left( \frac{E - k'_2}{L - 1} \right)^{\alpha_1},
\]

where \( k'_2 \) is uniquely determined by (7). If he moves to industry 2, he earns

\[
(1 - \alpha_2)\lambda A(2)k''_{\alpha_2},
\]

where \( k''_{\alpha_2} \) is uniquely determined by

\[
\alpha_1 A(L - 2) \left[ \frac{E - 2k''_{\alpha_2}}{L - 2} \right]^{\alpha_1-1} = \alpha_2 \lambda A(2)k''_{\alpha_2-1}.
\]

Using the argument with equation (2) in the proof of Proposition 2, we have \( k''_{\alpha_2} > k'_2 \), which in turn implies

\[
(1 - \alpha_2)\lambda A(2)k''_{\alpha_2} > (1 - \alpha_2)\lambda A(1)k'_2 \alpha_2
\]

\[
> (1 - \alpha_1)A(L) \left( \frac{E}{L} \right)^{\alpha_1} > (1 - \alpha_1)A(L - 1) \left( \frac{E - k'_2}{L - 1} \right)^{\alpha_1},
\]

where the second inequality is because \( E > E^* \) as we have shown and the last inequality holds when \( k'_2 > \frac{E}{L} \), which holds if and only if \( \frac{\alpha_1 A(L - 1)}{\alpha_2 \lambda A(1)} < (\frac{E}{L})^{\alpha_2-\alpha_1} \). Consequently, this second agent finds it strictly profitable to also move to industry 2 when \( \frac{\alpha_1 A(L - 1)}{\alpha_2 \lambda A(1)} < (\frac{E^*}{L})^{\alpha_2-\alpha_1} \), which is equivalent to

\[
\left[ \frac{(1 - \alpha_1)A(L)}{(1 - \alpha_2)} \right]^{(1 - \alpha_2)/\alpha_2} \left[ 1 - \left( \frac{(1 - \alpha_1)A(L)}{\alpha_1 A(L - 1)} \right)^{1/\alpha_2} \right]^{1/\alpha_1} > (L - 1)^{\alpha_1-1} L^{1-\alpha_1}.
\]
The above inequality is true if $L$ is sufficiently large, which we will assume true throughout the paper.

Then as implied by Lemma 1, the rest of the firms in industry 1 will enter industry 2 one by one until there is only one firm remaining in industry 1. The last firm has incentives to move to industry 2 if and only if $E > E^{**}$, which indeed holds. Consequently, when $E^{**} \leq E^* < E$, there is a unique subgame Nash equilibrium in which all the firms will be staying in industry 2. So the equilibrium outcome is identical to that in the static Nash equilibrium.

### 5.2 Different $A(n)$ Function

People may wonder whether our results depend on the fact that the externality becomes increasingly stronger when $A(n)$ takes the form of an exponential function, as we assume in our previous analysis. To address this question, suppose nothing changes except that now the function $A(n)$ becomes

$$A(n) = A_0 n^\xi,$$

so that the marginal increase in the "magnitude" of Marshallian externality is diminishing when more firms enter the same industry. We first characterize $E^*$.

The function (9) becomes

$$\alpha_1 (L - 1)^{\xi + 1 - \alpha_1} E^{* \alpha_1 / \alpha_2 - 1} \left[ 1 - \frac{(1 - \alpha_1) L^{\xi - \alpha_1}}{\lambda (1 - \alpha_2)} E^{* \alpha_1 / \alpha_2 - 1} \right]^{\alpha_1 - 1} \right].$$

It implies

$$\frac{\partial E^*}{\partial \lambda} < 0; \lim_{L \to \infty} E^* (\lambda, L, \alpha_1, \alpha_2, \xi) = 0.$$

Also, suppose $\xi \geq \alpha_1$, then

$$\frac{\partial E^*}{\partial L} > 0, \lim_{L \to \infty} E^* (\lambda, L, \alpha_1, \alpha_2, \xi) = \infty.$$

and the intuition is that, the larger the population, the cheaper the labor and also the stronger the Marshallian externality in the current industry, hence the weaker the incentive to deviate away from the less capital-intensive industry.

And

$$\frac{\partial E^*}{\partial \xi} > 0.$$

Next we characterize $E^{**}$. (14) becomes

$$= \alpha_2 \lambda^{1/\alpha_2} \left[ \frac{(1 - \alpha_1) L^{\xi - \alpha_1}}{1 - \alpha_2} \right]^{(\alpha_2 - 1)/\alpha_2}.$$

(23)
from which we conclude: [1] $\frac{\partial E^*}{\partial L} > 0$ and $\lim_{\lambda \to \infty} E^* (\lambda, L, \alpha_1, \alpha_2, \xi) = \infty$, when $\alpha_1 \leq \frac{1}{2}$; [2] suppose $\xi \geq \alpha_2$, then $\frac{\partial E^*}{\partial L} < 0$ and $\lim_{\lambda \to \infty} E^* (\lambda, L, \alpha_1, \alpha_2, \xi) = 0$; [3] $\frac{\partial E^*}{\partial \xi} < 0$. We can see that the main properties of functions $E^* (\lambda, L, \alpha_1, \alpha_2, \xi)$ and $E^{**} (\lambda, L, \alpha_1, \alpha_2, \xi)$ are almost the same as in the previous analysis. This implies that all the qualitative features in the previous analysis will remain intact.

6 Conclusion

In this paper we develop a growth model with multiple industries to study how industries evolve as capital accumulates endogenously when each industry exhibits Marshallian externality (increasing returns to scale). We show that, in the long run, the laissez-faire market equilibrium is Pareto optimal when the time discount rate is sufficiently small or sufficiently large. When the time discount rate is moderate, there exists a very rich set of multiple dynamic market equilibria, some of which are Pareto dominated. This may help explain why diverse patterns of industrial development are observed in the real world. To ensure the economy achieve Pareto efficiency, it would require the government to first identify the industry target consistent with the endowment structure and then to coordinate in a timely manner, possibly for multiple times. However, industrial policies may make people worse off than in the market equilibrium if the government picks an industry which deviates too far away from the comparative advantage of the economy even if the industry exhibits Marshallian externality. This may help explain why industrial policies succeeded in some countries but failed in others and why sometimes industrial upgrading may take place even without government support.

Different from the literature, we highlight that the mere existence of Marshallian externality in an industry is insufficient to justify government support for that industry. Instead, a crucial prerequisite for successful industrial policies is to first identify the "right" industry target in time, that is, the industry which is most consistent with the capital endowment of the economy. Moreover, we show that the "right" industry target may endogenously shift over time as the economy develops, therefore successful industrial policy may require sequential and timely "pushes" instead of "once-and-for-all" intervention, as typically argued in the existing literature. To illustrate the importance of "right identification", we show that when the government identifies the wrong industrial target and "pushes" it accordingly, the economy may be even worse off than the laissez-faire market equilibrium even though the target industry does exhibit Marshallian externality.

For the sake of analytical simplicity, just like the standard models in the literature, our model also assumes away uncertainty associated with the identification of the right "industrial target". We also ignore the case where there are multiple industries with similar capital intensities but insufficient number of investors (either because of the credit constraint or the scarcity of qualified
entrepreneurs). Thirdly, industry-specific productivity growth is not incorporated into our analyses. It would be interesting to explore those and many other issues in the future.
References


Appendix 1:

When $E_6 > E_2$, we could have

$$E_2 < E_6 < \min\{E_1, E_4\}. $$

or

$$E_2 < \min\{E_1, E_4\} < E_6 < \max\{E_1, E_4\}. $$

or

$$ E_2 < \min\{E_1, E_4\} < \max\{E_1, E_4\} < E_6. $$

More specifically,

(a) when

$$E_2 < E_6 < \min\{E_1, E_4\},$$

there are following possibilities

$$\max\{E_3, E_5\} < E_2 < E_6 < \min\{E_1, E_4\} \quad (24)$$

$$\min\{E_3, E_5\} < E_2 < \max\{E_3, E_5\} < E_6 < \min\{E_1, E_4\} \quad (25)$$

$$E_2 < \min\{E_3, E_5\} < \max\{E_3, E_5\} < E_6 < \min\{E_1, E_4\} \quad (26)$$

Under (24), we have when $E \in (0, E_3)$, there is a unique equilibrium, in which all stay in industry 1; when $E \in [E_3, E_2]$, there are two equilibria: all in industry 1 and all in industry 2; when $E \in (E_2, E_6)$, there is a unique equilibrium, in which all stay in industry 2; when $E \in [E_6, E_4]$, there are two equilibria: all in industry 2 and all in industry 3; when $E \in (E_4, \infty)$, there is a unique equilibrium, in which all stay in industry 3.

Under (25), we have when $E \in (0, E_2)$, there is a unique equilibrium, in which all stay in industry 1; when $E \in [E_2, E_6]$, there is a unique equilibrium, in which all stay in industry 2; when $E \in (E_6, E_4]$, there are two equilibria: all in industry 2 and all in industry 3; when $E \in (E_4, \infty)$, there is a unique equilibrium, in which all stay in industry 3.

Under (26), we have when $E \in (0, E_2]$, there is a unique equilibrium, in which all stay in industry 1; when $E \in (E_2, E_3)$, there is no equilibrium; when $E \in (E_3, E_6)$, there is unique equilibrium; in which all stay in industry 2; when $E \in [E_6, E_4]$, there are two equilibria: all in industry 2 and all in industry 3; when $E \in (E_4, \infty)$, there is a unique equilibrium, in which all stay in industry 3.

(b) When

$$E_2 < \min\{E_1, E_4\} < E_6 < \max\{E_1, E_4\}. $$
there are following possibilities

\[ E_2 < \min\{E_1, E_4\} < \max\{E_3, E_5\} < E_6 < \max\{E_1, E_4\} \quad (27) \]

\[ E_2 < \max\{E_3, E_5\} < \min\{E_1, E_4\} < E_6 < \max\{E_1, E_4\} \quad (28) \]

\[ \max\{E_3, E_5\} < E_2 < \min\{E_1, E_4\} < E_6 < \max\{E_1, E_4\} \quad (29) \]

Under (27), we have when \( E \in (0, E_2] \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_3) \), there is no equilibrium; when \( E \in [E_3, E_6) \), there is unique equilibrium: industry 2; when \( E \in (E_6, E_4] \), there are two equilibria: all in industry 2 and all in industry 3; when \( E \in (E_4, \infty) \), there is a unique equilibrium, in which all stay in industry 3.

Or we have when \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in [E_2, E_6] \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_6, E_4] \), there are two equilibria: all in industry 2 and all in industry 3; when \( E \in (E_4, \infty) \), there is a unique equilibrium, in which all stay in industry 3.

Or we have when \( E \in (0, E_2] \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_6] \), there is no equilibrium, when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_5 < E_2 < E_4 < E_3 < E_6 < E_1 \).

Under (28), we have when \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in [E_2, E_6] \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_6, E_4] \), there are two equilibria: all in industry 2 and all in industry 3; when \( E \in (E_4, \infty) \), there is a unique equilibrium, in which all stay in industry 3.

Or we have when \( E \in (0, E_2] \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_3) \), there is no equilibrium; when \( E \in [E_3, E_6) \), there is unique equilibrium: industry 2; when \( E \in (E_6, E_4] \), there are two equilibria: all in industry 2 and all in industry 3; when \( E \in (E_4, \infty) \), there is a unique equilibrium, in which all stay in industry 3.

Or we have when \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in [E_2, E_4] \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_3 < E_2 < E_5 < E_4 < E_6 < E_1 \).

Or we have when \( E \in (0, E_2] \), there is a unique equilibrium, in which all stay
in industry 1; when \( E \in (E_2, E_3) \), there are no equilibria; when \( E \in [E_3, E_4] \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_2 < E_3 < E_5 < E_4 < E_6 < E_1 \).

Under (29), we have when \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in (E_2, E_4) \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( \max\{E_3, E_5\} < E_2 < E_4 < E_6 < E_1 \).

Under (29), we have when \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in (E_2, E_4) \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_2 < E_3 < E_4 < E_5 < E_6 \).

Or we have when \( E \in (0, E_3) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria: all in industry 1 and all in industry 2; when \( E \in (E_2, E_4) \), there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_2 < E_3 < E_4 < E_5 < E_6 \).

Or we have when \( E \in (0, E_2) \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_3) \), there are no equilibria; when \( E \in [E_3, E_4] \),
there is a unique equilibrium, in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_2 < E_4 < E_5 < E_6 \).

Or we have when \( E \in (0, E_2] \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_2 < E_1 < E_4 < E_5 < E_6 \).

Or we have when \( E \in (0, E_2] \), there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_6) \), there is no equilibrium; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when

\[
E_2 < E_1 < E_4 < E_5 < E_6 \text{ and } E_3 \leq E_5.
\]

Or we have when \( E \in (0, E_3), \) there is a unique equilibrium, in which all stay in industry 1; when \( E \in [E_3, E_2] \), there are two equilibria 1 and 2; when \( E \in (E_2, E_4) \), there is unique equilibrium; in which all stay in industry 2; when \( E \in (E_4, E_6) \), there are no equilibria; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when \( E_3 < E_2 < E_4 < E_5 < E_6 \).

Or we have when \( E \in (0, E_2], \) there is a unique equilibrium, in which all stay in industry 1; when \( E \in (E_2, E_6) \), there is no equilibrium; when \( E \in [E_6, \infty) \), there is a unique equilibrium, in which all stay in industry 3. For example, when

\[
E_2 < E_1 < E_4 < E_5 < E_6 \text{ or when } E_2 < E_4 < E_1 < E_3 < E_6 \text{ and } E_5 \leq E_3.
\]
Figure 1
Figure 2

Figure 3
Figure 4
Figure 5