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**Scenario Analysis Tool
for Assessment and
Monitoring of
Government Guarantees:
A Guidance Note**

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Abstract

Central to making good decisions about debt guarantees is assessing their expected and possible fiscal cost, a task that many governments still struggle with. This paper therefore describes a relatively simple scenario analysis method for estimating potential payments from the government when a beneficiary faces difficulty with debt payments. The basic version of the approach estimates the payments in a given scenario to inform risk management decisions. The paper then extends the method for situations where more rigor is required or where the economic guarantee fee is to be estimated. The final part of the paper extends the method to assess the combined loss of multiple guarantees when simultaneously subjected to negative economic conditions.

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Scenario Analysis Tool for Assessment and Monitoring of Government Guarantees

Lilia Razlog, Chris Marrison, Tim Irwin

Introduction

Risk quantification is important to inform risk management decisions during the initial structuring, pricing, and approval of a guarantee and later during the monitoring of the guarantee as part of a portfolio¹. In the initial stage managers need to assess the scale of potential future government payments under alternative structures to decide whether the government is prepared to take-on the risk of making such payments. If the government decides that the potential risk out-weights the social-good, the next stage is to estimate the average, or expected, loss on the guarantee. Depending on the policies, fees and reserves can be linked to this average loss. After approval, the ongoing monitoring of the portfolio guides decisions about the potential losses to be paid from sources such as reserves, calls on the budget or increased borrowing. This monitoring also guides decisions as to whether to take on new risks. In assessing the portfolio of risks from the guarantees, it is also important to understand the correlation between losses on the guarantees' portfolio and losses in other government operations (see Marrison 2001).

There are several well-established methodologies for estimating risk and Appendix 1 gives a brief overview of the approaches which are most commonly considered for guarantees. The best approach to use in any given situation depends on factors such as project complexity, the novelty of the situation, the availability of data and whether the estimate of the average loss, potential loss or correlated loss is most important in supporting the decision being made. Ideally several estimation approaches should be used in conjunction to get alternative views of the risk. This paper describes the use of one particular method to quantify the risk of government guarantees on debt: the scenario analysis approach, which is also known as stress-testing. The same approach can also be used to assess the risk of companies benefiting from on-lending.

Scenario analysis is particularly useful for unusually structured companies, for unusual stress situations, or for projects where there are no scorecards available for credit grading. The main advantage of scenario analysis is that it is a direct approach to quantifying the amount of loss, this means that it can also be used for cases where the company is already in distress and the task is to estimate the amount of payment rather than the probability of payment. The result of a scenario analysis is a currency amount payable by the government in that scenario. These results are more easily communicated to management than the results from statistical metrics. Scenario analysis also provides forward looking multi-year results for any given scenario. Rather than just assigning a single grade or looking at the first few years, with scenario analysis it is relatively easy to look over the life of the guarantee and understand the sources of risk.

Scenario analysis can also include cross-asset and cross-government correlations. By testing all assets under the same scenario conditions, the results for multiple projects and state-owned enterprises can be added to give a consolidated picture. The main challenge in using scenario analysis is to associate probabilities with each scenario to get statistical metrics such as the expected loss, so a large part of this

¹ The paper was peer reviewed by Philip Anderson, Asli Senkal and Mike Williams. Comments on earlier drafts of this paper were received from experts of the World Bank's MTI Global Macroeconomic and Debt Practice.

paper is given to discussion and examples of how to estimate the probability of a given stress, or conversely, how to define a stress that has a given probability.

The approach described in this paper modifies the usual use of scenario analysis in three ways:

- It uses a form of scenario model that has been cut down to be as easy as possible to use whilst still retaining the core calculations and results.
- It associates probabilities with losses to give an estimate of the average expected loss on the guarantee. This can be linked to the guarantee fee.
- It describes a framework in which losses across multiple guarantees can be correlated by linking them to the same underlying macro-economic conditions.

This core model is designed to be used by government analysts who need to assess the risk of guarantee proposals but have minimal time, information or resources. The model can also be used as a quick “first-cut” analysis or to cross-check results from other models. Two Excel template models have been created that embody the methodologies described. Those Excel models can be used as described in this paper or used as the basis for making tailored models.

The paper is structured as an application manual rather than a theoretical description of a methodology. The body of the text gives the main concepts and numerical examples so that readers can see how the concepts can be used in practice. The appendices act as extended footnotes and references to give technical details for those readers who want to extend the analysis.

There are four parts to the paper. The first gives an overview of the scenario testing methodology and describes the calculation framework used in the subsequent parts. The second part describes the use of the model for assessing the loss from a single guarantee in a single scenario. The third part describes how to set up a succession of increasingly severe scenarios and associate probabilities with each scenario to get the average expected loss to be covered by fees. The last part covers the use of the approach to assess the correlation between the losses of multiple guarantees.

Overview of the Scenario-Analysis Approach

Scenario analysis forecasts the financial performance of a project or a state-owned enterprise under a set of future conditions, typically including a base-case scenario and various cases of concern such as negative economic conditions and negative company-specific performance. The impact of the stress factors on the project will depend on the details of the project’s operations. For example, a mining operation may be positively impacted by a rise in commodity prices, but an airline with unhedged fuel costs will be negatively impacted by the same increase in commodity prices. As another example, construction-cost over-runs are usually of concern but may have little impact on the government if the equity sponsor is strong and has contracted to pay for any cost over-runs or has a fixed-price contract with the construction company. Appendix 2 lists types of risks that are typically assessed with scenario analysis.

The overall process for a scenario analysis can be described as follows:

Obtain the company’s or project’s base case nominal cashflow forecast for gross operating income, operating costs, and debt costs. Typically, this forecast is in quarterly or annual steps. The forecast can come from budget plans, project proposals, or if no other information is available, the base-

case forecast can be assumed to be the same as the average performance over the last 3–5 years of the company’s activity. For the simplest model only four line-items are required: gross income, operating costs, debt principal payments and interest payments². Operating costs should be interpreted broadly as including all those items that need to be subtracted from the gross income before paying the debt, that includes day-to-day operational costs, admin costs, financial costs such as hedges and any capital costs that are not funded from debt or equity. More complex scenario models can be used if more detailed information is available, for example, by separating fuel costs from staff costs or other factors relevant for a specific project.

Define a specific scenario or stress for economic factors that are of importance to the project such as an increase in commodity prices, an increase in debt costs, a change in FX rates, or a fall in demand due to an economic downturn.

Define a specific scenario or stress for idiosyncratic company-specific factors such as construction cost over-run, over-estimation of demand, or a failure to cut operating costs.

For the stressed scenario, forecast the stressed operating income and stressed costs to obtain the net operating income (NOI).

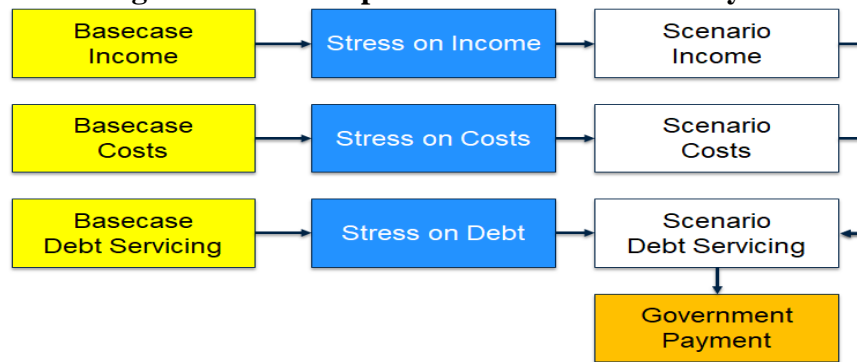
Compare the NOI with the stressed required debt payments and if any of the debt is guaranteed, assume that any shortfall in paying the debt will require a payment from the Government.³

These components of a scenario analysis are illustrated in Figure 1.

² The cashflows should only include the operating plan that is under analysis. For example, if there are future expansion plans that are optional, they should not be included because they can be part of a separate decision later. But if the current plan requires additional future capital expenditure, that should be included in the cashflows. If the future capital expenditure is financed from net operating income, the expenses should be subtracted from the operating costs because it implies there will be less free cash to make the debt payments. If the future capital costs are to be funded by debt, it should be reflected in increased debt payments. If the future costs are to be funded by equity, this would have no effect on operating costs or debt payments, but in that case it would be wise to stress test the possibility that the equity sponsor defaults on the cash injection and instead debt needs to be used.

³ This assumes that the government will step-in and make a payment to prevent default and therefore will not be called upon to pay the total outstanding debt. This is also making the simplifying assumption that there is not a later recovery of payments made by the government on behalf of the guaranteed entity. Such a repayment may come from future profits or from the sale of collateral. In most cases guaranteed entities that go into distress continue to be in distress and are unable to make later repayments. Similarly selling collateral is likely to worsen the financial position of the entity and just require government support in other forms. Given these considerations, we make the conservative and simplifying assumption that there are no future recoveries of payments made. In some situations, recoveries may be possible e.g., from otherwise healthy companies who are given a guarantee when there is a severe short-term shock such as the 2008 financial crisis or 2020 covid crisis. In such cases we have seen governments give guarantees or funds in exchange for shares in future profits. If important, repayments could easily be added to the model.

Figure 1. The Components of a Scenario Analysis



Scenario analysis has the advantage of being very flexible and can be used quickly in a very direct way to get a broad estimate of the credit risk. The models described here are have only the core, essential elements, but if required, the complexity of the template model can be increased to get more a specific view of the risks for an individual company.

Alternative Approaches to Building Scenario Analysis Models

Broadly, there are three ways to link the cashflows to the risk factors. The first is to have a detailed bottom-up cashflow model for the project where factors such as interest rates, fuel prices and construction costs appear explicitly and are used in the rest of the model to calculate the cashflows. In most cases such models are built and held by the company or project sponsors for their own budget-planning purposes. It may be possible for the government to get a copy of the model or give the guaranteed entity the stress cases and get them to run the model. The key outputs to look for are the net operating income and debt service required. For any year where the net income gets close to or falls below the debt service required, the government should expect to pay the shortfall (although in some cases the project may have a reserve account to make some of the payment). For government analysts there are three difficulties with this bottom-up approach: typically, the model is not available to the government, the model is complex, and the model is not standard from one project to the next so it is difficult to compare projects and get portfolio results.

The second approach to linking the cashflows to the risk factors is to create a moderately detailed cashflow model by getting the forecasts of the base case income, costs and debt servicing, and then using footnotes from the annual report or project proposal document to break the main cashflows into elements that depend on changes in the risk factors. For example, a power plant may have annual costs of \$100 million, and in the foot notes you find that 60% of the cost is fuel, 32% is operating staff and 8% is maintenance equipment and parts. Further, you may find that 75% of the fuel has a fixed price hedge contract. You can then construct a model which has \$45 million fixed fuel costs, \$15 million variable fuel costs, \$32 million operating staff and \$8 million maintenance. For the stress, multiply the variable fuel cost by the degree of commodity price change, increase operating staff cost by inflation and, if necessary, increase the number of staff e.g., if the forecast costs assume staff reductions, assume the reductions don't happen. The result (in millions) would be:

$$\begin{aligned} \text{Scenario Cost} = & \$45 + \$15 (\text{Scenario Fuel Price} / \text{Base Fuel Price}) \\ & + \$32 (\text{Scenario CPI} / \text{Base CPI}) + \$8 \end{aligned}$$

For government analysts, the main difficulty with this approach is the complexity of interpreting the footnotes and building-up the model for the costs, income and debt servicing.

A third approach is to just look at the totals of gross income, operating costs and debt costs and multiply each of those by a stress factor as discussed later. This approach blends all the risks together into the cashflow-multipliers and has a relatively simple cashflow model. This is the approach taken in this paper. The main difficulty in defining this approach is to set the balance between accuracy and ease of use. So far as possible the scenario analysis approach has been “boiled down” to retain its essential elements whilst requiring the smallest amount of input from the analyst. This “boiling down” requires many small assumptions and approximations. To avoid cluttering the main discussion, the effects of these assumptions are covered in footnotes and appendices for those readers who want to go deeper into the analysis or who want to adapt the method to include the details. This cashflow-multiplier approach is explained below and is used in the template models. In many ways this cashflow-multiplier approach shifts the focus of analysis from the cashflows to the stress multipliers.

A Cashflow Multiplier Model for Scenario Analysis

Mathematically, a cashflow-multiplier model can be formulated as follows to give the resulting government payments on the debt in each year:

Gross Income	= Base Gross Income $\times M_Income$
Operating Cost	= Base Operating Cost $\times M_Cost$
Net Operating Income	= Gross Income - Operating Cost
Principal Payment	= Base Principal $\times M_Principal$
Interest Payment	= Base Interest $\times M_Interest$
Debt Payment	= Principal Payment + Interest Payment
Government Payment	= Percent $\times \max [0, Debt Payment - NOI]^4$

Where

M_Income is the multiplier on gross income.

M_Cost is the multiplier on operating costs.

M_Principal is the multiplier on the debt principal payments.

M_Interest is the multiplier on interest payments.

Percent is the portion of debt guaranteed by the government

Essentially these seven lines describe the whole model. As inputs for each project we only need the forecast for four cashflows. The cashflows are:

- Base Gross Income
- Base Operating Cost
- Base Principal Payment
- Base Interest Payment

As a simple numerical example consider an entity with the following profile in year five (cash amounts are in millions):

⁴ In this simplified approach we are only considering cashflows within each year. We are not considering the effect of pre-existing cash accounts, equity injections or the build-up of reserves from earlier years of excess income. If these amounts are significant for a project, their effect can be approximated by adding them to the income, e.g., existing cash balances can be added to year-one income, equity injections can be added to the year of injection and excess income that is held in a reserve can be subtracted from one year and added to a later year when it is needed.

Base Gross Income	\$100
Base Operating Cost	\$30
Base Principal Payment	\$25
Base Interest Payment	\$20
Percent Guaranteed	100%

In the base case the net operating income (NOI) is \$70 and comfortably covers the debt payments of \$45. Later the paper discusses approaches for quantifying the amount for each multiplier, but for now consider a scenario in which demand is 10% less than expected so $M_Income = 0.9$, operating costs are 10% higher than expected so $M_Cost = 1.1$, construction-cost over-runs increase debt by 20%⁵, the loan is denominated in foreign currency and FX rates move 15% so $M_Principal = 1.2 \times 1.15 = 1.38$. Finally, interest rates rise 1% and the floating rate on the loan goes from 6% to 7% so $M_Interest = 1.38 \times 7\%/6\% = 1.61$. The cashflows are then:

Gross Income	= \$100 × 0.9	= \$ 90.00
Operating Cost	= \$30 × 1.1	= \$ 33.00
Net Operating income	= \$90 – \$33	= \$ 57.00
Principal Payment	= \$25 × 1.38	= \$ 34.50
Interest Payment	= \$20 × 1.61	= \$ 32.20
Government Payment	= 100% × max [0, 34.50 + 32.20 - 57.00]	= \$ 9.70

With this moderate set of stresses the loss is \$9.70 in the given year. If we double all the variations⁶ the loss comes out to be \$43.10. Appendix 3 discusses the case where there is also a minimum revenue guarantee from the government.

If the user has an intuitive sense for likely values for the stress multipliers, the values can be directly entered into the models and the loss estimates obtained immediately as in the example above. This estimate may be sufficient to determine the credit worthiness of the company or for a first analysis. In the Excel template Model One, these calculations for a single stress can be carried out using just the “Input Cashflows” tab⁷

Box 1 gives a case study of how this scenario analysis was used to evaluate the risk of a requested government guarantee for a university extension.

⁵ Typically, the guarantee will have a nominal sum attached to it and would not formally cover an increased amount of debt. But if there is a cost over-run, the practical options at that stage are to either scale-back the project (and therefore the income), make a direct government payment to cover the increased cost, obtain more equity or increase the debt. In this analysis we do not consider reducing the scope of the project because that would change the amount of goods and services delivered. Instead we assume the project goes ahead as planned with additional financing. We do not assume a direct injection of government funds because this is much less likely than giving a guarantee to obtain more debt. The analysis does allow the possibility of the costs being paid by some combination of debt and equity.

⁶ i.e., demand is 20% less, costs are 20% higher, construction over-runs 40%, FX moves 30%, rates increase by 2%.

⁷ The other tabs are used to assess multiple scenarios and estimate the averaged loss, as described in the next section. If the user only wants to assess one scenario at a time, the other tabs can be ignored.

Box 1: Ugandan University Project

The government was asked to consider a request for a guarantee on a loan to be used to build an extension to a university. The extension was to house medical training facilities, the loan was to be provided by an ExIm bank, and it was to be repaid from the additional fees paid by the new medical students. If the net income from those fees was insufficient, the university would use surplus funds from its current operations to pay the loan.

At the time of application, the university already had some debt. The university provided historical financial data, a description of the new project and base case projected cashflows.

The Government's cashflow analysis looked separately at the current operations of the university and the projected operations of the new medical center and for each the base-case forecasts were established for the gross income, costs, and debt servicing. In the base-case scenario all of the debt could be serviced. Combinations of the following stresses were then applied:

For the existing university operations:

- Assume that growth of student numbers is less than forecast and is only the growth rate experienced historically.
- Assume that the growth of student numbers is flat.
- Assume revenues fall to the worst level seen over the last few years. As a stress, assume revenues fall twice as far.
- Assume costs increase to the highest level seen over the last few years. As a stress, assume costs increase twice as far.

For the new medical facilities:

- Assume the growth in student numbers takes twice as long to reach full capacity.
- Assume that only 90 percent of the projected fees can be charged or collected.
- Assume the government only sponsors half as many new students as requested.
- Assume that construction costs and therefore debt increase by 20, 50, or 100 percent (no construction contract had yet been signed, so costs were still quite uncertain).
- Assume the debt is in a foreign currency and assume a strengthening of the foreign currency relative to domestic currency equal to the worst case seen in the last 5 and 10 years, respectively.

Each factor was stressed individually to get a sense of the which factors were most important and then all the factors were stressed together.

First the extent of the stress was moderate to see if the proposal was strong enough to withstand average project problems, and then the factors were stressed more severely to understand the likely government payments if there were significant problems.

Based on this analysis, more research was done on those factors that were found to be the most significant in causing a government payment to better understand the uncertainty in those factors and see if the uncertainty could be reduced, by for example having a construction contract for an agreed price. The analysis also gave the MoF a quantitative basis to inform the line ministry of the likely cost of the guarantee and the size of the potential future payments required.

This section described how to obtain loss estimates with assumed levels for the stress multipliers. If a deeper analysis is required, the next section describes a more structured approach for setting the cashflow-multipliers.

Setting Stress Levels to Estimate the Loss for a Single Guarantee with a Single Scenario

Defining the Severity of the Scenario

In thinking about the extent to which factors should be changed, it is first useful to consider what is meant by a stress. This is most easily communicated by thinking of several cases with increasing severity and decreasing probability. For example, a set of stresses may be defined as follows:

Base Case: nothing goes wrong and everything goes according to forecast.

Average Case: normal things go wrong for this type of company or project, i.e., there is a 50% chance of conditions being better or worse than this average case.

Moderately bad Case: there is a “moderate” probability of conditions being as bad as this. Moderate could be for example be defined to be a one-in-six chance, i.e., a condition that occurs for one-in-six companies or one-in-six years (16%) equates to one standard deviation for a Normal distribution).

Worst Case: there is a “small” probability of conditions being as bad as this. Small could for example be defined to be a one-in-fifty chance, i.e., one-in-fifty companies or one-in-fifty years (one-in-fifty equates to two standard deviations for a Normal distribution).

In the Base Case the cashflow-multipliers on the forecasts equal one. For the other cases we need to set the values of the multipliers. Broadly, there are two approaches to setting the multipliers on the cashflows. One is to look at the cashflows as a whole, the other is to look at the underlying factors that could cause cashflows to vary.

Setting Multipliers for the Cashflows as a Whole

Looking at the cashflow as a whole works well if there is historical information on the cash flows, for example if the guarantee is for an established operation such as a state-owned enterprise. In this case the multipliers can be based on historical performance, e.g., looking at the worst fall in income over the last few years, or looking at past forecasts vs. actual performance, or looking at the standard deviation of income and costs over the last few years. It may also be possible to look at the cashflows for similar companies, for example when assessing a proposed new hospital, it may be possible to look at previous hospitals and compare actual performance to forecast performance.

If there is insufficient relevant information on the uncertainty of cashflows as a whole, then the alternative is to look at the underlying factors that drive the cashflows and understand how those factors may change.

Setting Multipliers Based on Underlying Risk Drivers

In the section above we simply multiplied the key cashflows. The number used for that multiplication may come from looking at the past variability in the cashflows (for example with a state-owned enterprise) or from intuition if the analyst or line-ministry knows of similar companies or projects. If a more detailed analysis is required, the next step is to identify the underlying risk factors driving the uncertainty of the cashflows.

The underlying risk drivers are those factors which affect the cashflows either directly or indirectly. For example, debt payment cashflows will depend on the debt outstanding, which in turn may depend on the construction cost over-run. The process of estimating the effect on cashflows has three steps:

- Identify the risk drivers, i.e., the underlying factors that cause variations in the cashflows
- Link the risk drivers to the cashflows
- Quantify the possible variation in the risk drivers

The following sections address each of these steps.

Identifying the Risk Drivers

Risk drivers can be idiosyncratic company-specific risks such as misestimates of costs, or they can be systemic macro-economic risks such as changes in inflation, oil prices or rainfall. The risk drivers applicable to a given company or project can be identified using a combination of the following approaches:

Use the risks listed in Appendix 2 as a “menu” and identify whether the company in question is subject to each of these risks.

Read the company financial statements or project description and look for explicit and implicit assumptions, for example how certain is the construction cost or the demand forecast or are there any optimistic assumptions about how future costs will be lower and future income higher than past performance.

If there is a financial model supplied with the guarantee application, look at the line items for costs such as fuel and the assumptions for factors such as interest rates, FX rates and inflation.

Talk to line-ministry experts and consultants to use their experience, advice and data.

Look at historical examples of problems with similar companies and projects in similar sectors and other countries.

Linking the risk drivers to the Cashflows

The link between the risk drivers and the cashflows can be thought of in two parts: the proportion of the cashflow affected by the driver, and the sensitivity of that portion to the risk driver. For example, consider an operation for which 60 percent of the costs are fuel, 20 percent of those fuel costs are under a fixed price contract, and the other 80 percent depends on the spot market. In this case the proportion is 60% and the sensitivity is 0.8, i.e.,

$$\text{Cost Multiplier Due to Oil}^8 = 1 + 0.6 \times 0.8 \times \text{Increase in Oil / Expected Oil Cost}$$

This can be generalized to say

$$M_F = 1 + p \times b \times dF/F$$

⁸ Note that commodity prices are normally quoted in USD, so if the project’s cashflows are not in USD, wherever there is sensitivity to commodity prices there is also sensitivity to FX rates, so the combined effect would be multiplied by the relative change in FX. For example, if 60% of the costs were oil, the combined multiplier in this example would be well approximated by $M = (1+0.6 \times 0.8 \times d\text{Oil}/\text{Oil}) \times (1+0.6 \times 0.8 \times d\text{FX}/\text{FX})$

where

M_F is the multiplier on the cashflow due to factor F
p is the portion or percentage of the cashflow affected by F
b is the sensitivity of the cashflow to changes in F
dF is the change in the factor
dF/F is the relative change in the factor

In many cases p and/or b will equal zero or one. For example, typically interest rates do not directly drive operational costs so $p = 0$ for the link between interest rates and operating costs. Conversely, all of the operating costs may be linked to inflation so p and b would both equal one for CPI.

Two cases are slightly more complicated. One is the link between construction cost-over-runs and debt payments. If debt is expected to fund only part of the base-case construction costs, but debt funds all of the cost over-runs, the percentage increase in debt will be greater than the percentage increase in cost. This is discussed in Appendix 4. The result is that the multiplier on the principal amount, expressed in the company or project's currency, depends on the cost increase and on the extent to which the loan is denominated in a foreign currency, i.e.:

$$M_{Principal} = \text{Multiplier due to cost increase} \times \text{Multiplier due to FX}$$

where the Multiplier due to FX is given by:

$$\text{Multiplier due to FX} = 1 + (p_{FX}) \times (\Delta FX / FX_{Base})$$

Here

p_{FX} is the proportion of the debt that is denominated in a foreign currency
 ΔFX is the change in the FX rate
 FX_{Base} is the FX rate assumed in the basecase (or the current FX rate if the basecase rate is not explicitly given)

The other slightly complicated link is between interest payments and changes in interest rates. The multiplier on interest payments depends on the change in the loan amount and the change in the interest rates:

$$M_{Interest} = M_{Principal} \times [(1 - p_{Floating}) + p_{Floating} \times M_{Rate}]$$

Where $p_{Floating}$ is the proportion of the debt that has a floating rate.

M_{Rate} is the rate on the loan in the scenario divided by the rate in the basecase and is given by:

$$M_{Rate} = \text{Scenario Loan Rate} / \text{Base Loan Rate}$$

For most purposes this description of the multiplier for the rate is sufficient. However, for the purpose of putting the interest rate sensitivity in the model in the same format as the other sensitivities we can go a step further to modify this equation into the form:

$$M_F = 1 + p \times b \times dF/F$$

This is done by breaking the rate into the underlying floating rate and the added margin:

$$\begin{aligned}M_Rate &= (\text{Scenario LIBOR} + \text{Margin}) / (\text{Base LIBOR} + \text{Margin}) \\M_Rate &= (\text{Base LIBOR} + \text{Margin} + \text{Rate Change}) / (\text{Base LIBOR} + \text{Margin}) \\M_Rate &= [1 + \text{Rate Change} / (\text{Base Total Rate})]\end{aligned}$$

This last line lets us express M_Rate in terms of the change in the economic scenarios, divided by the total (or “all-in”) rate on the loan in the base case.

To make the concepts more concrete, consider the following numerical examples:

M_Income would be 0.7 if actual revenues were assumed to be 30 percent less than forecast. The 30% could for example be made up of a 20% shortfall in sales, passenger or traffic volume and a price that was 10% less than expected, e.g., due to lower inflation, or pressure from the users.

M_Cost would be 1.2 if half the cost of the operation was fuel and fuel prices rose 40 percent. If in addition the other half of the cost was maintenance, and maintenance increased by 10%, the total multiplier would be 1.3. M_Cost would be 1.36 if in addition inflation was 5% higher than the base forecast so that CPI was 1.05 times the base-case.

$M_Principal$ would be 1.4 if the company or project was 50 percent funded by debt in the base-case, then there was a 20 percent construction cost overrun, and all the overrun was paid by increasing debt (see Appendix 4 for derivation). In addition, if the loan was denominated in a foreign currency and there was a 30 percent appreciation relative to the local currency $M_Principal$ would increase by a further factor of 1.3 to 1.82.

$M_Interest$ would be 2.55 if the loan was floating rate (for example LIBOR) with an expected all-in rate of 5 percent but then LIBOR increased by 2 percent. This creates a multiplier of 1.4⁹, which when combined $M_Principal$ of 1.82 would put $M_Interest$ at 2.55.

This last example is interesting because it shows how the accumulation of moderate stresses can lead to a large stress for the project: a 20% cost over-run, a 30% FX appreciation and a 2% rate increase led to interest payments more than doubling.

Quantifying the Possible Variation in Macro-Economic Risk Drivers

For the systemic macro-economic risk factors such as commodity prices, GDP, inflation, exchange rates, and interest rates, it is relatively easy to get historical time series data from sources such as the OECD, IMF or World Bank. For most of these factors we are interested in relative changes, for example between 1990 and 1991 LIBOR went from 15% to 12% whereas from 2015 to 2016 LIBOR fell from 0.39% to 0.31%. Both of these represent a relative change of -20%. We use relative changes for two reasons: one is that it easily allows us to use multipliers on the cashflows, the other is that the size of changes are generally related to the current level¹⁰.

⁹ $M_Rate = [1 + 2\% / 5\%]$

¹⁰ More formally, relative changes are generally more statistically stationary over time, for example the standard deviation of the relative annual change in LIBOR was around 20% in both 1990 and 2015.

Time series can be converted to relative change by calculating the ratio $R\{t\} = (F\{t\}-F\{t-1\})/F\{t-1\}$ where $R\{t\}$ is the relative change at time t , and $F\{t-1\}$ is the level of the factor at the previous time step. The time steps should be the same as the cashflow model, e.g., yearly.

For global factors such as commodity prices, the shocks can be defined by looking back over the worst relative changes in the last 20 to 30 years. For national factors such as exchange rates, there may have been no local crisis in the last few years, so it may be necessary to look at what has happened to other countries and test the case “if the same happens here”. This historical stress test is easy to communicate to people who are not familiar with statistics. The message is “if prices and rates move as they did in the past crisis, then this is the amount we will need to pay on the guarantee”.

Rather than looking at specific historic events, an alternative is to base the stresses on the standard deviation of the historical time series, i.e., by calculating the standard deviation of $R\{t\}$ ¹¹. Let us define the base case to be no change, the average case to be the expected change, the moderately bad case to be a one standard deviation change, and the worst case to be two standard deviations. The multipliers for the factors can be defined as:

Base	Multiplier of 1
Average	Multiplier according to the average forecast
Bad	The average case plus or minus one standard deviation
Worst	The average case plus or minus two standard deviations

The “plus or minus” depends on the direction that causes stress. For example, an airline will be stressed by increasing oil prices, whereas an oil refinery is typically stressed from falling prices, so choose to add or subtract depending on the nature of the guaranteed company.

Quantifying the Possible Variation in Idiosyncratic Risk Drivers

The idiosyncratic factors such as construction costs or staff levels are particularly prone to over-optimism for new projects where the originators or line ministries cannot foresee all difficulties and have a tendency to present their prospective project in an optimistic light. Estimates of the uncertainties in estimating future costs and demand may be obtained by looking at previous similar companies or projects or from experts in the line ministries. There are also several studies of how actual outcomes have compared with forecast outcomes for different types of projects.

For toll roads, Bain (2009) looks at historical evidence of over-optimism in estimating traffic volumes. For a sample of over 100 toll roads, the actual traffic was on average only 77% of the forecast traffic, with a standard deviation of 26%. If we assume the error is Normally distributed, there is a 50% chance of the traffic being less than 77% of the forecast, a one-in-six chance of actual traffic being less than 51% of the

¹¹ Most relative time series have an asymmetric log-normal probability distribution, e.g., the probability of doubling equals the probability of halving, so a more refined analysis is as follows. Take the natural log of R , calculate the standard deviation, do the multiples of standard deviation, and then take the exponent, i.e.,

$$\text{Worst Increase} = \text{Average} \times \exp[\text{average}\{L\} + 2 \times \text{stdev}\{L\}]$$

and

$$\text{Worst Decrease} = \text{Average} \times \exp[\text{average}\{L\} - 2 \times \text{stdev}\{L\}]$$

where $L(t)=\ln[R(t)]$.

forecast, and a one-in-fifty chance of being less than 25% of the forecast. Therefore, for a toll road it is reasonable to test these multipliers on the forecast for traffic revenue:

Average Case:	multiply by 0.77
Moderately bad Case:	multiply by 0.51 (i.e., $0.77 - 0.26$)
Worst Case:	multiply by 0.25 (i.e., $0.77 - 2 \times 0.26$)

For project cost and revenue estimates, Flyvbjerg et al. (2003) and Priemus et al (2008) analyze projects in the transport sector. For example, they found that rail projects outside North America and Europe had an average cost over-run of 65% with a standard deviation of 50%. Therefore, for a rail project it is reasonable to test these multipliers on the forecast for construction costs:

Average Case:	multiply costs by 1.65
Moderately bad Case:	multiply costs by 2.15 (i.e., $1.65 + 0.50$)
Worst Case:	multiply costs by 2.65 (i.e., $1.65 + 2 \times 0.50$)

As an initial proxy, it is also reasonable to assume the same multipliers for operating and maintenance costs. The effect of a cost over-run will be to increase either the equity contribution, the debt, or both, depending on the contract terms and whether the equity partner defaults as discussed in Appendix 4. Chen et al (2016) looked at the time and cost over-run characteristics of US commercial buildings, public infrastructure and industrial plants. They give the following average cost over-runs:

Commercial buildings:	6.5%
Public infrastructure:	5.8%
Industrial plants:	12.6%

In aggregate, the cost over-runs have a standard deviation of 21%. So, for a well-run facilities-project, with signed fixed-price¹² contracts, the following would be reasonable stress tests for cost-over-runs:

Average Case:	multiply costs by 1.06
Moderately bad Case:	multiply costs by 1.27 (i.e., $1.06 + 0.21$)
Worst Case:	multiply costs by 1.48 (i.e., $1.06 + 2 \times 0.21$)

In addition to these studies, a search of the internet for “cost overrun” will find other studies that may be more relevant to the project in question (the quality of the study and its underlying data needs to be carefully assessed before deciding whether to use it). For example, searching for "cost overrun" “wind turbine”, results in several studies on cost over-runs for electricity projects. For example (Statista, 2019) gives the average cost over-runs for electricity infrastructure projects as shown in table 1.

¹² “Fixed price” normally means that the contractor will deliver the building as specified in the design for the price specified in the contract. However, the contractor will increase the price if there is any change away from the assumptions in the initial design.

Table 1: Average Cost Over-runs for Electricity Projects

Type of Project	Average Cost Over-run
Nuclear reactor	117.3%
Hydro-electric dam	70.6%
Thermal plant	12.6%
Transmission	8.0%
Wind farm	7.7%
Solar facility	1.3%

The sections above described the principle steps for scenario analysis:

- Establish the cascade of equations linking income and costs to shortfalls in debt payment
- Identify the sources of risk
- Link the risk drivers to cashflows
- Set the multiplier for the degree of stress of the risk drivers

The following sections describe an Excel model that implements these calculations.

Use of the Template Model to Assess a Single Guarantee in a Single Scenario

This section describes how the principles are applied in an Excel template model. This model can be used directly or used as the basis for developing more tailored models.

In Model One, to assess the loss on a single guarantee in a single scenario it is only necessary to use the “Input Cashflows” tab¹³. This tab allows the user to input the cashflow forecasts and stress multipliers directly and calculates the consequent loss. The results are not linked to any other tab, so this Input Cashflows tab allows users to easily enter and test any multipliers of interest.

The “Input Cashflows” tab¹⁴ requires the input of financial projections for four major cashflow items:

- Gross Operating Income to Company
- Operating Expenses
- Debt Principal Payment Required
- Debt Interest Payment Required

As discussed in Appendix 3, if there is a revenue guarantee it can also take projections for:

- Direct Support
- Direct Receipts

¹³ The other tabs are used to assess multiple scenarios and estimate the averaged loss, as described in the next section. If the user only wants to assess one scenario at a time, the other tabs can be ignored.

¹⁴ The model contains an “Input Cashflows” tab and a “Basecase” tab. The Input Cashflows tab allows the user to test any stress and see the consequent payments. The Basecase tab takes the four unstressed basecase cashflows that were entered at the top of the Input Cashflows tab, assumes there is no additional stress and calculates the consequent government payments. The results of the Input Cashflows tab are standalone, whereas the Basecase tab is ne of the feeds into the Collected tab to estimate the economic fees.

Model One also allows the complexity of having the debt of the new project being supported by an existing company. This is applicable for example if an existing hospital wants to add a new facility and is relying on the income from both the new facility and the existing hospital to repay the loan. The equations for income, costs and debt payments are the same for the existing company as for the new project but the amounts for the cashflows and multipliers can be different. To calculate the amount that the government must pay to support the debt, the combined income and costs from the new project and existing company are compared with the combined debt outstanding. The user can leave the cashflows blank for the existing company if there is no parent company, or if the parent company has not guaranteed the debt payments.

Model One can be used during the approvals process and an example of the workflow is at Appendix 5. In the years after the guarantee has been approved, Model One can continue to be used to monitor the risk of individual companies in a range of possible scenarios. For companies that are not in trouble, the model could be updated quarterly or annually with the current income, cost and debt projections. The uncertainties can also be updated, for example once construction is completed there is much less uncertainty in the required debt payments because the loan balance is fixed.

Box 2 gives an example of the model being applied to a transport project.

Box 2: Kosovo Transport Project

Model One was applied by the Kosovo Debt Management Group to assess the risk on an outstanding guarantee to a transport project. The basecase cashflows were entered for gross income, operating costs, debt principle repayments and interest costs on the Input Cashflows tab (these basecase cashflows were obtained from the original model provided during project approval).

The cashflows were then stressed to reflect the following risks:

- Income was reduced due to the possibility of lower than expected passenger volumes
- Costs were increased to reflect higher than expected operating costs and an increase in fuel costs
- Interest payments were increased after the point where the loan became floating rate
- Income and costs were stressed to be less than expected due to low inflation, which means that inflation is less of a help in increasing revenues to pay back the fixed portion of the loan.
- There was no stress on the debt amounts due to capital costs because the project was already operational (and even during start-up, the capital risk was small because the capital costs were mostly from a fixed price contract for standard transport equipment)
- There was no stress on the debt amounts due to FX as they were in the same currency as the income and costs

The result was a projected payment required from government. In setting the stresses, two levels were used. The first was the conservative, non-optimistic case, which corresponds to the mean stress. The second was a moderately bad case which was set such that there was only a one-in-six chance that actual outcome would be worse than this stress. This corresponds to a stress equivalent to the mean plus one standard deviation, if the stresses were Normally distributed. The difference in the stress multipliers between the mean case and the bad case therefore corresponds to one standard deviation. This was used to guide the setting of the stress levels on the Stress Multipliers tab. Having populated the Stress Multipliers tab, the Averaged Loss was read from the Collected tab.

Estimating the Fee for a Guarantee by using Multiple Scenarios

Estimating Averaged Loss Across Several Scenarios

The previous sections discussed the approach of taking base-case cashflows and stressing them to estimate the loss in single scenario. That single scenario approach can be used to say, “Given this project structure, if these stresses occur, this is how much we will need to pay”. This next section discusses how probabilities can be used to combine multiple scenarios and estimate the Averaged Loss¹⁵.

The Averaged Loss equates to the economic fee that would be charged if the government was to be fully compensated for the taking on the risk, i.e., to say “Given this project structure, and a range of possible outcomes, this is the average amount we should expect to pay, and therefore the amount we should collect in fees to break-even on average”.

The Averaged Loss is the average amount that the government will be required to pay. It is calculated by multiplying the amount paid in a stress by the probability of that stress, and then summing up the result for all stresses. For example, if there was a 96 percent chance of no loss, and a 4 percent chance of losing \$100, the Averaged Loss would be \$4:

$$\text{Averaged Loss} = 96\% \times 0 + 4\% \times \$100 = \$4$$

Note that the Averaged Loss is typically quite a small number because it includes all those cases where there is zero loss, but in this example if something does go wrong, the loss given distress will be \$100, not \$4.

In most cases there are more than two possible outcomes. For example, if there was a 3 percent chance of paying \$50 and a 1 percent chance of paying \$100, the Averaged Loss would be \$2.5, i.e.,

$$\text{Averaged Loss} = 96\% \times \$0 + 3\% \times \$50 + 1\% \times \$100 = \$2.5$$

The amount charged as a guarantee fee should reflect this estimate of the Averaged Loss. A fully commercial bank or insurance company would ensure that the fee equaled the Averaged Loss, plus administrative costs, plus a profit for taking the risk. To the extent that the government charges less than a commercial rate, it is effectively providing a subsidy. If the Averaged Loss was estimated accurately and the fee equaled the Averaged Loss, then over many guarantees, over the long-term, the total fees received would equal the total payments made by the government.

Using scenario analysis, the Averaged Loss can be estimated by setting a series of stresses of decreasing probability and increasing severity, calculating the loss, and then summing up the probability-weighted losses:

$$\text{Averaged Loss} = \text{sum} \{ \text{Probability of Scenario}^{16} \times \text{Loss in Scenario} \}$$

¹⁵ In statistical terminology this is the Expected Loss. However, the term Averaged Loss is used here because in non-technical usage “expected” implies that this is the most likely outcome and can mis-lead non-technical audiences to think that the potential loss amounts are small.

¹⁶ Note that the probabilities must sum to one. Strictly speaking, the probability of an exact scenario happening is close to zero, for example the probability of fuel prices exactly doubling is infinitesimally small. So as used here, the Probability of Scenario could be more precisely defined as the probability of a scenario occurring with similar loss levels, so each modelled scenario is representing a class, band, set, or cloud of actual scenarios.

As a numerical example, for some company, the result could look like Table 2:

Table 2. Example of Estimating Averaged Loss from Scenario Analysis

Scenario Name	Probability of Scenario (%)	Loss to Government (\$)	Probability-weighted Loss (\$)
#Base	25	0	0.00
#1	25	0	0.00
#2	20	4	0.80
#3	15	8	1.20
#4	10	17	1.70
#5	4	32	1.28
#6	1	57	0.57
		Averaged Loss	5.55

In this table, the first row is the base case, labeled #Base. This is the case where nothing goes wrong, and for most projects there will be a low probability of the actual case being better than the base¹⁷. Scenario #1 is where there are ordinary problems, for example average cost-over-runs. In each subsequent scenario the conditions are set to be worse until we get to a case that is so bad that there is only a 1% chance that the actual outcome could be worse than this. For each scenario we use a cashflow model to estimate the loss. In the base case and the relatively benign scenario (#1) the loss is zero, but in the worst case (here Scenario #6) the loss rises to \$57.

We can consider each of these individual scenarios to be representing a set of scenarios of similar severity and loss¹⁸, so the probability we assign to each scenario is actually the probability of having a scenario with similar loss, for example we take scenario #4 to represent all the possible scenarios with losses between scenario #3 and scenario #4, and therefore the probability associated with scenario #4 is the probability of the actual outcome being between scenario #3 and scenario #4.

If the analyst has a sense of the types of scenarios that could occur and their associated probabilities, then this calculation is straight-forward: enter each scenario in the cashflow model, record the result, multiply by the probability, and then sum up the results to get the averaged loss. However, it is often difficult to estimate the probability of a scenario, so the next section outlines a method for creating scenarios and estimating the probability based on the standard deviation of the underlying risk factors.

¹⁷ For example, if actual toll-road traffic is on average 75% of the forecast traffic, and there is a 25% standard deviation in the error between forecast and actual traffic, there is only a 16% chance that the actual traffic volume will be better than the basecase, i.e., that the actual traffic will be more than 100% of the forecast. In contrast, if the main risk factor is a commodity or market price, the probability of being better than the basecase may be close to 50% if the forecast is unbiased.

¹⁸ The probability of the actual outcome being exactly the same as a scenario is extremely small, but here we are looking at the probability of the actual outcome being “similar” to the scenario, where we define “similar” according to the loss level.

Linking Stress Levels to Probabilities Estimated from a Normal Distribution

This approach to estimating the probability of a scenario is to make a series of scenarios by multiplying the standard deviation of the risk factors and then assigning the probability according to the Normal probability distribution. The steps are as follows:

- Identify the most important factors determining the profitability of the project
- Link the factors, F, to the cashflow-multipliers, M
- Estimate the cashflow-multipliers for the average case, e.g., where there are average cost over-runs. Call this multiplier “A” (for average). There will be a different value for each cashflow, e.g., *A_Income*, *A_Cost*, *A_Principal*, and *A_Interest*.
- Estimate the cashflow-multipliers for the one standard deviation case, i.e., where the underlying factors move by an additional one standard deviation in the worse direction. Call this addition to the cashflow-multiplier “S”. If standard deviation information is not available, think of this scenario as being a moderately bad case but not a worst case. Mathematically there is a 16% probability of the outcome being worse than one standard deviation, i.e., a one-in-six chance of the actual outcome being worse than this moderately bad case, so estimate the cashflow-multiplier (A+S) to be the level that could occur with a one-in-six chance, and thereby estimate $S = \text{Moderately Bad} - A$.
- Make a series of scenarios based on the average and a number of standard deviations, e.g., for 0, 0.5, 1.0, 1.5, 2.0, and 2.5 standard deviations, e.g., the worst case is $A + 2.5 \times S$.
- Estimate the probability of the scenarios based on the number of standard deviations and the Normal Distribution. In Excel this function is $P = \text{NORMDIST}(x, 0, 1, 1)$ where x is number of standard deviations and P is the probability of the outcome being not as severe as x .

The result of this process is illustrated in Table 3 which creates six cashflow-multipliers and the associated probability. The probability is the probability that the scenario is worse than the actual outcome, for example in this table there is a 99% chance that the actual outcome will have lower loss than Scenario #6, or conversely there is a 1% chance of the outcome being worse than Scenario #6.

Table 3. Estimating Scenario Probability based on Number of Standard Deviations

Scenario Name	Number of Standard Deviations	Cashflow Multiplier	Probability that Scenario is worse than Actual Outcome	Probability of Scenario
#Base	$(1-A)/S^{19}$	$M = 1$	PBase = NORMDIST $\{(1-A)/S,0,1,1\}$	PBase
#1 (average)	0	$M = A$	50%	50% - PBase
#2	0.5	$M = A \pm 0.5 S$	69%	69% - 50%
#3	1.0	$M = A \pm 1.0 S$	84%	84% - 69%
#4	1.5	$M = A \pm 1.5 S$	93%	93% - 84%
#5	2.0	$M = A \pm 2.0 S$	98%	98% - 93%
#6	2.5	$M = A \pm 2.5 S$	99%	99% - 98% + (1-99%)

The \pm symbol is used because we want to either add or subtract S according to the direction that most stresses the project. For example, for actual cost versus expected cost we add S, however for actual demand compared forecast demand, we subtract S.

The basecase is where nothing goes wrong and the cashflow-multiplier M equals one, i.e., it is the case where things go better than on average. For example, if the average cost over-run was 1.1, and the standard deviation was 0.2, it would require a move of -0.5 standard deviations for basecase to actually happen. The probability (P_{Base}) of the actual outcome being better than, or as good as, the basecase is given in Excel by the function NORMDIST $\{-(1-A)/S,0,1,1\}$.

The probability of each scenario is taken to be the probability that the actual outcome will be better than the scenario, minus the probability of the previous scenario. For example, the probability of Scenario #5 is 5%, because we are saying that there is a 5% probability that the actual outcome will have a loss level between Scenario #4 and Scenario#5 and we are taking the loss level of Scenario #5 to represent all these possible cases²⁰.

Note that for the last scenario, in this case Scenario #6, we include both the probability of the loss-level being between Scenario #5 and #6, and the probability of the loss level being more than Scenario #6²¹. Appendix 6 is a technical note discussing the effective assumptions being made when obtaining the cashflow multipliers from the standard deviations.

¹⁹ $M=A + n \times S$, where n is the number of standard deviations. In the basecase $M=1$, therefore $1=A + n \times S$, from which n for the basecase must be $(1-A)/S$.

²⁰ This assumption slightly over-counts the risk because we are taking all the possible loss levels between Scenario #4 and Scenario #5 and representing them by the higher loss level of Scenario #5. This assumption tends to offset some of the assumptions discussed in Appendix 5.

²¹ In this assumption we are effectively saying that all the actual scenarios that have a loss level worse than #6 are being represented by Scenario #6. This is an undercounting of the risk. The extent of this undercounting is small so long as the amount of stress in Scenario #6 is large enough that there is only a small probability of the actual outcome being worse than #6. Practically, this means ensuring that the last scenario has at least two standard deviations of stress in the risk factors.

Example of Estimating the Probability of Scenarios

As a numerical example, consider an initial analysis for a generic toll road. First, we will quantify the underlying risk factors, and from that derive the multipliers for A and S.

Bain (2009), estimates the average traffic volume for toll roads to be 76 percent of the forecast volume with a standard deviation of 26 percent. For simplicity, let us assume there are no significant effects from inflation and tariff shortfalls.

Assume the road is funded 50 percent by debt, and that any cost over-run is paid from debt, i.e., the increase in the amount of debt equals the increase in the amount of construction costs. Flyvberg et al (2002) estimate that road projects have an average cost over-run of 20 percent, with a standard deviation 30 percent. For the initial analysis we will assume that the uncertainties in operating costs are the same as the uncertainties in construction costs because most of the operating costs are repairs.

Let us further assume that this project is funded by floating rate loans in local currency, so there is interest-rate risk, but no FX risk. For interest rates, from 30 years of historical data the short-term interest rates for an example country fluctuate by a factor of 32 percent year-to-year (e.g., a change from 2% to 2.64% would be a one standard deviation move). For the initial analysis we will use only the standard deviation of the interest rate over one year. Table 4 summarizes the statistics of the underlying risk drivers for this project.

Table 4. Toll Road Example Summary of Uncertainties for Underlying Risk Factors

Factor	Mean of Factor	One Standard Deviation move in Worst Direction
Traffic Volume	76%	-26%
Costs	120%	30%
Interest rates	100%	32%

The next step is to use these uncertainties in the underlying risk factors to estimate cashflow-multipliers as follows:

The Income is directly linked to traffic volumes i.e., for M_{Income} A equals 0.76 and S equals -0.26. Note that to define the stresses we subtract the standard deviation because it increases the risk for this factor.

Assume operating costs increase in the same way as the construction costs, i.e., the multiplier A for M_{Costs} equals 1.2 and S equals 0.3.

The debt multiplier increases twice as much as the cost multiplier because of the 50% leverage i.e., for $M_{Principal}$ A equals 1.40 and S equals 0.60²².

The interest multiplier is the same as the debt multiplier but with additional uncertainty for interest rates. In this case let the basecase all-in total rate on the debt be 5 percent with LIBOR at 2 percent in the basecase. Assume that rates are expected to stay flat, so in the average case the multiplier is one. A one standard deviation move by a factor of 1.32 would increase LIBOR to 2.64 percent, thereby increasing the scenario's

²² Appendix 3 discusses the link between cost over-run and debt multipliers.

all-in rate to be 5.64 percent so in the one-standard deviation case the multiplier due to floating rate interest rates is $5.64/5.00$ i.e., M_Rate for the one standard deviation case is 1.13. Given this, $M_Interest$ equals 1.40 and S equals 0.86^{23} .

The resulting averages and standard deviations for the cashflow-multipliers are given in Table 5.

Table 5. Toll Road Example: Average and Standard Deviation for the Cashflow-Multipliers

Cashflow Multiplier	A	S
<i>M_Income</i>	0.76	-0.26
<i>M_Cost</i>	1.20	0.30
<i>M_Principal</i>	1.40	0.60
<i>M_Interest</i>	1.40	0.86

From the average and standard deviation of the cashflow-multipliers we construct the series of scenarios by taking different numbers of standard deviations as in Table 6. For the basecase we set the cashflow-multipliers equal to 1. In other words, the basecase assumes that the actual traffic volume will be better than the average. For the income multiplier in the basecase to equal one implies that there has been a move of -0.92 standard deviations from the average case of 0.7624. We use this with the NORMDIST function to calculate the probability of the basecase scenario.

For Scenario #1 we just take the average, and for Scenario #2 onwards we add multiples of the standard deviation. For each multiple we estimate the associated probability by assuming a Normal distribution. For example, there is an 84 percent chance that the actual project performance will be better than Scenario #3 which has a one standard deviation risky move.

To summarize, for the basecase we set the cashflow-multipliers to 1.00, calculate the implied number of standard deviations, N and thereby the implied probability. For the other scenarios we set the number of standard deviations and calculate the cashflow-multipliers from A and S. In all cases we estimate the probability based on the number of standard deviations.

²³ S for the multiplier is estimated from the one standard deviation case minus the average case. The one standard deviation case includes the change in the debt amount and the change in the interest rates: $(A+S)=(1.4+0.6)\times 1.13=2.26$, $A = 1.4$, therefore $S=0.86$ for interest payments

²⁴ As earlier, $N = (\text{Mean}-1)/\text{StDev}$ for the basecase multiplier to equal 1.00

Table 6. Toll Road Example Cashflow Multipliers for Scenarios of Increasing Severity

Scenario	Number of Standard Deviations (N)	Probability that Scenario is worse than Actual Outcome	Probability of Scenario	<i>M_Income</i>	<i>M_Cost</i>	<i>M_Principal</i>	<i>M_Interest</i>
Base	-0.92 ²⁵	0.18	0.18	1.00	1.00	1.00	1.00
# 1	-	0.50	0.32	0.76	1.20	1.40	1.40
# 2	0.50	0.69	0.19	0.63	1.35	1.70	1.89
# 3	1.00	0.84	0.15	0.50	1.50	2.00	2.38
# 4	1.50	0.93	0.09	0.37	1.65	2.30	2.88
# 5	2.00	0.98	0.05	0.24	1.80	2.60	3.35
# 6	2.50	0.99	0.02	0.11	1.95	2.90	3.84

These cashflow-multipliers are then applied to the cashflow amounts to calculate the government’s payment per year per scenario. For example, assume that the cashflow model estimates the losses in Table 7. Multiplying by the probability of each scenario gives the weighted loss per scenario and the averaged loss across all scenarios.

Table 7. Toll Road Example Calculation of Averaged Loss

Scenario	Probability of Scenario	Loss to Government (\$)	Probability Weighted Loss
Base	0.18	0	0
# 1	0.32	0	0
# 2	0.19	4	0.76
# 3	0.15	8	1.20
# 4	0.09	17	1.53
# 5	0.05	32	1.60
# 6	0.02	57	1.14
Averaged Loss			6.23

Template Model for Averaged loss

In Excel Model One, the StressMultipliers tab contains the cashflow-multipliers to be used for the basecase and for six additional scenarios. The Input Cashflows tab is where the company’s financial information is entered. The tabs called Basecase and Scenario1 to Scenario 6 are identical to Input Cashflows except that they copy the financial information from Input Cashflows and they have different numbers for looking up risk factors from the StressMultipliers tab. The results from Basecase and each of the Scenarios tabs are linked to the Collected tab. The Collected tab weights the Scenarios’ losses with their associated probabilities to calculate the Averaged Loss. The Averaged Loss is then given in several forms: as an

²⁵ Calculated from $(1-A)/S$ of the worst factor, i.e., *M_Income*

amount per year, as a net present value and as a percentage of the loan balance²⁶. This percentage would be the guarantee fee if the fee was charged per year and covered the full cost of the Averaged Loss²⁷.

Estimating the Correlated Loss for a Portfolio of many Guarantees in a Single Scenario

Linking Losses to Macro-economic Risks

In the sections above we looked at a single guarantee under multiple scenarios to inform decisions on approval, structuring, and pricing. In this section we look at portfolios containing multiple guarantees. A portfolio of multiple guarantees poses a particular risk to a government if losses on those guarantees occur at the same time as each other, i.e., if all companies tend to default at the same time. This is especially risky if the losses occur at the same time as other fiscal problems. As a high-level example, consider a portfolio of a guarantee to a mine plus a guarantee to an airline. As commodity prices rise, the mine gains income and the airline gains costs, so the potential loss to the mine becomes lower and the potential loss to the airline becomes higher, i.e., the risks are negatively correlated. Alternatively, a portfolio of guarantees to a hydro project and to agriculture will tend to suffer losses at the same time if there is a drought. These drought-related losses will probably also be correlated with lower tax receipts and increased costs for government relief efforts.

The extent of this correlation can be estimated by simultaneously subjecting all the guarantees to the same adverse economic and market conditions. This can be done in the same framework as used above, but breaking the cashflow stress multipliers into idiosyncratic and systemic components. The idiosyncratic components are risks that are unique to the company and not correlated with other companies, e.g., mis-estimation of costs or demand. The systemic components are driven by changes in the economy and market such as increased fuel costs, increased interest rates or lower demand due to depressed economic conditions. Effectively this links the macro-economics of the country to the micro-economics of each guaranteed company.

To illustrate the separation of idiosyncratic and systemic risks, let us take the specific example of the income for a toll road. The multiplier on income can be a combination of the idiosyncratic mis-estimation of expected demand, multiplied by a systemic fall in demand due to a poor economy, multiplied by a fall in tariff due to lower than expected inflation:

$$M_Income = (1 + \text{Estimation Error}) \times \text{GDP Effect} \times \text{CPI Effect}$$

In this example let's say that for every 1% fall in GDP, the demand falls 2%. If we use the terms GDPs for the scenario GDP and GDPo for the Base GDP, we have:

$$\begin{aligned} \text{GDP Effect} &= (1 + 2 \times \text{GDP relative change}) \\ \text{GDP Effect} &= (1 + 2 \times [\text{GDPs} - \text{GDPo}] / \text{GDPo}) \\ \text{GDP Effect} &= (1 + 2 \times [\text{GDPs} / \text{GDPo} - 1]) \end{aligned}$$

²⁶ The loan balance is estimated from the input debt repayments. Alternatively, if the loan balance per year is known, then analyst can over-write the estimate.

²⁷ A fully economic fee (e.g., charged by a bank) would also include a cost of risk (cost of capital) and administrative costs. To the extent that the government charges less, this is effectively a subsidy.

As a further complication, let's say that 30% of the use was precontracted with a trucking company, so only 70% of the demand was dependent on GDP:

$$\begin{aligned} \text{GDP Effect} &= 30\% + 70\% \times (1 + 2 \times [\text{GDPs}/ \text{GDPo} - 1]) \\ \text{GDP Effect} &= 1 + 70\% \times 2 \times [\text{GDPs}/ \text{GDPo} - 1] \end{aligned}$$

We can generalize this to say that the percentage of income affected by GDP is represented by “p” and the sensitivity to changes in GDP is represented by “b”. Then

$$\text{GDP Effect} = 1 + p \times b \times [\text{GDPs}/ \text{GDPo} - 1]$$

We can derive a similar equation for the CPI effect, then returning to the equation for the overall income for this example we have:

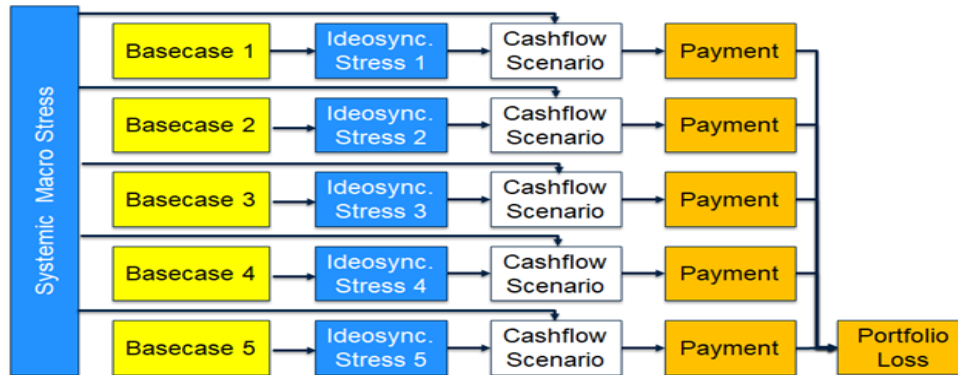
$$\begin{aligned} M_Income &= (1 + \text{Estimation Error}) \\ &\quad \times (1 + p_g \times b_g \times [\text{GDPs}/ \text{GDPo} - 1]) \\ &\quad \times (1 + p_i \times b_i \times [\text{CPIs}/ \text{CPIo} - 1]) \end{aligned}$$

where p_g is the percentage of income affected by GDP, b_g is the sensitivity of the effected portion to GDP, p_i is the percentage of income affected by CPI, and b_i is the sensitivity of the effected portion to CPI. In general, each scenario cashflow is the base cashflow multiplied by the idiosyncratic risk and multiplied by the effect of all the market factors. The resulting set of equations for M_Income , M_Cost , $M_Principal$ and $M_Interest$ are shown in detail Appendix 7 along with an example of setting the parameter values.

Description of Model Two for Multiple Guarantees

The purpose of Model 2 is to assess the potential loss on a portfolio of guarantees responding to a systemic economic scenario. Model 2 is similar to Model 1 and takes the same basecase cashflows for its inputs, but instead of having one project with multiple scenarios, it has multiple companies with one scenario at a time. Model Two can be used with the same company basecase cashflows as Model 1, but linking the cashflows to the general economy. The illustration in Figure 6 shows how the model is structured to feed the systemic risk and idiosyncratic risks into the cashflows for multiple companies, estimate the payment to each project, and then sum up the payments across the portfolio. In this illustration each basecase is for a different project, e.g., Basecase 1 is the basecase income, cost and debt payments for project 1, Basecase 2 is the basecase income, cost and debt payments for project 2 etc.

Figure 6. Testing Multiple Companies in a Single Macro Scenario to Estimate the Combined Portfolio Loss



In Model Two the alternative macro-economic scenarios are entered on the Macro-Scenarios tab. For each company the basecase cashflows, idiosyncratic stresses, scenario cashflows and government payments are calculated on the “Company” tab. The sensitivities of each company to the macro-economic conditions are also entered on the Company tabs.

The template model is set up containing only two Company tabs. To add more companies, simply add a new blank sheet, copy all of the Company 1 tab onto the new sheet, and link the results for “Total payments by Government” to the Collected Results sheet so that the loss is added to the loss of the other companies. The Collected Results tab is also the central place for setting which scenario number is to be run.

The template models are delivered in Excel so that government analysts can follow the calculations, and so that, if desired, analysts can use the models as the basis for making more sophisticated models. For analysts wanting to add detail to the models, Appendix 8 describes how to make several immediate enhancements.

Conclusion

This paper has presented a relatively simple but flexible model that can be adapted to give estimates of losses, fees and portfolio correlations. The models can be used as stand-alone tools or in connection with macro-modeling tools, when shock factors can be assessed as part of a more systemic analysis. In addition to being useful as a relatively direct approach to quantifying the loss amounts, these models can also be used as the starting point to make more detailed models, as a check on other models or as an alternative view of the risks to complement other approaches.

The methodology and the credit risk assessment models can be used as part of the process of evaluating loan guarantee proposals, on-lending proposals, and regular monitoring of the outstanding portfolio of government guarantees and on-lending.

Appendix 1: Different Risk Measurement Approaches

Risks associated with a guarantee can be assessed based on the standard credit-risk evaluation techniques (Marrison 2002). There are four principal approaches to credit risk quantification for guarantees:

Credit grading scorecards – This approach typically looks at a series of financial and business factors for the entity, assigns a numerical value to each factor, weights each factor, and then calculates the weighted sum. The sum is then typically related to a numerical or letter grade. If there is not a large enough sample of historical defaults, the weights are set by experts looking across multiple similar projects and adjusting the weights until each project has a score reflecting either the experts' judgement of the credit quality for each project or an external standard such as a letter grade assigned by a rating agency.

Statistical models – When there is a sufficient amount of historical data on credit events for similar companies or projects, it is possible to use statistical techniques to compute a financial distress index from a set of weighted profitability and balance sheet ratios. Weights are obtained through a discriminant analysis and can be adapted to the nature of the entity being evaluated. This approach is relatively easy to use but requires the availability of ample historical data for calibrating the model. A limitation is that its predictability is good only over a short horizon e.g., one to two years. Statistical models and scorecards are normally built to estimate the probability of default. To estimate the average expected loss (and economic fees), a second set of models is needed to estimate the loss amount given default.

Scenario analysis – Scenario analysis is a commonly used technique in which analysts ask, “what would happen if”. At one extreme, the analysis may be simply a conversation and consideration of future possibilities. At the other extreme it may use a detailed cashflow projection model with stresses applied to market factors such as commodity prices and company-specific factors such as construction cost over-runs. Such analysis gives an idea of the sensitivity of the entity to changes in the assumptions and the extent to which it can absorb risks before guarantee payments are required. The scenario approach can also be used in cases where the company is already in distress to estimate the amount and timing of the required payments.

Cashflow simulation – In scenario analysis, as discussed above, a few “handpicked” scenarios are tested. In cashflow simulation similar cashflow models are used but the scenarios are selected randomly based on assumed probability distributions for the risky variables (for example, revenues, traffic flows, exchange rate). The distributions are ideally based on historical data. The cashflow simulation techniques offer the advantage of capturing project- and context-specific risks but are complex and technically demanding. For large projects, where there is a large amount of money at stake, cashflow simulation models may be worthwhile, but their complexity means that if not properly built, the results can be misleading, so the models need to be carefully vetted, preferably by a third party.

Appendix 2: Types of Risks Faced by Different Types of Companies

Different types of projects and companies have different types of risks. The risks can also vary over time. The greatest variation over time is between initial planning and initial operation. For new projects there are the risks of construction-cost over-runs, delays, and over-optimism in estimating customer demand. For a project or company that is already in operation, the main risks are about efficiency, changes in the market and credit risk from contracted counterparties.

List of risks according to type of project

Examples of risks for different types of companies are given below.

Power generation

Additional risks if operations have not started

Construction cost over-run and delay, especially if a fixed price contract has not been agreed²⁸

Over-optimistic forecast of technical efficiency, e.g., increased fuel requirement, lower power generation, or lower availability

Over-optimistic forecast of operational costs, e.g., more staff

Delays, increased costs or reduced income due to the need to take into account the findings of environmental impact analysis, or mitigation of the concerns of local residents.

Delays, increased costs or reduced income due to incorrect projections if the technology or implementation are new (e.g., Panama Canal vs. Suez Canal).

Risks during operations

Increase in unhedged fuel costs

Unexpected changes in tariffs, e.g., government influence to minimize increases

Higher than expected electricity theft

Lower than expected bill collection rates

Default or slow payment by large customers

Default on supply or offtake agreements

Change in FX rates affecting costs of debt, equipment and staff

Increase in interest rates on floating rate debt

Lower technical efficiency than forecast, e.g., lower power generation or availability

Lower operational efficiency than forecast, e.g., more staff

Increased maintenance costs

Unexpected inflation (if for example, inflation is lower than expected, revenues may not increase sufficiently to cover fixed supply and debt costs)

Failure to produce, e.g., due to drought for hydro project

²⁸ If no contract is yet signed, the actual costs could be far higher than initially estimated. Even with a fixed price contract there can be some cost increases e.g., if changes are needed to the specifications or if the contract has clauses saying that the price is fixed only if ground conditions are as expected.

Transport projects such as toll-roads or SOEs such as airlines

Additional risks if operations have not started

Construction cost over-run and delay

Over-optimistic forecast of demand or operational costs

Risks during operations

Lower demand or prices due to economic downturn

Change in FX rates or interest rates affecting costs of debt

Increased maintenance or fuel costs

Unexpected inflation

Facilities such as hospitals and schools

Additional risks if operations have not started

Construction cost over-run and delay

Over-optimistic forecast of demand or operational costs

Risks during operations

Lower demand or prices due to economic downturn

Change in FX rates or interest rates affecting costs of debt

Increased maintenance costs

Unexpected inflation

Contract default by a counterparty who has agreed to lease part of the facilities

Water supply or treatment (in many ways the risks are similar to electricity production)

Additional risks if operations have not started

Construction cost over-run and delay, especially if a fixed price contract has not been agreed²⁹

Difficulty with secondary construction e.g., distribution

Over-optimistic forecast of operational costs, e.g., more staff needed or more breakdowns

Risks during operations

Increase in fuel costs

If users pay directly for the service, there may be delinquencies or refusal to increase tariffs

If payment is from an intermediary (e.g., municipality), the municipality may refuse to pay or be unable to pay

Higher than expected water theft

Default or slow payment by large customers

Default on offtake agreements

Change in FX rates affecting costs of debt, equipment and staff

²⁹ If no contract is yet signed, the actual costs could be far higher than initially estimated. Even with a fixed price contract there can be some cost increases e.g., if changes are needed to the specifications or if the contract has clauses saying that the price is fixed only if ground conditions are as expected.

Increase in interest rates on floating rate debt

Unexpected inflation (if for example, inflation is lower than expected, revenues may not increase sufficiently to cover fixed supply and debt costs)

Failure to produce, e.g., due to drought

Checklist of Typical Risks across all Project Types

Capital cost over-run causing an increase in debt payments

Construction time over-run delaying income

Increase in operating effort e.g., more maintenance or staff than forecast

Lower production or higher consumption due to technical problems

Abandonment of project by construction company or equity sponsor, leaving government to operate the plant and pay back debt

Default of supply agreements, forcing supply to be bought at higher market values

Default of offtake agreements, forcing production to be sold at lower market values

Removal of expected subsidies or other government support

Lower demand than forecast

Lower prices or tariff than forecast

Higher fuel or other feedstock costs

Change in FX rates affecting income, operating costs or the cost of servicing debt

Higher or lower than expected inflation, commodity prices, or interest rates

Uninsured events e.g., flood, fire, strike, civil disturbance or other force majeure

Climate risks such as uninsured flooding, insufficient water for hydro, low agricultural production, increased cooling costs

There are also a whole series of “event risks” such as fires and earthquakes, but these can be discounted from the analysis IF they will be covered by adequate insurance that will pay the debt in case of such an event.

Appendix 3: Inclusion of Revenue Guarantees

In the body of the paper we only considered government payments on debt guarantees. In addition, governments sometimes make commitments such as minimum-revenue guarantees. For example, in a toll road project, the government may guarantee a minimum revenue amount based on the forecast traffic volumes. The net cashflow for such a guarantee is the difference between the full amount of the forecast income from traffic and the actual income from traffic. Such payments can be captured in the scenario models by adding lines for Direct Payment and Direct Revenue as follows:

$$\begin{aligned}\text{Direct Support} &= \text{Base Direct Support} \times M_{\text{Support}} \\ \text{Direct Revenue} &= \text{Base Direct Revenue} \times M_{\text{Revenue}} \\ \text{Net Support Payment} &= \text{Direct Support} - \text{Direct Revenue}\end{aligned}$$

For example, if a toll road is projected to earn \$100, but actually earns \$60, the government will make a net support payment of \$40.

$$\begin{aligned}\text{Direct Support} &= \$100 \times 100\% \\ \text{Direct Revenue} &= \$100 \times 60\% \\ \text{Net Support Payment} &= \$40\end{aligned}$$

Note that if a revenue guarantee is used in combination with a debt guarantee, the effect is to reduce the uncertainty in the project income. The revenue guarantee shifts much of the risk from the debt guarantee and it is important to not double count the risk. For example, in a project without a revenue guarantee, if the traffic volume halves the income halves and there is a risk of not paying some of the debt. However, in the same project but with a revenue guarantee, the government makes up the shortfall in income, there is a net loss on the revenue guarantee but there is no call on the debt guarantee. In the first case the loss of revenue is first absorbed by the company before impacting the debt. In the second case it directly affects the government payments.

Bringing together the equations for guarantee payments and direct support, the estimate of loss is given by:

$$\begin{aligned}\text{Gross Income} &= \text{Base Gross Income} \times M_{\text{Income}} \\ \text{Operating Cost} &= \text{Base Operating Cost} \times M_{\text{Cost}} \\ \text{Net Operating Income} &= \text{Gross Income} - \text{Operating Cost} \\ \text{Principal Payment} &= \text{Base Principal} \times M_{\text{Principal}} \\ \text{Interest Payment} &= \text{Base Interest} \times M_{\text{Interest}} \\ \text{Govt Debt Payment} &= \text{Percent Guaranteed} \times \max[0, \text{Principal} + \text{Interest} - \text{NOI}] \\ \text{Direct Support} &= \text{Base Direct Support} \times M_{\text{Support}} \\ \text{Direct Revenue} &= \text{Base Direct Revenue} \times M_{\text{Revenue}} \\ \text{Net Support Payment} &= \text{Direct Support} - \text{Direct Revenue} \\ \text{Total Govt. Payment} &= \text{Govt. Debt Payment} + \text{Net Support Payment}\end{aligned}$$

Appendix 4: Linking Increases in Debt Principal to Construction Cost Over-runs

In the basecase, the capital costs are funded by the expected equity contribution plus the expected debt. Any increase in cost must be funded from a mixture of debt and equity. The proportional increase in the amount of debt depends on the following three factors:

- e: the percentage of cost over-run funded from equity
- d: the percentage of the project expected to be funded from debt in the basecase
- c: the percentage by which the costs increase

Multiplying the basecase cost (Co) by d gives the expected debt outstanding (Do):

$$Do = d \times Co$$

Adding the cost increase, less the amount paid from equity gives the actual debt amount (D):

$$D = d \times Co + c \times Co \times (1-e)$$

The multiplier on principal is the ratio of D/Do :

$$\begin{aligned} M_{Principal} &= [d \times Co + c \times Co \times (1-e)] / (d \times Co) \\ &= [d + c \times (1-e)] / d \\ &= 1 + c \times (1-e) / d \end{aligned}$$

For example, if the project is planned to be 60% funded from debt, and all the cost-over-run is absorbed by equity, then $d = 60\%$, $e = 100\%$ and $M_{Principal} = 1$. Conversely, if the equity defaults, costs increase 40% and debt is used to finish the project, $d = 60\%$, $e = 0\%$, $c = 40\%$ and $M_{Principal} = 1.67$, i.e., a 40% increase in costs leads to a 67% increase in debt payments.

Appendix 5. Example of Workflow using Model One

This appendix describes a way in which the models may be used as part of the approvals and structuring process:

The risk assessment team receives the documentation for the guarantee request and ensures that the documentation includes the projected base case forecasts for gross income, operating costs, and debt payments to populate the model. If the information is missing, the team requests it from the applicant.

The team reviews the project proposal to assess whether the information is reasonable and reliable, if possible, with cross-checks. Then, along with the line ministry, the team identifies the factors that could change the financial performance of the project, for example cost over-runs, a shortfall in demand, and changes in other government decisions and subsidies.

The risk assessment team enters the data into the model and performs an initial stress-test by multiplying the income, costs, and debt payments according to the possible changes in the factors. From this initial stress, the team will get an idea of the range of costs created by the guarantee and how sensitive the project is to changes in circumstances. This preliminary analysis can be used to direct the collection of further information from the applicant, for example about the current state of contract negotiations, examples of similar projects elsewhere, and whether there are options to reduce the risk in the project by, for example, changing the currency of the loan, hedging commodity prices, or building the project in stages. This preliminary analysis can also be used to identify which risk factors are the most important and therefore require more research. For example, if oil prices are a key risk, the team can look at the historical variability of oil prices and as a moderate stress take a change of one standard deviation, and as a harder stress look at the worst case in the last ten years.

If there is insufficient time, if it is a minor project, or if the initial analysis clearly shows that the project is too risky, the approval decision may be based on this initial quantification of the risk along with other factors such as the governance structure of the entity and past moral hazard behavior. Alternatively, the process may be iterative, with the team getting better information on the project and better information on the range of uncertainty in the primary risk drivers.

If a more thorough analysis is warranted, the next step is to use Model One to estimate the economic cost of the Averaged Loss. As discussed in the earlier section on setting the stressed scenarios and the section on estimating the Averaged Loss, this is done by testing increasingly severe scenarios and estimating their probabilities. The resulting estimate of Averaged Loss is the average amount that the government is likely to pay. This Average Loss can be used to set the guarantee fee and is the minimum amount³⁰ that a commercial organization, like a bank, would charge as a fee to take –on the expected payments.

Knowing the estimate of Average Loss, the ministry can then set the fee amount. The difference between the Average Loss and the actual fee charged is an effective subsidy (or to be precise the minimum amount of the subsidy since it excludes the costs of administration and risk-bearing), and this subsidy amount can be compared with the benefits and with other subsidies to decide whether the guarantee is an effective use of public money. In this way, the fee negotiations go hand-in-hand with the approvals process, because if

³⁰ A commercial organization would also need to charge a fee for the administrative time and a fee to cover the cost of the capital that must be held in case the losses are higher than expected.

the fee is sufficient to cover most of the risk, there is very little cost to the government and the guarantee can be approved. If the future economic cost of the actual risk is expected to be much higher than the fees, this burden needs to be balanced more carefully against the actual benefits.

Appendix 6. Technical Note on Assumptions Embedded in the Standard Deviation Approach

There are four main assumptions in using this approach:

- We are assuming that the probability distribution of the risk factors is Normally distributed. This is a reasonable assumption for macro-economic factors, and moderately bad scenarios, but less reasonable for factors such as extreme cost-over-runs.
- We are assuming that the estimates of mean and standard deviation are not overly optimistic
- We are assuming that beyond one standard deviation the effects are additive.
- We are assuming perfect correlation between all the factors, i.e., that all factors go bad at the same time.

Assumptions 1, 2, and 3 tend to undercount the risk whereas assumption 4 tends to overcount the risk. Assumption 3 deserves further explanation. As an example, consider a cashflow-multiplier that is 100 percent affected by two underlying risk factors, Y and Z such that $M=Y \times Z$, and assume Y and Z both have a mean of 1.2 and a standard deviation of 0.15. We then have the following:

Case with average movement in both:	$= 1.2 \times 1.2 = 1.44$
Case with one std. dev. movement in both	$= 1.35 \times 1.35 = 1.82$
Case with two std. dev. movement in both:	$= 1.50 \times 1.50 = 2.25$

However, our approach of using the standard deviation of the cashflow-multiplier would be:

A	$= 1.44$
A + S	$= 1.82$ therefore S = 0.38
A + 2 × S	$= 2.21$

In this example the additive approximation gives a two standard-deviation cashflow-multiplier of 2.21 whereas the full multiplication gives 2.25. This discrepancy arises when several factors multiply each other. There are two ways to treat this discrepancy. One is to accept the discrepancy and consider that this undercounting of risk is offset somewhat by the assumption of 100 percent correlation which tends to overcount the risk. The other alternative is to not use the approach of multiplying by the number of standard deviations of M, but instead multiply the individual risk factors by the number of standard deviations and calculate the resulting cashflow-multiplier directly.

Appendix 7. Equations Linking Macro-economic Scenarios to Cashflows

In general, each scenario cashflow is the base cashflow multiplied by the idiosyncratic risk and multiplied by the effect of all the market factors. Mathematically this can be written as:

$$CF_{c,Scenario} = CF_{c,Base} \times I_{c,Scenario} \times \prod_{f=1}^n \left(1 + p_{c,f} \times b_{c,f} \times \left(\frac{F_{Scenario}}{F_{Base}} - 1 \right) \right)$$

Where

CF_c ,	is a cashflow c , e.g., income, expenses, debt payment, or interest payment
$I_{c,Scenario}$	is the multiplier on cashflow c for the idiosyncratic risk in the given scenario
$\prod(\dots)$	is the symbol for multiplication
f	is the label for each macro-economic factor and n is the number of factors
$p_{c,f}$	is the percentage of cashflow c that is affected by factor f
$b_{c,f}$	is the sensitivity of cashflow c to factor f
$F_{Scenario}$ and F_{Base}	are the levels of macro-economic factor f in the scenario and basecase

For example, let us take the case where there are five macro-economic factors: GDP, CPI, Interest Rates, FX, and Commodity Prices. The multiplier for income would then be³¹:

$$\begin{aligned} \underline{M \text{ Income}} = I_{income,Scenario} & \\ & \times \left(1 + p_{income,GDP} \times b_{income,GDP} \times \left(\frac{GDP_{Scenario}}{GDP_{Base}} - 1 \right) \right) \\ & \times \left(1 + p_{income,CPI} \times b_{income,CPI} \times \left(\frac{CPI_{Scenario}}{CPI_{Base}} - 1 \right) \right) \\ & \times \left(1 + p_{income,FX} \times b_{income,FX} \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right) \\ & \times \left(1 + p_{income,Price} \times b_{income,Price} \times \left(\frac{Price_{Scenario}}{Price_{Base}} - 1 \right) \right) \end{aligned}$$

To make this a little more concrete, as an example, let us consider an airline with the following parameters:

100% of income falls by 4% if GDP falls 1%:

$$p_{income,GDP}=100\%, \quad b_{income,GDP} = 4$$

100% of income falls by 1% if CPI falls 1%:

$$p_{income,CPI}=100\%, \quad b_{income,CPI} = 1$$

60% of ticket sales are from overseas:

$$p_{income,FX}=60\%, \quad b_{income,FX} = 1$$

Income is unaffected by commodity prices:

$$p_{income,Price} = 0, \quad b_{income,Price} = 0$$

³¹ Note that Income is not generally directly dependent upon interest rates, unless the entity is for example a bank where there is a direct linkage between rates and income.

Numerically the equation then becomes:

$$\begin{aligned}
 \underline{M\ Income} = I_{income,Scenario} & \times \left(1 + 4 \times \left(\frac{GDP_{Scenario}}{GDP_{Base}} - 1 \right) \right) \\
 & \times \left(1 + 1 \times \left(\frac{CPI_{Scenario}}{CPI_{Base}} - 1 \right) \right) \\
 & \times \left(1 + 0.6 \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right) \\
 & \times \left(1 + 0 \times \left(\frac{Price_{Scenario}}{Price_{Base}} - 1 \right) \right)
 \end{aligned}$$

For M_Cost the equations are virtually the same as for income but the values for p and b are different. For example, let us assume that 30% of the airline's costs depend on demand (driven by GDP), 20% of costs change with inflation (staff), 20% of costs depend on FX (spare parts and technical assistance), and 50% of the costs depend on commodity (oil) prices. M_Cost would then be:

$$\begin{aligned}
 \underline{M\ Cost} = I_{Cost,Scenario} & \times \left(1 + 0.3 \times 4 \times \left(\frac{GDP_{Scenario}}{GDP_{Base}} - 1 \right) \right) \\
 & \times \left(1 + 0.2 \times \left(\frac{CPI_{Scenario}}{CPI_{Base}} - 1 \right) \right) \\
 & \times \left(1 + 0.2 \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right) \\
 & \times \left(1 + 0.5 \times \left(\frac{Price_{Scenario}}{Price_{Base}} - 1 \right) \right)
 \end{aligned}$$

From these equations we see that the change in economic conditions causes the changes in income and costs to be correlated, but not exactly the same because the percentages and sensitivities are different.

The equation is simpler for M_Principal because the principal payment amounts in local currency terms are only dependent on the debt outstanding and FX rates for any portion of the debt that is denominated in foreign currency. The debt outstanding primarily depends on idiosyncratic risks such as cost over-runs. This gives:

$$\underline{M\ Principal} = I_{principal,Scenario} \times \left(1 + p_{principal,FX} \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right)$$

Here $p_{principal,FX}$ is just the percentage of the debt that is denominated in FX.

M_Interest is directly affected by M_Principal because interest payments are directly linked to the outstanding amount and effects of FX, but in addition interest payments may increase if part of the debt is priced at a floating rate, e.g., at LIBOR+4%. The effect of changes in interest rates needs to be treated a little differently from the other factors. This small difference in treatment means that in the cashflow model we need to be sure to include the special calculation for interest rates, and it also means that the sensitivity is parameterized a little differently.

This is because we want to know the ratio of the scenario interest rate divided by the base interest rate but the base interest rate is not directly given in the economic scenario. For other economic factors, the basecase is described by the macro-economic forecast in the basecase, but for interest rates base and scenario rate includes the margin or spread above forecast for LIBOR. For example, if the loan is priced at LIBOR a margin of 4% and LIBOR doubles from 2% to 4%, the all-in total rate moves from 6% to 8%, i.e., by a factor of 1.33, not by a factor of 2. Looking only at the effect of interest rates, and assuming that all the debt is floating rate, we have:

$$\begin{aligned}
 M_Rate &= \text{Scenario Loan Rate} / \text{Base Loan Rate} \\
 M_Rate &= (\text{Scenario LIBOR} + 4\%) / (\text{Base LIBOR} + 4\%) \\
 M_Rate &= (\text{Base LIBOR} + 4\% + \text{Rate Change}) / (\text{Base LIBOR} + 4\%) \\
 M_Rate &= [1 + \text{Rate Change} / (\text{Base total rate})]
 \end{aligned}$$

This last line lets us express M_Rate in terms of the change in the economic scenarios, divided by the total (or “all-in”) rate on the loan in the basecase. Combining the change due to principal amount and the change due to rates gives $M_Interest$:

$$\begin{aligned}
 M_Interest &= M_Principal \times \left(1 + p_{floating} \times \left(\frac{LIBOR_{Scenario} - LIBOR_{Base}}{Base\ Total\ Rate} \right) \right) \\
 M_Interest &= M_Principal \times \left(1 + p_{floating} \times \left(\frac{1}{Base\ Total\ Rate} \right) \times (LIBOR_{Scenario} - LIBOR_{Base}) \right)
 \end{aligned}$$

This is very similar to the equations for income and cost. Here $p_{floating}$ is the proportion of debt that is floating (rather than having the rate fixed). The sensitivity is $\left(\frac{1}{Base\ Total\ Rate} \right)$ where the total rate is the interest rate on the floating part of the loan in the basecase. However, unlike the income and cost equations, we use the absolute difference between the scenario and the base, rather than dividing by the base scenario to get the relative difference.

To summarize, the section below pulls together the equations for the cashflow-multipliers to be applied when combining idiosyncratic and systemic risks.

$$\begin{aligned}
 \underline{M\ Income} &= I_{income,Scenario} \\
 &\times \left(1 + p_{income,GDP} \times b_{income,GDP} \times \left(\frac{GDP_{Scenario}}{GDP_{Base}} - 1 \right) \right) \\
 &\times \left(1 + p_{income,CPI} \times b_{income,CPI} \times \left(\frac{CPI_{Scenario}}{CPI_{Base}} - 1 \right) \right) \\
 &\times \left(1 + p_{income,FX} \times b_{income,FX} \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right) \\
 &\times \left(1 + p_{income,Price} \times b_{income,Price} \times \left(\frac{Price_{Scenario}}{Price_{Base}} - 1 \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 \underline{M\ Cost} &= I_{cost,Scenario} \\
 &\times \left(1 + p_{cost,GDP} \times b_{cost,GDP} \times \left(\frac{GDP_{Scenario}}{GDP_{Base}} - 1 \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \times \left(1 + p_{cost,CPI} \times b_{cost,CPI} \times \left(\frac{CPI_{Scenario}}{CPI_{Base}} - 1 \right) \right) \\ & \times \left(1 + p_{cost,FX} \times b_{cost,FX} \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right) \\ & \times \left(1 + p_{cost,Price} \times b_{cost,Price} \times \left(\frac{Price_{Scenario}}{Price_{Base}} - 1 \right) \right) \end{aligned}$$

$$\underline{M_Principal} = I_{principal,Scenario} \times \left(1 + p_{principal,FX} \times \left(\frac{FX_{Scenario}}{FX_{Base}} - 1 \right) \right)$$

$$\underline{M_Interest} = \underline{M_Principal} \times \left(1 + p_{floating} \times \left(\frac{1}{Base\ All-in\ Rate} \right) \times (LIBOR_{Scenario} - LIBOR_{Base}) \right)$$

Where

$p_{income,GDP}$ the percentage of demand or prices that depends on GDP, e.g., for an oil refinery, 100% of demand may depend on GDP, whereas for a hospital the percentage may be zero.

$b_{income,GDP}$ the extent to which GDP effects the portion depends on GDP

$p_{income,CPI}$ the percentage of price that depends on CPI, e.g., for an oil refinery, if the commodity price is in nominal terms, there is no additional effect of CPI, however if commodity prices are in real terms, p would be 100%. Similarly, for a company with a fixed price contract, there would be no effect, but for a business selling to consumers, 100% of the income is probably affected.

$b_{income,CPI}$ the extent to which CPI effects the portion that depends on CPI, typically 100%

$p_{income,FX}$ the percentage of price that depends on FX, e.g., for an oil refinery, if the commodity price is in US dollar terms, the percentage will be the proportion of production that is sold on the spot market. For a university that had half its students from overseas, with tuition fixed in FX, half the income would depend on FX.

$b_{income,FX}$ the extent to which FX effects the portion that depends on FX, typically 100%

$p_{income,Price}$ the percentage of price that depends on commodity prices, e.g., for an oil refinery the percentage will be the proportion of production that is sold on the spot market. An electricity generating plant may expect to increase its prices to cover the consequent increase in generation costs, e.g., 50%.

$b_{income,Price}$ the extent to which commodity prices effects that portion which depends on commodity prices, typically 100%.

$p_{cost,GDP}$ the percentage of costs that depends on GDP, e.g., for a transportation project where costs depend little on demand, this could be close to zero. For manufacturing processes where costs partly depend on production levels, this percentage will equal the expected percentage of variable costs.

$b_{cost,GDP}$ the extent to which GDP effects the portion depends on GDP, typically the same as for demand (if the actual driver of cost increase is the increase in production costs, then that portion of the costs will increase at the same rate as demand. For example, if 40% of costs depend on production and a 1% increase in GDP causes a 2% increase in demand, $p_{cost,GDP}$ will be 40% and $b_{cost,GDP}$ will be 2.0.

$p_{cost,CPI}$ the percentage of costs that depends on CPI, e.g., staff costs.

$b_{cost,CPI}$ the extent to which CPI effects the portion that depends on CPI, typically 100%

$p_{cost,FX}$ the percentage of costs that depend on FX, e.g., for overseas components, staff or commodities.

$b_{cost,FX}$ the extent to which FX effects the portion that depends on FX, typically 100%

$p_{cost,Price}$ the percentage of cost that depends on commodity prices, e.g., for a factory with a fixed-cost supply contract for part of its feedstock, p would be the percentage bought on the spot market.

$b_{cost,Price}$ the extent to which commodity prices effects that portion which depends on commodity prices, typically 100%.

$p_{principal,FX}$ the percentage of debt outstanding that is denominated in FX

$b_{principal,FX}$ typically, 100%

$p_{interest,rates}$ this is the same as $p_{floating}$ i.e., the proportion of debt that has a floating interest rate

$b_{interest,rates}$ this is the sensitivity of interest costs to changes in rates, i.e., $\left(\frac{1}{Base\ Total\ Rate}\right)$

To use this approach, for each guaranteed company, values need to be set for the idiosyncratic risks and for each of the percentages (p) and sensitivities (b). Note that the idiosyncratic risk is independent from the macro-economic risk, so a reasonable scenario could be to set the idiosyncratic factors to their base-case (i.e., equal to 1) and then stress the macro-economic factors or vice versa. However, the more conservative approach is to assume that the idiosyncratic problems occur at the same time as the economic problems and stress them all together. As a numerical example, Tables 7a, 7b and 7c give typical values for a transport project and an explanation of how each value was derived.

Table 7a: Example Parameter Values for Idiosyncratic Risks

Cashflow Multiplier	Idiosyncratic Multiplier
M_Income	0.7
M_Cost	1.2
M_Principal	1.6
M_Interest	1.6

Explanation of values set for idiosyncratic risks in the example:

The idiosyncratic multiplier on income is the actual income being 70% of the forecast income
 The idiosyncratic multiplier on costs is the actual staff level being 20% more than forecast
 The idiosyncratic multipliers on principal and interest payments are due to increased debt to pay for construction cost over-runs.

Table 7b: Percentage of Cashflows Affected by each Macro-economic Factor

Cashflow Multiplier	Relative GDP Level	Relative CPI Level	Interest Rate Difference	Relative FX Level	Commodity Price Level
<i>M_Income</i>	1.0	1.0	0	0.2	0.0
<i>M_Cost</i>	0.5	0.6	0.0	0.3	0.2
<i>M_Principal</i>	0.0	0.0	0.0	0.4	0.0
<i>M_Interest</i>	0.0	0.0	0.5	0.4	0.0

Explanation of values set for the percentages of each cashflow that is affected by each macro-economic variable:

M_Income

GDP 100% of revenues for this project are affected by GDP
 CPI 100% of revenues depend on inflation
 Interest rates Interest rates do not directly affect income
 FX 20% of revenues in this project are denominated in FX
 Commodities No income directly depends on commodity prices

M_Cost

GDP 50% of costs for this project are affected by demand
 CPI 60% of costs vary with inflation, 10% are fixed and 30% depend on FX
 Interest rates Interest rates do not directly affect operating costs
 FX 20% of costs in this project depend on commodity prices plus 10% depend on FX
 Commodities 20% of costs in this project depend on commodity prices, e.g., oil.

M_Principal

GDP Required principal payments do not depend on GDP
 CPI Required principal payments do not depend on inflation
 Interest rates Interest rates do not directly affect principal payments
 FX 40% of the loans are denominated in a foreign currency
 Commodities Required principal payments do not depend on prices

M_Interest

GDP	Interest payments do not depend on GDP
CPI	Interest payments do not depend on inflation
Interest rates	50% of the loans have floating rates
FX	40% of the loans are denominated in a foreign currency
Commodities	Interest payments do not depend on prices

Table 7c: Sensitivity of Cashflows to each Macro-economic Factor

Cashflow Multiplier	Relative GDP Level	Relative CPI Level	Interest Rate Difference	Relative FX Level	Commodity Price Level
<i>M_Income</i>	4.0	1.0	-	1.0	-
<i>M_Cost</i>	4.0	-	-	1.0	1.0
<i>M_Principal</i>	-	-	-	1.0	-
<i>M_Interest</i>	-	-	1/6%	1.0	-

Explanation of values set for the sensitivities of each cashflow to each macro-economic variable:

In this table most of the sensitivities are 1.0, i.e., the portion of the cashflow affected by the macro factor changes directly in line with the factor, e.g., the 40% of debt costs that are affected by FX increase exactly in line with FX so the sensitivity (b) equals 1.0.

One exception is the relationship to GDP, for which demand changes 4% for a 1% change in GDP. The other exception is interest rates, where the relative sensitivity to an interest rate change is one divided by the basecase rate (in this case 6%).

Appendix 8. Potential Enhancements to the Excel Template Models

The template models were developed to be easy to use and transparent. Part of the tradeoff was to take out details that added some additional accuracy but made the models more difficult to use. Once an analyst is comfortable with the base models, they may wish to modify them to take into account more details of an individual deal. This appendix describes how to modify the models to take the following into account:

- Reserve accounts
- Reimbursement of Guarantee Payments
- Log-Normal probability distribution of risk factors

Treatment of Reserve Accounts

The models assume that there are no cash reserves at the beginning of each year and the debt must be paid from the net operating income each year. In reality companies and projects will generally start with a cash balance and keep some cash in reserve at the end of each year, rather than draining all excess cash to the owners. If the concern is to only include the initial reserve amount, this can be done by simply adding to the cash to the first year's gross income. If the concern is to capture excess operating cash to be used each year, this can be done by creating a cashflow line-item called "Cash reserve" after the "Net cashflow after debt servicing" with the following formula:

$$\text{Cash reserve} = \min (\text{Net cashflow after debt servicing}, \text{Required balance})$$

Where Required balance is a new input describing the amount to be kept in the account at the end of each year before disbursing excess cashflows to the owners. Add this "Cash reserve" amount to the "Scenario Gross Operating Income to Company" in the following year.

Treatment of Reimbursement of Guarantee Payments

Many guarantee contracts have a clause which requires the guaranteed company to reimburse the government for any debt payments made by the government on behalf of the company. For simplicity, the template models ignore this reimbursement and instead make the conservative assumption that if a company misses a payment in one year it is unlikely to repay in following years. Alternatively, to include reimbursements, add the following:

- A line-item which sums "Debt payments made by Government for Guarantee on Debt" for all years from the beginning until the previous year
- A line-item which calculates the minimum of the "Net Cashflow after Debt Servicing" or the sum of debt payments previously paid.
- Add this amount to the "Debt payments made by Government for Guarantee on Debt"

Log-Normal probability distribution of risk factors

For estimating the probability-weighted averaged loss, Model One makes the simplifying assumption that the probabilities associated with each scenario correspond to a Normal probability distribution, however, in many cases the distribution is more likely to be skewed, e.g., with a Log-normal distribution. For example, unanticipated cost over runs are more likely than cost under-runs, especially for large complex projects. There is little difference between Normal and Log-normal distributions in cases where the

standard deviation is less than the mean and where the stress is moderate. For example, an 80% probability stress will generally be very similar whether the distribution is Normal or Log-normal, whereas a 99% stress will be much larger for the Log-normal distribution.

In this scenario analysis framework, the place where the distribution is primarily used is in defining the six stress levels in Model One when estimating the expected loss. If the distribution is assumed to be Normal, we can create a ladder of stresses e.g., Multiplier = Average + N × Standard deviation, where N = 0, 0.5, 1, etc. We can then simply use the NORMDIST function to get the associated probability.

A similar approach can be used for Log-normal distributions, but we must first reflect the Log-normal parameters to the underlying Normal distribution. This is done with the following equations:

$$mn = LN\{(ml^2)/SQRT\{ml^2+sl^2\}\}$$

$$sn =SQRT\{LN\{1+(sl^2)/(ml^2)\}\}$$

$$xn= LN\{xl\}$$

where

mn is the mean of the underlying Normal distribution

ml is the mean of the observed Log-normal distribution

sn is the standard deviation of the underlying Normal distribution

sl is the standard deviation of the observed Log-normal distribution

xn is the point at which we wish to evaluate the probability on the underlying Normal distribution

xl is the point at which we wish to evaluate the probability on the observed Log-normal distribution

We can then use the function NORMDIST(xn,mn,sn,1) to give the probability of an event being less than xl for a Log-normal distribution with mean ml and standard deviation sl. These results for the probability can then be used at the top of the “Collected” tab of Model One. As an illustration of the potential difference in the probabilities, table 8a shows the probabilities for three log-normal distributions with increasing standard deviations and compares the probabilities with a Normal distribution.

Table 8a: Comparison of Normal and Log-normal Probabilities.

Mean of Log	1.100	1.100	1.100	Normal
St Dev of Log	0.100	0.250	0.500	
Mean of x	0.091	0.070	0.001	0
Stdev of x	0.091	0.224	0.433	1
pr>m	52%	54%	59%	50%
pr>m+0.5 s	70%	72%	75%	69%
pr>m+1.0 s	84%	85%	86%	84%
pr>m+1.5 s	93%	92%	92%	93%
pr>m+2.0 s	97%	96%	96%	98%
pr>m+2.5 s	99%	98%	98%	99%

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