Primary Commodity Prices
Co-movements, Common Factors and Fundamentals

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Abstract

The behavior of commodities is critical for developing and developed countries alike. This paper contributes to the empirical evidence on the co-movement and determinants of commodity prices. Using nonstationary panel methods, the authors document a statistically significant degree of co-movement due to a common factor. Within a Factor Augmented VAR approach, real interest rate and uncertainty, as postulated by a simple asset pricing model, are both found to be negatively related to this common factor. This evidence is robust to the inclusion of demand and supply shocks, which both positively impact on co-movement of commodity prices.
Primary Commodity Prices: Co-movements, Common Factors and Fundamentals

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1. Introduction

Movements in commodity prices matter for countries’ external and internal balances as well as their respective fiscal and monetary policies. It is therefore not surprising that the nature of such movements, and their determinants, have attracted so much attention in both academic and policy circles. Earlier research focused on the historical trends of primary commodity prices relative to the price of manufactured goods in the examination of the Prebisch (1950) and Singer (1950) hypothesis, as recently revisited by Harvey et al. (2010). Attention has also focused on commodity prices’ time series properties (for example, Cuddington, 1992, Deaton, 1999, and Cashin, et al., 2000). Both aspects carry important welfare implications: while a declining trend in commodity prices supports the hypothesis of the so-called ‘resource curse’ for commodity-abundant developing countries, the degree of volatility and persistence of commodities prices affects the design and effectiveness of stabilization policies.

Another relevant feature of commodity prices is their tendency to co-move. Understanding such co-movement is just as important, because of the welfare implications for both commodity importers and exporters. Indeed, a synchronized increase in commodity prices is likely to place commodity import dependent countries under considerable inflation pressure (see Borensztein and Reinhart, 1994). If co-movements are due to substitution effects, they further foster export concentration in commodity producing countries. In both cases, the ability to diversify shocks to the current account, to manage domestic imbalances and to resist inflation pressures will be constrained.

The contemporaneous and dramatic upsurge in commodity prices in the 2000s has prompted a new search for the fundamentals that make commodity prices co-
move. Among the alternatives, Frankel (2008) and Calvo (2008) have discussed the role of the real interest rate; Wolf (2008) and Svensson (2008) have mentioned the importance of shifts in global supply and demand. Further, Krugman (2008) has argued that the increase in oil prices, providing an incentive to produce biofuels, is responsible for the increase in food prices. Little effort, however, has so far been devoted to disentangling these hypotheses from an empirical standpoint.

In this paper, we attempt to progress the empirical evidence on primary commodity prices along different dimensions. First, we examine the extent and nature of price co-movements between primary commodities.\(^1\) In order to perform our empirical analysis we exploit the information embedded in annual historical prices. Specifically, we analyze 24 commodity price series observed for more than 100 years of data from 1900 to 2008. Such a low frequency should reduce the noise to signal ratio and largely eliminate the influence of speculation on commodity prices, allowing us concentrate on “fundamental” price co-movements. We first diagnose the overall co-movement in the panel, using the test statistic suggested by Ng (2006) and then apply the Bai and Ng (2004) Panel Analysis of Nonstationary and Idiosyncratic Components (PANIC) to identify potential common factors in commodity prices. These methods are attractive since they include statistical tests of co-movement, taking account of the time series properties of the data. Our findings highlight a sizeable degree of correlation in the data and detect the existence of a common factor.

We next investigated the relationship between commodity prices and macroeconomic determinants. Using a Factor Augmented Vector Auto Regression (FAVAR) approach,\(^2\) we relate the identified common factor in commodity prices to

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\(^1\) This mirrors much recent empirical work on co-movement of economic variables, for example Kose et al. (2003), Monacelli and Sala (2009) and Ciccarelli and Mojon (2010).

\(^2\) Such an approach has been applied by Bernanke et al. (2005) in the examination of US monetary policy.
their macroeconomic fundamentals. Here, we draw on a stylized theoretical model that postulates the role of the real interest rate, as suggested by Frankel (2008) and Calvo (2008), and uncertainty, as indicated by Beck (1993, 2001). Furthermore, we assess whether our results are robust to alternative measures of risk or other factors, such as demand and supply shocks, as suggested by Svensson (2008), Wolf (2008) and Krugman (2008).

This paper proceeds as follows. Section 2 reviews the relevant empirical literature on commodity prices. Section 3 posits our stylized model of fundamental determinants of commonalities. Section 4 presents the data and the empirical evidence on the co-movements in commodity prices. Section 5 relates the common factor in commodity prices to its determinants. Section 6 concludes.

2. Related Empirical Literature on Commodity Prices

Movements in commodity prices are important for the welfare of both developing and developed countries (see, among the others, Lu and Neftci, 2008, Frankel, 2008, and Daude et al., 2010). This importance has spawned a considerable academic literature with a primary focus on their time series properties. Seminal empirical work in this area can probably be dated back to Prebisch (1950) and Singer (1950) and their controversial thesis (PST) of a declining long-term trend in the terms of trade of commodity exporters, that provided justification for import substitution policies as an appropriate tool for development. An extensive literature ensued that

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3 On the different impact of commodity prices for developing and developed countries, Frankel (2008) notes how, on the one hand, the low levels of commodity prices in the late 1980s and in the 1990s may have played a role in some of the financial crises in commodity exporters emerging markets, deteriorating their current accounts. On the other hand, he also notes, they have acted like a positive supply shock for industrial countries, such as the US, lowering input prices and inflation and allowing high growth and employment.
focused on the historical relationship between the price indices of primary commodities and manufactured goods.\(^4\)

Furthermore, Deaton (1999) has stressed the importance of looking at the time series properties of individual commodities and their co-movement, rather than price indices, in order to assess the different impact of commodity prices on developing and industrial countries, and assess the need for stabilization policies.\(^5\) A strand of literature has subsequently investigated these properties. With respect to their degree of persistence, Cashin, Liang and McDermott (2000), for example, calculate median unbiased half lives of 60 commodity prices observed monthly between 1957 and 1998. However, they find that shocks are typically long lasting, and conclude that stabilization schemes may be more costly than beneficial. Cashin, et al. (2000) report “typical” commodity prices half lives in the range of 5 years.\(^6\) With respect the issue of co-movement, Cashin, McDermott and Scott (2002) find evidence of synchronization in the prices of related commodities.\(^7\)

As mentioned above, the surge in commodity prices in the 2000s has renewed the interest for the co-movement of commodity prices and their determinants. Mollick et al. (2008), for example, investigate the role of globalization in the terms of trade of relative prices and test whether US relative prices are affected by international prices. They do find a decreasing trend in relative prices, but argue that this trend is not

\(^5\) In particular, Deaton (1999) underlines how industrial countries, who on average are net importers of a large range of commodities, perform very differently from less developed countries, who often export only a limited range of primary goods. Further, he argues that while world demand (imports) may determine common shocks to a wide range of prices, the impact of shocks to the world supply may differ from good to good, causing relative prices to differ.
\(^6\) For other studies of commodity prices see, inter alia, Bleaney and Greenaway (2001), MacDonald and Ricci (2001), Chen and Rogoff (2002) and Chen, Rogoff and Rossi (2008).
\(^7\) Prominent work on the co-movement of commodity prices from Pindyck and Rotemberg (1990) suggests substantial price co-movement beyond macroeconomic fundamentals and argues looking at monthly data that this is due to commodity speculation. In this paper, we look at long spans with a lower frequency in the attempt to limit the extent of noise or speculation in the data.
related to globalization or international integration. On this evidence, they conclude that policies aiming at increasing or decreasing the degree of integration with the world economy would not be effective at modifying this long term trend. Cuddington and Jerrett (2008) and Jerrett and Cuddington (2008) search for the presence of super-cycles (20-70 years cycles) in a set of metal goods prices and use correlation and principal components analysis to investigate their degree of concordance.

A parallel and lively debate, also spurred by the recent price boom, has revolved around the determinants of commodity prices. In this respect, Frankel (2008) purports the role played by the real interest rate on bonds as follows: a rise in the real interest rates provides an incentive to intensify mining in an effort to invest the proceeds. As the supply of natural resources is increased in consequence, their price should drop. At the same time, higher rates of return on bonds will reduce speculative demand for commodities and hence further cut their price. Furthermore, a rise in interest rates reduces inventory demand and commodity prices. Similarly, Calvo (2008) argues that the increase in commodity prices mostly stems from the combination of low central bank interest rates, the growth of sovereign wealth funds and the consequent lower demand for liquid assets. However, he argues that this relationship is only temporary as prices will adjust in the long run.

While presenting the case for interest rates, Frankel (2008) and Svensson (2008) also underline the role of risk in explaining primary commodity movements. Without considering the role of interest rates, the relevance of risk was previously considered also by Beck (1993, 2001), who discussed ARCH effects and possibly GARCH in Mean effects in commodity prices, finding mixed evidence. The importance of uncertainty for economic outcomes, and particularly for investment, has also been suggested by Dixit and Pyndick (1994).
The recent interest in commodity prices movements has led to additional suggestions as to their determinants. Svensson (2008) argues that global demand and supply shocks may be important for commodity prices. The importance of global demand as a determinant of commodity prices has also been highlighted by Wolf (2008). He emphasizes the increasing demand from emerging market economies such as China and India, as they become more prominent in world trade of commodities. Finally, according to Krugman (2008) as inventory holdings have not surged in recent years, speculation is a less convincing rationale for common or idiosyncratic movements in commodity prices. Instead, Krugman believes that a resource shortage is the main determinant of increases in the prices of primary commodities. Consistent with this view, the increase in oil prices may explain the contemporaneous increase in other commodities, such as foodstuff, via both cost effects on the energy intensive agriculture sector and substitution effects due to the increasing production of biofuels.

A recent empirical literature has investigated the determinants of fluctuations in commodity prices. This literature only partly relates to the above explanations. Akram (2009), for example, uses structural VAR models of quarterly data from 1990 to 2007 to separately look at the impact of real interest rates changes on the real price of crude oil, industrial raw materials, food and metals. Vansteenikste (2009) extracts a dynamic principal component from a large set of monthly commodity prices for the recent period and tests its potential determinants using IV regressions. She finds that co-movement was highest in the 1970s and 1980s, declined in the 1990s and has recovered somewhat during the early years of this century. Also, she finds evidence that supply, global demand, exchange rate and real interest rate are important. Lombardi, Osbat and Schnatz (2010) also look at the quarterly commodity prices from the middle of the 1970s to 2008 and apply a FAVAR to assess the role of real
interest rates, crude oil price, US dollar real effective exchange rates, and world industrial production separately for each of the non-oil 15 commodities in the sample. They find evidence in favor of industrial production and exchange rates, but no robust evidence of oil or interest rate effects.

Our study differs from past empirical work in many aspects. First, we take a long run historical perspective and look at commonalities in commodity prices over more than a century of yearly data. In this respect, our work tries to assess the contemporary explanations for commodity price movements against the historical evidence. The use of low frequency and long span data increases further the noise to signal ratio in terms of “fundamental” co-movement and reduces the extent of “excess” co-movement due to speculation, as suggested by Pyndick and Rotemberg (1990). Second, accounting for the time series properties of the data using non-stationary panel methods, we utilize Uniform Spacings and the PANIC approach to investigate and summarize such co-movements and integrate these methods in a FAVAR analysis. Third, we integrate Frankel (2008) with Beck (1993, 2001) and look at both the role of interest rates and risk as determinants of commonalities in commodity prices. Further, we control for the role of global demand and supply shocks along the lines discussed by Svensson (2008), Wolf (2008) and Krugman (2008).

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8 Somehow paradoxically, following Pyndick and Rotemberg (1990), an extensive literature has concentrated on identifying “excess co-movement” of commodity prices, or co-movement “beyond” what is granted by fundamentals, rather than the determinants of fundamental co-movement.
3. A Stylized Model of Commodity Prices

Frankel (2008), Svensson (2008) and Calvo (2008) emphasize the theoretical link between real interest rates and real commodity prices. Svensson (2008) suggests that if commodities prices are determined just like any other asset, they should amount to discounted present values of future returns. A rise in real rates will raise the discount factor, the present value of future returns will fall and so will today’s commodity prices. Intuitively, and consistent with the sign of this relationship, a rise in real interest rates on bonds will lead to less speculation in commodities as economic agents look to invest in fixed income assets. We firstly set out the importance of real rates of return and then proceed to consider other potentially important determinants.⁹

We can set out a simple asset pricing model in which real commodity prices are dependent upon real interest rates, along the lines of Frankel (1986, 2008). We abstract from expected storage costs by assuming they are constant.¹⁰ Due to arbitrage, the expected return on commodity prices in the next period will be equal to the real rate of return. Where the current period expect price of commodity prices in the next period is denoted by \( E_t CP_{t+1} \) and the real rate of return is \( r_t \) for a risk free asset, for example a US Treasury Bill. Algebraically, the risk neutral rational valuation formula is as follows:

\[
\left[ E_t CP_{t+1} - CP_t \right] / CP_t = r_t, \tag{1}
\]

which can be re-arranged to give:

\[
CP_t = \left[ E_t CP_{t+1} \right] / (1 + r_t). \tag{2}
\]

⁹ Frankel (2008) proposes and identifies empirically a role for real interest rates on commodity prices operating through inventories, although only for oil. As far as we are aware, inventory data was not available for the historical time period we consider in this paper.

¹⁰ This choice is also due to the fact that this data for such a long span is not available.
Equation (2) suggests a negative relationship between bond returns and commodity prices. To solve for expectations we first take logs, where $\ln(CP_t) = cp_t$ and assuming that approximately $R_t = \ln(1+r_t) \approx r_t$, we have:

$$cp_t = E_t cp_{t+1} + \gamma R_t$$  \hspace{1cm} (3)

Equation (3) suggests that commodity prices are based upon their future values and the real interest rate, where $\gamma < 0$ is a constant parameter. The negative coefficient on $R_t$ is the main rationale suggested by Frankel (2008) and Svensson (2008) why permissive monetary policy in the 2000s lead to high commodity prices. Whilst equation (3) has intuitive appeal, it requires further specification to make it empirically operational. Therefore, we forward iterate the log of real commodity prices, which, ruling out an explosive solution, is based upon current and future values of the real interest rate

$$cp_t = \gamma E_t \sum_{k=0}^{\infty} \delta^k R_{t+k} ,$$  \hspace{1cm} (4)

where real interest rates are an AR(1) process, $R_t = \delta R_{t+1}$, and note that $E_t R_{t+1} = \delta^k R_t$

this gives us the following equation, since $\sum_{k=0}^{\infty} \delta^k = (1-\delta)^{-1}$:

$$cp_t = \gamma \sum_{k=0}^{\infty} \delta^k R_t = \gamma \frac{1}{(1-\delta)} R_t = c R_t = f(R_t) ,$$  \hspace{1cm} (5)

Where $f(.)$ is the relationship between the log of commodity prices and real interest rates.

However, Frankel (2008) and Svensson (2008) acknowledge that interest rates are not the only potential determinant of commodity prices. Svensson (2008) further suggests that interest rates and commodity prices may be jointly determined by other factors. Some papers have emphasized a role for risk in the determination of commodity prices. Beck (1993), based upon Muth’s (1961) model of commodity prices with rational expectations, suggests that commodity prices may exhibit conditional
heteroscedasticity. Such heteroscedasticity is explained by the storable nature of many commodities: price variance in one period is transmitted to inventory variance, which in turn causes price variance in the next period. When investors are risk averse, volatile commodity prices (i.e. high risk) will reduce inventory and, therefore, their prices. Dixit and Pindyck (1994) would also suggest a role for uncertainty as a determinant of price, but with an opposite sign. Investment in the production of primary commodities may be irreversible. Consequently price uncertainty would increase the option value of waiting, the opportunity costs of deciding to invest in production and the price of commodities.

The simple model presented above can be reformulated to accommodate risk averse investors by including a time varying risk premium, $\rho_t$, in equation (1). The return to commodities is then as follows:

$$\left[E_t(CP_{t+1} - CP_t)\right]/CP_t = r_t + \rho_t$$

(6)

This subsequently yields a relationship between commodity prices, real interest rates and the risk premium. We do not assume that the estimated coefficients on risk, $\rho_t$, and risk free return, $R_t$, will be equivalent.

$$c_{Pt} = f(R_t, \rho_t)$$

(7)

How can this risk be modeled? Engle, Lilien and Robins (1987) model asset prices using an ARCH in mean approach. This has also been adopted for individual commodities by Beck (2001), although with mixed success. Here, we extend Beck’s approach by simultaneously considering the role of risk and interest rates as potential determinants of commonalities in commodity prices. Specifically, we utilize stock market risk, based on the standard deviation of monthly stock prices. 11

11 We go beyond the work of Beck (2001) by presenting evidence on whether commodity prices can be represented by a GARCH in mean process. We were unable to examine whether this approach was robust to demand and supply shocks hence we present our GARCH results as a robustness exercise. Another alternative, such as the VIX index, was unfortunately unavailable for such a long span of data.
We consequently examine the role of real commodity prices, real interest rate and risk in a VAR system. This is similar to the approach adopted by Bernanke et al. (2005) in the sense that they utilize the information from a factor of economic activity variables and incorporate this into a VAR when examining the impact of monetary policy on output and prices. We are primarily interested in the relationship between commonalities in commodity prices, real interest rates and risk. Following the notation of Bernanke et al. (2005), within the FAVAR the transition equation is as follows:

\[
\begin{bmatrix}
F_t \\
X_t
\end{bmatrix} = \Phi(L) \begin{bmatrix}
F_t \\
X_t
\end{bmatrix} + v_t
\]

Equation (8) consists of the common factor \(F_t\), the matrix \(X_t\) which contains the real interest rate and risk term and a lag polynomial (i.e. \(\Phi(L)\)) with lag order four. As an identification scheme for the residuals \(v_t\) is required, we use Koop et al. (1996) Generalized Impulses responses to examine the impact of risk and real interest rates on real commodity prices; our results are thus not sensitivity to the ordering of the VAR, as would be the case with an identification scheme based on the Cholesky decomposition. Before we implement the FAVAR, we investigate commonalities in prices.

### 4. Commonalities in Commodity Prices

As discussed above, we aim at investigating the co-movements in commodity prices and how these co-movements relate to determinants of commodity prices discussed in the literature. In order to pursue this objective, we first employ the

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12 Frankel (2008) examines the impact of risk and demand on inventory levels of oil. We examine the impact on commodity prices. All variables were stationary, see Table A2 in the Appendix.
13 Bernanke et al. (2005) identify a negative relationship between nominal interest rates and NAPM commodity prices. See also Sims (1992) for an examination of monetary policy with a role for commodity prices to ameliorate price puzzles.
Uniform Spacings approach of Ng (2006) and statistically assess the degree and pervasiveness of co-movement between commodity prices. Further, we use the factor model from Bai and Ng (2004) to examine the time series properties of the common and idiosyncratic elements and to investigate whether co-movement can be summarized by common factors. Finally, we consider potential determinants of such common elements, in real terms, using a Factor Augments VAR following Bernanke et al. (2005) and GARCH in Mean following Beck (2001).

4.1 Data

In order to reduce the noise to signal ratio in the data, we have run this investigation on a long span of yearly data. The quality of the historical data used in empirical work is one of the main disputes in the literature on commodity prices. This dispute was settled by Grilli and Yang (GY, 1988), who collected and summarized in a single trade weighted index 24 prices of internationally traded non-fuel commodities for the period 1900 to 1986. Since, their dataset (or variants) has become the most widely used for empirical research on historical commodity prices. The original data from GY has been recently revised and updated to 2003 by Pfaffenzeller et al. (2007).

In order to investigate the behavior of historical commodity prices up to the most recent period, including the recent swings in prices, we have updated this data to 2008. This gives us a panel of time dimension \( T=109 \) and cross sectional dimension \( N=24 \). Please refer to the Data Appendix for a description of how the data has been updated and for some descriptive statistics.

14 Grilli-Yang’s commodity price index \( (GY_t) \) is a simple arithmetic mean to individual commodity prices \( (CP_{it}) \), with weights \( (a_i) \) based upon trade shares \( GY_t = \sum a_i CP_{it} \). Cuddington and Wei (1992) show how adopting a geometric average (i.e. \( GY_t = \Pi CP_{it}^{a_i} \)) may affect a resultant commodity price index.
Table 1. Commonalities of Commodity Prices

<table>
<thead>
<tr>
<th>Panel A: Uniform Spacings Analysis</th>
<th>( \hat{\theta} )</th>
<th>Number of small correlation pairings</th>
<th>Small svr</th>
<th>Large svr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.308</td>
<td>85</td>
<td>0.145</td>
<td>4.504*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: PANIC Analysis</th>
<th>Factor</th>
<th>Idiosyncratic</th>
<th>Information Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IC1</td>
<td>IC2</td>
<td>IC3</td>
</tr>
<tr>
<td>-0.003, 0.835, -1.660, -1.693, -2.698 [0.335, 0.162, 0.118, 0.107, 0.059]</td>
<td>1.310</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Notes: Panel A presents evidence on the degree of cross sectional correlation. \( \hat{\theta} \) is the proportion of correlations that are small. Ng (2006) Spacings Variance Ratio test statistic (svr) provides evidence of whether correlation is significantly different from zero, distributed as standard normal, therefore the critical value is 1.65 (significant at 5% with asterisk, *). First order serial correlation is removed following Ng (2006), using an AR(1) approach. There are \( n = N(N-1)/2 = 276 \) correlations, for \( N = 24 \). Panel B examines the stationarity properties of our panel time series by examining the unit root of factor nonstationarity and idiosyncratic nonstationarity. We use annual data on commodity prices from 1900 to 2008 (\( N = 24, T = 109 \)). We identify the factor structure using an information criterion from Bai and Ng (2002). For the factor unit root test, we reject the null hypothesis of a unit root for large negative (less than \(-2.89\)) and for the idiosyncratic component we reject the null hypothesis of a unit root for large positive values of the test statistic.

4.2 Uniform Spacings Analysis

To assess the pervasiveness of co-movement in commodity prices, we first apply the Ng’s (2006) Spacings Variance Ratio (SVR) test. This test partitions a series of linear relationships in a panel data set into subsamples of ‘small’ and ‘large’ correlations. The proportion of small correlations is represented by \( \hat{\theta} \). If the SVR test statistic is significantly different from zero, pairings in a particular subsample are significantly correlated. If the small group correlations are significant, so must the subsample of pairings with large correlations. If only the subsample of large correlations is significant, then we have only a degree of correlation in the data.

We find evidence of a significant degree of correlation in our panel (see Panel A, Table 1). The SVR for the large group of correlations, which represent about 70% of the data (i.e. \( \hat{\theta} = 0.31 \)), has a test statistic of 4.504, when the 5% critical value is 1.65, and hence allows us to reject the null hypothesis of no significant correlation in our 24 commodity price indexes. For the remaining 30% of correlations in the small
group we cannot reject the null hypothesis of no linear association. Similarly, a q-q plot of correlations in Figure 1 visually shows a significant amount of correlation pervasiveness in commodity prices, with some degree of heterogeneity. The substantial fraction of significant correlations suggests that common factors may lie at the root of this co-movement. This is more formally explored in the next section.

Figure 1. Uniform Spacings Correlations

Notes: This figure shows the q-q plot of the cross-sectional correlations of the 24 commodity prices. Deviations from the 45% line denote evidence of correlation in the data. If all correlations are nonzero the intercept is shifted upwards above 0.5. Homogeneity in correlations means the q-q plot should be flat over a subset. More frequent and greater correlation means a move above the 45% line.

4.3 Commonalities in the Level of Commodity Prices

As the uniform spacings correlations point to a sizeable (albeit incomplete) correlation structure, we next test for the evidence of common factors using the Bai and Ng (2004) PANIC approach. Therefore, we assume that our commodity price series \((CP_i)\) consist of both a common factor \((F_t)\) and an idiosyncratic component \((u_{it})\):

\[
CP_{it} = c_i + \lambda_i F_t + u_{it},
\]  

(9)

---

15 Principal components analysis has also, recently, been adopted by Jerrett and Cuddington (2008) and Vansteenkiste (2009) to investigate commodity prices. In a different setting, Clark (2006) and Monacelli and Sala (2009) identify a common component in prices using disaggregate consumer price data. Factors models have also been utilised recently by Bernanke et al. (2005), Stock and Watson (2008), Mojon and Ciccarelli (2009), Kose et al. (2008) and Byrne et al. (2009).
with factor loadings \((\lambda_i)\) and cross sectional fixed effects \((c_i)\) for \(i=1,...,24\).

To identify the appropriate number of common factors, we use the information criteria from Bai and Ng (2002). Results presented in Panel B of Table 1 show that information criteria IC1 and IC2 suggest that there are at least 5 common factors in the data and the third information criterion, IC3, suggests that there is only one. Since Bai and Ng (2002) regard this criterion as more reliable in the presence of cross sectional correlation, we conclude in favor of the latter.

**Figure 2. First Principal Component of Commodity Prices**

![Graph showing the first principal component of commodity prices](image)

*Notes*: This figure presents the principal component \((pc)\) of historical commodity prices and compares it to the Grilli-Yang \((gy)\) Index.

Figure 2 plots the principal component from PANIC against the Grilli–Yang Index. While both series broadly describe a similar pattern, there appears to be one difference: the principal component tends to exceed the GY index when prices and volatility are high, such as during 1975 and 1980, and in the 2000s. The opposite seems to be the case in periods where prices and volatility are lower. The GY index, hence, under/overestimates the overall impact of commodity price increases in volatile/tranquil times. This result, somehow, confirms the importance to look at individual commodities and their co-movement, as suggested by Deaton (1999).

Three statistics are presented to assess the importance of the factor for each commodity price series (see Table 2). The first statistic, equal to the ratio of the
standard deviations of idiosyncratic component (i.e. $\sigma_{\text{uit}}$) and of the differenced commodity price data (i.e. $\sigma_{\Delta \text{CPit}}$), would equal one if all variation were idiosyncratic (i.e. $R^2 = \sigma_{\text{uit}} / \sigma_{\Delta \text{CPit}}$). The second statistic, equal to the ratio between the standard deviations of the common factor (i.e. $\sigma_{Ft}$) and that of the idiosyncratic component (i.e. $\sigma_{\text{uit}}$), would be zero if all variation were idiosyncratic (i.e. $\sigma_{Ft} / \sigma_{\text{uit}} = 0$). Finally, we present the factor loadings ($\lambda_i$) from equation (1); they should be larger for series that are more related to the common factor.

Table 2. Cross Sectional Heterogeneity in Commodity Prices

<table>
<thead>
<tr>
<th>Commodity</th>
<th>$\sigma_{\text{uit}} / \sigma_{\Delta \text{CPit}} = R^2$</th>
<th>$\sigma_{Ft} / \sigma_{\text{uit}}$</th>
<th>$\lambda_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.915</td>
<td>0.307</td>
<td>-0.301</td>
</tr>
<tr>
<td>Bananas</td>
<td>0.919</td>
<td>0.198</td>
<td>-0.248</td>
</tr>
<tr>
<td>Beef</td>
<td>0.910</td>
<td>0.191</td>
<td>-0.235</td>
</tr>
<tr>
<td>Cocoa</td>
<td>0.975</td>
<td>0.166</td>
<td>-0.215</td>
</tr>
<tr>
<td>Coffee</td>
<td>1.000</td>
<td>0.023</td>
<td>-0.057</td>
</tr>
<tr>
<td>Copper</td>
<td>0.728</td>
<td>0.581</td>
<td>-0.863</td>
</tr>
<tr>
<td>Cotton</td>
<td>0.879</td>
<td>0.421</td>
<td>-0.342</td>
</tr>
<tr>
<td>Hides</td>
<td>0.996</td>
<td>0.036</td>
<td>0.043</td>
</tr>
<tr>
<td>Jute</td>
<td>0.975</td>
<td>0.223</td>
<td>-0.281</td>
</tr>
<tr>
<td>Lamb</td>
<td>0.970</td>
<td>0.083</td>
<td>-0.075</td>
</tr>
<tr>
<td>Lead</td>
<td>0.944</td>
<td>0.354</td>
<td>-0.597</td>
</tr>
<tr>
<td>Maize</td>
<td>0.798</td>
<td>0.774</td>
<td>-0.527</td>
</tr>
<tr>
<td>Palm</td>
<td>0.923</td>
<td>0.517</td>
<td>-0.445</td>
</tr>
<tr>
<td>Rice</td>
<td>0.688</td>
<td>1.610</td>
<td>-1.128</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.819</td>
<td>0.608</td>
<td>-0.653</td>
</tr>
<tr>
<td>Silver</td>
<td>0.403</td>
<td>2.545</td>
<td>-2.031</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.149</td>
<td>1.770</td>
<td>-3.759</td>
</tr>
<tr>
<td>Tea</td>
<td>0.976</td>
<td>0.153</td>
<td>-0.163</td>
</tr>
<tr>
<td>Timber</td>
<td>0.597</td>
<td>0.405</td>
<td>-0.396</td>
</tr>
<tr>
<td>Tin</td>
<td>0.685</td>
<td>0.640</td>
<td>-1.041</td>
</tr>
<tr>
<td>Tobacco</td>
<td>0.999</td>
<td>0.022</td>
<td>-0.028</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.773</td>
<td>0.747</td>
<td>-0.508</td>
</tr>
<tr>
<td>Wool</td>
<td>0.973</td>
<td>0.283</td>
<td>-0.154</td>
</tr>
<tr>
<td>Zinc</td>
<td>0.929</td>
<td>0.402</td>
<td>-0.674</td>
</tr>
</tbody>
</table>

Notes: [1] ratio of standard deviation of idiosyncratic component to standard deviation of differenced data, if all variation idiosyncratic then tends to one (i.e. $R^2 = \sigma_{\text{uit}} / \sigma_{\Delta \text{CPit}}$); [2] standard deviation of common to idiosyncratic component, if all variation idiosyncratic then this tends to zero (i.e. $\sigma_{Ft} / \sigma_{\text{uit}} = 0$); and [3] The factor loadings ($\lambda_i$) from equation 1. We highlight in bold cross sectional units where the ratio of the common to the idiosyncratic variability is greater than 0.5, indicating that the majority of variation is explained by the factor.
Table 2 shows that the common factor explains more than half of the variation in the prices of Copper, Silver, Wheat and Sugar, and a substantial degree of the variation in the prices of Maize, Palm Oil, Rubber, Silver and Tin. The prices of Tobacco, Hides and Lamb, however, seem to be less tied to the common factor and, hence, show more idiosyncratic behavior. While this finding points to some degree of heterogeneity in the co-movement of commodity prices, as also suggested by the SVR statistic, nevertheless a large degree of the historical variability of the majority of commodity prices seems to be determined by a non-stationary common factor.

We next investigate to what extent this common factor can be related to macroeconomic fundamentals.

5. The Common Factor in Commodity Prices and Its Determinants

In this section, we examine the relationship between the principal component of the 24 commodity prices and the macroeconomic determinants discussed in the literature. Since Frenkel (2008) and Svensson (2008) refer to real prices, we deflate the identified principal component using the US CPI. This transformation also ensures that our non-stationary principal component becomes stationary in real terms, as it can be seen from Table A2. While we focus on the role of the real interest rate and risk as determinants of commodity prices, we also investigate the role of global demand, as proxied by the growth rate of US real GDP, and of real crude oil prices, interpreted here as a measure of supply shocks.

Bivariate Scatter Plots

Figure 3 provides bivariate scatter plots between these four variables and the real principal component in commodity prices. While a negative relationship seems to
emerge with risk and return, the real commodity prices component seems to be positively correlated with US economic growth and the real price of oil.

**Figure 3. Real Commodity Prices and Macroeconomic Determinants**

*Impulse Responses*

Figure 4 presents Impulse Responses from our FAVAR. Impulse responses of commodity prices to real interest rate shock are displayed in the left hand side charts, while the responses to a one standard deviation innovation in risk are presented on the right. We find that a real interest rate shock has a significant negative impact in the first five years, while an increase in risk leads to an immediate and significant decrease in the level of commodity prices during the first four years. Our findings are
consistent with the predictions of our simple theoretical model. They hence not only confirm the view of Frankel (2008) and Calvo (2008) that interest rate have a (temporary) adverse impact on commodity prices, but also are consistent with the ideas of Beck (1993) and Dixit and Pindyck (1994), that risk and uncertainty are associated with movements in commodity prices.

**Figure 4. Determinants of Commonalities in Primary Commodities in FAVAR**

![Graph showing response of PC to R and RISK](image)

*Notes:* This figure presents the response of real primary commodities (PC) to a generalized one standard deviation innovation in real interest rates (R) and risk (RISK). Identification of the residuals of the VAR is by Koop et al. (1996). Also included are ± two asymptotic standard error bands.

**Robustness Checks**

We consider several robustness exercises related to the measurement of risk; generated regressors problems; robustness to demand and supply shocks; and the nature of oil prices shocks. We first examine the sensitivity of our results to an alternative measure of risk, derived as the conditional variance of commodity prices based on an IGARCH in mean specification. The GARCH in mean approach is not only robust to the generated regressors problem (see Pagan and Ullah, 1996), but it also captures the risk term. Impulse responses of our alternative risk measure retain the finding that uncertainty has a negative impact on commodity prices. Model
specifications tests and impulse responses to a VAR based on this alternative measure of risk are presented in Table A3 and Figure A3 in the Appendix.

Our results also remain robust to the inclusion of additional explanatory variables suggested by Svensson (2008), Wolf (2008) and Krugman (2008). Figure 5 indicates that real interest rates and risk are both related to the price of commodities in such an extended FAVAR system, with the same sign, approximate magnitude and statistical significance as before. While, as expected, global demand shocks and oil price shocks are associated with increases in relative commodity prices, their incorporation in our extend VAR system does not weaken the established link with real interest rates and risk. Finally, as real oil prices are found to be nonstationary (see Table A2), we both consider model specifications with real interest rates in levels and first differences. Impulse responses of both model specifications are found to be qualitatively similar (see Figure A4). Although return and risk both appear to have a greater effect in magnitude than supply and demand shocks, especially in the initial period. Therefore, while it has been suggested by Svensson (2008), Wolf (2008) and Krugman (2008) that demand and supply factors are important for commonalities for commodity prices, we are of the view that they remain driven in the short-term by responses to real interest rate and risk.
6. Conclusions

The recent surge in commodity prices has brought new momentum to the spirited debate in academic and policy circles on large swings in commodity prices and their determinants. In this paper, we draw attention to the co-movement of commodity prices. We address the issue of co-movement using the correlation methodology of Ng (2006) and the nonstationary Panel factor model from Bai and Ng (2004). We find evidence of co-movement of commodity prices within a methodology that acknowledges that the time series may be nonstationary.

The above analysis returns significant evidence of co-movement in commodity prices and, importantly, it identifies a common factor. Building on a
simple asset pricing model of commodity prices, we empirically relate this common factor in real terms to the real interest rate, as also recently suggested by Frankel (2008) and Svensson (2008), and to risk, as previously suggested by Beck (1993, 2001). Within a Factor Augmented VAR framework, we support a negative relationship between real interest rate and real commodity prices, with shocks to the real interest rates being absorbed within a five years period. Risk, captured by a measure of stock market uncertainty, is also negatively related to commodity prices, as set out in a simple asset pricing model, but with a shorter term impact. Our results are robust to the inclusion of shocks to proxies for global demand and supply, which appear positively related to the common factor in commodity prices. Hence, we cannot discount the views of Svensson (2008), Wolf (2008) and Krugman (2008), although the initial period impact of global demand and supply factors are smaller than those of the real interest rate and risk.

Importantly, our results confirm the relevance of the real interest rate for commodity prices and are consistent with the view that looser monetary policy may lead to higher commodity prices. Hence, to the extent that prices are important for stabilization policies, monetary policy should be cognizant of its impact on commonalities in commodity prices.
References


A1. Data Appendix

Commodity Prices

The dataset of Pfaffenzeller, Newbold and Rayner (PNR, 2007) for the period 1990-2003 has been upgraded to 2008 using a splicing methodology and Indices of Market Prices and Unit Values from the IMF commodity price September 2009 tables. In particular, series have been chosen to match PNR data. When more than the exact denomination was not present, the series with the highest correlation with the PNR series have been chosen. These are in Index Number Units.

Aluminum: PNR and IMF series for Aluminum [Code 15676DRDZF ].
Bananas: PNR and IMF series for Bananas Lat/Amer.US.P. [24876U.DZF]
Beef: PNR and IMF series for Beef All Orig.US Ports [19376KBDZF]
Cocoa: PNR and IMF series for Cocoa Beans [22374R.DZF]
Coffee: PNR and IMF series for Coffee Other Milds (New York) [38676EBDZF]
Copper: PNR and IMF series for Copper UK (London) [11276C.DZF]
Cotton: PNR and IMF series for Cotton US Liverpool [11176F.DZF]
Hides: PNR and IMF series for Hides U.S. (Chicago) [11176P.DZF]
Jute: PNR and IMF series for Jute Bangladesh (Chitt-Chal) [51376X.DZF]
Lamb: PNR and IMF series for Lamb New Zealand (London) [19676PFDZF]
Lead: PNR and IMF series for Lead U.K. (London) [11276V.DZF]
Maize: PNR and IMF series for Maize U.S. (Gulf Ports) [11176J.DZF]
Palm Oil: PNR and IMF series for Palm Oil Malaysia (U.K.) [54876DGDZF]
Rice: PNR and IMF series for Rice Milled 5% Broken [57876N.ZF]
Rubber: PNR and IMF series for Rubber Malaysian Rss #1 [57876L.ZF]
Silver: PNR and IMF series for Silver U.S. (New York) [11176Y.DZF]
Sugar: PNR and IMF series for Sugar Caribbean (N.Y.) [00176IADZF]
Tea: PNR and IMF series for Tea Unit Value [52474S.DZF]
Timber: PNR and IMF series for Timber from the World Bank
Tin: PNR and IMF series for Tin All Origins (London) [11276Q.DZF]
Tobacco: PNR and IMF series for Tobacco U.S. (All Markets) [11176M.DZF]
Wheat: PNR and IMF series for Wheat [21374D.DZF]
Wool: PNR and IMF series for Wool Australia-N.Zeal (UK) 50s [11276HDDZF]
Zinc: PNR and IMF series for Zinc [21874T.DZF]

Macroeconomic Determinants

1. US Real Interest Rate: deflated by the US Consumer Price Index.
   (i) Short run Interest rates \( (r_{TB,j}) \) are Municipal Bond Yields (1901-1919) and US Treasury Bill Yields from BEA (1920-1947) <http://www.census.gov/compendia/statab/hist_stats.html>. And from IMF IFS (1948 to 2008). Real Rates are from ex post inflation (based upon \( CPI_t \)).
3. Oil Prices (\( OIL_t \)) Log ratio of Crude Petroleum Prices to \( CPI_t \) from Officer (2009).
## Data Appendix

### Table A1. Descriptive Statistics of Commodity Prices

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Maximum</th>
<th>Minimum</th>
<th>COV(CP&lt;sub&gt;t&lt;/sub&gt;, CP&lt;sub&gt;t-1&lt;/sub&gt;)</th>
<th>a&lt;sub&gt;t&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>1.225</td>
<td>15.843</td>
<td>60.613</td>
<td>-44.977</td>
<td>0.946</td>
<td>5.1</td>
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<tr>
<td>Bananas</td>
<td>2.858</td>
<td>9.985</td>
<td>33.565</td>
<td>-34.383</td>
<td>0.976</td>
<td>0.9</td>
</tr>
<tr>
<td>Beef</td>
<td>3.702</td>
<td>21.911</td>
<td>82.589</td>
<td>-60.102</td>
<td>0.978</td>
<td>5.1</td>
</tr>
<tr>
<td>Cocoa</td>
<td>2.222</td>
<td>26.805</td>
<td>111.019</td>
<td>-60.867</td>
<td>0.938</td>
<td>2.7</td>
</tr>
<tr>
<td>Coffee</td>
<td>2.536</td>
<td>25.108</td>
<td>78.844</td>
<td>-60.779</td>
<td>0.909</td>
<td>10.3</td>
</tr>
<tr>
<td>Copper</td>
<td>2.782</td>
<td>19.307</td>
<td>60.482</td>
<td>-46.914</td>
<td>0.958</td>
<td>5.9</td>
</tr>
<tr>
<td>Cotton</td>
<td>1.559</td>
<td>17.697</td>
<td>44.603</td>
<td>-49.415</td>
<td>0.937</td>
<td>4.3</td>
</tr>
<tr>
<td>Hides</td>
<td>1.767</td>
<td>26.198</td>
<td>71.480</td>
<td>-100.493</td>
<td>0.963</td>
<td>2.3</td>
</tr>
<tr>
<td>Jute</td>
<td>2.199</td>
<td>22.753</td>
<td>60.263</td>
<td>-78.318</td>
<td>0.894</td>
<td>0.2</td>
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<tr>
<td>Lamb</td>
<td>4.119</td>
<td>23.221</td>
<td>83.080</td>
<td>-64.558</td>
<td>0.987</td>
<td>0.9</td>
</tr>
<tr>
<td>Lead</td>
<td>2.882</td>
<td>20.750</td>
<td>69.403</td>
<td>-56.149</td>
<td>0.899</td>
<td>1.3</td>
</tr>
<tr>
<td>Maize</td>
<td>2.364</td>
<td>23.388</td>
<td>70.526</td>
<td>-92.227</td>
<td>0.918</td>
<td>6.8</td>
</tr>
<tr>
<td>Palm</td>
<td>2.090</td>
<td>24.600</td>
<td>72.736</td>
<td>-69.510</td>
<td>0.865</td>
<td>8.3</td>
</tr>
<tr>
<td>Rice</td>
<td>1.924</td>
<td>19.894</td>
<td>86.686</td>
<td>-57.797</td>
<td>0.871</td>
<td>3</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.575</td>
<td>29.354</td>
<td>102.311</td>
<td>-96.216</td>
<td>0.879</td>
<td>2.8</td>
</tr>
<tr>
<td>Silver</td>
<td>2.960</td>
<td>20.737</td>
<td>71.964</td>
<td>-67.731</td>
<td>0.888</td>
<td>1.7</td>
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<tr>
<td>Sugar</td>
<td>1.394</td>
<td>36.186</td>
<td>113.495</td>
<td>-134.935</td>
<td>0.724</td>
<td>7.3</td>
</tr>
<tr>
<td>Tea</td>
<td>1.711</td>
<td>16.604</td>
<td>55.900</td>
<td>-55.816</td>
<td>0.945</td>
<td>1.6</td>
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<tr>
<td>Timber</td>
<td>3.613</td>
<td>16.859</td>
<td>55.142</td>
<td>-46.785</td>
<td>0.984</td>
<td>12</td>
</tr>
<tr>
<td>Tin</td>
<td>3.185</td>
<td>20.999</td>
<td>57.432</td>
<td>-58.012</td>
<td>0.959</td>
<td>2.2</td>
</tr>
<tr>
<td>Tobacco</td>
<td>3.368</td>
<td>12.794</td>
<td>50.689</td>
<td>-33.059</td>
<td>0.991</td>
<td>2.9</td>
</tr>
<tr>
<td>Wheat</td>
<td>2.427</td>
<td>19.745</td>
<td>72.493</td>
<td>-51.085</td>
<td>0.932</td>
<td>8.1</td>
</tr>
<tr>
<td>Wool</td>
<td>1.551</td>
<td>21.576</td>
<td>77.049</td>
<td>-63.245</td>
<td>0.851</td>
<td>2.7</td>
</tr>
<tr>
<td>Zinc</td>
<td>2.943</td>
<td>22.017</td>
<td>94.742</td>
<td>-53.976</td>
<td>0.899</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>2.415</td>
<td>21.430</td>
<td>72.379</td>
<td>-64.056</td>
<td>0.920</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Mean, standard deviation, maximum and minimum are based on the commodity price index’s (CP<sub>t</sub>) annual growth rate (ΔCP<sub>t</sub>). COV(CP<sub>t</sub>, CP<sub>t-1</sub>) is the autocorrelation of the individual commodity prices. All Statistics based upon annual data.
Results Appendix

Unit Root Tests

Table A2. Unit Root Tests

<table>
<thead>
<tr>
<th>Variables</th>
<th>Level</th>
<th>First Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pc_t = \ln(\text{PC}_t / \text{CPI}_t)$</td>
<td>-2.971* [1]</td>
<td>-6.523* [3]</td>
</tr>
<tr>
<td>$OIL_t$</td>
<td>-1.864 [0]</td>
<td>-10.232* [0]</td>
</tr>
</tbody>
</table>

Notes: We use annual data on commodity prices 1901 to 2008 ($T=108$). The null hypothesis is of unit root, and we reject the null when we have test statistics which are less than the asymptotic critical value from Davidson and MacKinnon (1993) -2.86. Indicated by an asterisk at the 5% statistical significance level. Lag length is determined by Akaike Information Criteria in square brackets. $pc_t$ is the first principal component of the 24 commodities in the Grilli–Yang index, $R_t$ is the real short run US interest rate based upon ex post CPI inflation and yields on US Treasury Bills and Municipal Bonds, $DY_t$ is the growth rate logged US real GDP reflecting global demand and $OIL_t$ is a the log of real crude oil prices.

GARCH Estimates

The GARCH in Mean model is as follows.

$$pc_t = \mu + c \sigma_t^2 + a_t, \quad a_t = \sigma_t \varepsilon_t \quad \text{(A1)}$$

$$\sigma_t = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad \text{(A2)}$$

Where $pc_t$ is the real principal component of the price of commodities, $\sigma_t^2$ is the conditional heteroscedasticity, and we have the residual term $a_t$. A simple GARCH model [1] in Table A3 is mis-specified since $\alpha_1 + \beta_1 > 1$. Equation [2] has a GARCH in Mean, although is also mis-specified. Consequently we impose IGARCH in Mean, equation [3]. This is no longer mis-specified, there is a negative and significant GARCH in mean term. This is also robust to the inclusion of the real interest rate.

Table A3. GARCH-M Results for Commodity Prices

<table>
<thead>
<tr>
<th></th>
<th>[1]</th>
<th>[2]</th>
<th>[3]</th>
<th>[4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>5.469</td>
<td>5.462</td>
<td>6.038</td>
<td>6.088</td>
</tr>
<tr>
<td></td>
<td>(207.976)</td>
<td>(195.543)</td>
<td>(365.916)</td>
<td>(238.378)</td>
</tr>
<tr>
<td>$c \sigma_t^2$</td>
<td>0.040</td>
<td>-1.969</td>
<td>-2.145</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.355)</td>
<td>(-9.435)</td>
<td>(-7.429)</td>
<td></td>
</tr>
<tr>
<td>$R_t$</td>
<td>-0.025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.339)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_1 \sigma_{t-1}^2$</td>
<td>1.163</td>
<td>1.183</td>
<td>0.320</td>
<td>0.433</td>
</tr>
<tr>
<td></td>
<td>(3.606)</td>
<td>(3.574)</td>
<td>(4.929)</td>
<td>(6.558)</td>
</tr>
<tr>
<td>$\beta_1 \sigma_t^2$</td>
<td>-0.064</td>
<td>-0.082</td>
<td>0.680</td>
<td>0.567</td>
</tr>
<tr>
<td></td>
<td>(-1.061)</td>
<td>(-0.991)</td>
<td>(10.457)</td>
<td>(8.580)</td>
</tr>
</tbody>
</table>

Notes: $\mu$ is the constant in the mean equation. $c$ is the risk coefficient. $\alpha_0$ is the constant in the variance equation. $\alpha_1$ is the lagged residual. $\beta_1$ is the lagged variance in the variance equation. T-statistics in parentheses. GARCH-M is GARCH in mean estimation. IGARCH-M imposes the additional constraint that $\beta_1 = (1- \alpha_1)$. Incorporating $DY_t$ and $OIL_t$, the model did not converge.
Correlation between the conditional variance of commodity prices and stock market risk is 0.425.
Figure A3. Determinants of Commonalities in Primary Commodities

Notes: These graphs provide the response of primary commodities (pc) to a generalized one standard deviation innovation in real interest rates (R) and risk (risk). The latter is measured by the risk term from a GARCH in mean (see Table A3, equation [3]). Identification of the residuals of the VAR is by Koop et al. (1996). Also included are ± two asymptotic standard error bands.

Figure A4. Determinants of Commonalities in Primary Commodities

Notes: These graphs provide the response of primary commodities (pc) to a generalized one standard deviation innovation in real interest rates (R), risk (RISK), demand (DY) and supply (DOIL). Risk is measured by the risk term from the stock market. Identification of the residuals of the VAR is by Koop et al. (1996). Also included are ± two asymptotic standard error bands.