Strategic Information Revelation and Capital Allocation

Alvaro Pedraza Morales
Abstract

It is commonly believed that stock prices help firms’ managers make more efficient real investment decisions, because they aggregate information about fundamentals that is not otherwise known to managers. This paper identifies a limitation to this view. It shows that if informed traders internalize that firms use prices as a signal, stock price informativeness depends on the quality of managers’ prior information. In particular, managers with low quality information would like to learn about their own fundamentals by relying on the information aggregated in the stock price. However, in this case, the profitability of trading falls for informed speculators, who therefore reduce their trading volume, reducing the informativeness of prices. As a result, stock prices are not as useful in guiding capital toward its most productive use, leading to inefficient investment decisions. Using a sample of U.S. publicly traded companies between 1990 and 2010, the paper documents a positive correlation between the quality of managerial information and stock price informativeness. Contrary to the conventional view that less informed managers should rely more on stock prices when making investment decisions, the author finds no differences in the sensitivity of investment to stock prices for different levels of managerial information. The evidence suggests that while firms do learn from prices, the learning channel and its effects on real investment are limited.

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Strategic Information Revelation and Capital Allocation

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JEL Classification: G12, G23, C7, E22

Keywords: Asymmetric information, feedback effects, strategic interactions.

*Email address: apedrazamorales@worldbank.org. I am indebted to Pete Kyle and John Shea for their invaluable guidance. This study has benefited from the comments of Sushant Acharya, Jeronimo Carballo, Pablo Cuba, Laurent Fresard, Anton Korinek, Federico Manderlman, Luminita Stevens, Shu Lin Wee and Russ Wermers. I also thank seminar participants at the Western Economic Association International, Federal Reserve Bank of Atlanta, Federal Reserve Board of Governors, University of Maryland, World Bank, Bank of International Settlements and Bowdoin College for comments and suggestions.
1. Introduction

Do stock prices improve efficiency by directing capital towards more productive uses? A widely held view, dating back at least to Hayek (1945), is that stock prices are useful signals since they aggregate information about fundamentals that is not otherwise known to firms’ managers. In this sense, the stock price of a company might be informative to the manager when making a real investment decision. In a very practical way, informative prices enable superior decision-making (Fama and Miller (1972)).

In this paper I identify a limitation to this view. More precisely, in a model where informed traders in secondary markets internalize that stock prices are signals to firms’ managers, I show that trading volume and price informativeness depend on the quality of managers’ prior information. In other words, the amount of private information that is aggregated into the stock price through the trading process is a function of managers’ initial information. The learning channel is limited because prices are less informative for firms with low quality of managerial information, precisely the case in which managers would like to learn more from the stock market. This happens despite the fact that some market participants are endowed with perfect information. As a result, stock prices are not as useful in guiding capital towards its most productive use.

Aside from identifying a fundamental limitation to the allocational role of stock prices theoretically, I make two novel empirical contributions. Using a sample of U.S. publicly traded companies, I document a positive correlation between the quality of managers’ information and stock price informativeness. I find a stronger correlation for firms with higher institutional ownership, which further suggests the presence of strategic behavior. Also, contrary to the conventional view that less informed managers should rely more on stock prices when making investment decisions, I find no differences in the sensitivity of investment to stock prices for different levels of managerial information. The evidence suggests that while firms do learn from prices, the learning channel and its effects on real investment are limited.

I model learning from prices as follows. There is a continuum of publicly traded firms facing a real investment opportunity with uncertain net present value. Firms use stock prices to update their prior about their own fundamentals. Informed and noise traders submit demands for the firm’s shares in a secondary market. Informed traders are strategic in that they internalize the effect of their trades on both prices and on the firms’ inference problem.

Subrahmanyam and Titman (1999) argue that prices are useful to managers because they aggregate investors’ signals about future product demand.

This mechanism has received empirical support in the recent work of Durnev et al. (2004), Chen et al. (2007) and Bakke and Whited (2010).
The main result of the model can be summarized as follows: When firms’ managers are less informed a priori, informed traders realize that their expected trading profits are lower because the firm is less likely to undertake the project in the first place. When trading is costly, an informed speculator does not want to buy the stock of a firm with a potentially good investment project if the investment is likely to be cancelled. Similarly, a trader will not short sell a public firm with a negative NPV project if the firm is not likely to invest. Under these circumstances, traders with private information reduce their trading volume, and in turn prices are less informative about the fundamental. Investment sensitivity to price is lower in this case, as firms recognize that prices contain less information and are less inclined to rely on the stock price to update their prior. Overall, investment efficiency falls as the market signal is not as useful in helping managers distinguish between good and bad projects.

Stylized facts about speculative markets suggest that the best-informed traders are large. In the stock market, arbitrageurs with private information about merger prospects buy and sell significant percentages of the outstanding equity of publicly held companies. It is well recognized in existing literature that large traders take into account their effect on prices in choosing the quantities they trade (Grinblatt and Ross (1985) and Kyle (1985)). If an investor has superior information, attempts to use it will “publicize” some of the information and instantly reduce its value. Price-taking behavior would be irrational in this case. This paper extends this logic to argue that large traders internalize that prices are also signals to firms’ managers when making real investment decisions.

This paper is related to a growing literature that studies feedback effects on equilibrium asset prices.\(^3\) The basic idea of this literature is that if firms use market prices when deciding on their actions, traders should adjust their strategies to reflect this response. On the theoretical side, this paper is closely related to Bond et al. (2010). The authors suggest that when agents (e.g. directors, regulators, or managers) learn from stock prices, there is a complementarity between the agent’s direct sources of information and his use of market data. However, in their model, trading in the secondary market is not modeled explicitly. Instead, the stock price is set through a rational expectations condition, which is then used by the agent who is taking a corrective action. The main drawback in their model is that a rational expectations equilibria does not always exist, which limits the model’s predictions.\(^4\) In my model, trading by informed speculators and firms’ real investment decisions are the outcome of a strategic game. By assuming that informed speculators internalize the firms’ inference problem, I am able to show existence of equilibria for any level of managerial information, contrary to the non-existence result in Bond et al. (2010). In

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\(^3\)See Bond et al. (2012) for an excellent survey of this literature.

\(^4\)The authors interpret non-existence as indicating a loss of information transmitted by prices.
this sense, my model has a clear empirical prediction, namely that stock prices are less informative for managers with low quality of information. Furthermore, my model allows me to study informed trading behavior when there is learning from stock prices, a feature that is omitted in Bond et al. (2010).

Other papers studying feedback effects include Leland (1992), Dow et al. (2011), Goldstein and Guembel (2008), Edmans et al. (2011) and Goldstein et al. (2012). Leland (1992) shows that when insider trading is permitted, prices better reflect information and expected real investment rises. Dow et al. (2011) find that information production in secondary markets is sensitive to the ex-ante likelihood of the firm undertaking the project and Edmans et al. (2011) study asymmetric trading behavior between good and bad information. Most of these papers assume a discrete space for firms’ fundamentals, typically specifying two possible valuations for the investment project (i.e. high and low). My model assumes a continuous space of firms’ fundamentals, which allows me to study the interaction between the quality of managerial information, trading behavior, stock price informativeness and investment efficiency, issues that to my knowledge have been overlooked by the existing literature.

This paper provides a novel explanation for why markets are limited in their ability to aggregate information and guide real decisions. Shleifer and Vishny (1997) provide an alternative explanation based on limits to arbitrage, in which the slow convergence of prices to fundamentals may deter speculators from trading on their information. Other explanations rely on market frictions such as short selling constraints. For example, Diamond and Verrechia (1987) show that short selling constraints affect the speed of price adjustment to private information. In my model, the ability of prices to fully reflect fundamentals and to coordinate investment is crucially related to the precision of firms’ prior beliefs about their fundamental value.

Empirical evidence that price informativeness is high in well-developed financial systems and low in emerging markets is presented by Morck et al. (2000). The authors argue that in countries with well-developed financial markets, traders are more motivated to gather information on individual firms. My model offers an alternative interpretation of their results. I argue that low price informativeness may result from the failure of stock prices to aggregate information when feedback effects are present and ex-ante fundamental uncertainty is high, which is potentially the case for emerging markets.

Finally, this paper is related to the empirical literature that studies learning from prices. Durnev et al. (2004), Chen et al. (2007) and Bakke and Whited (2010) show that investment sensitivity to prices is higher for firms for which the stock price is more informative about fundamentals. My empirical work

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5 This result holds independently of whether the investment decision is value-increasing or value-decreasing for the firm.
6 Another example is Goldstein and Guembel (2008), who study price manipulation when traders are uninformed.
addresses the determinants of stock price informativeness. The evidence suggests that stock prices are less informative for firms with low quality information ex-ante. I also estimate a standard investment equation as in Chen et al. (2007), and show that, contrary to the conventional view, less informed managers do not rely more on stock prices to make investment decisions. Collectively, the evidence suggest that while secondary market are a useful source of information, they are limited in their ability to guide real decisions.

The rest of this document is organized as follows. Section 2 introduces the model economy. Equilibrium results are derived in section 3. This section includes a model extension where firms can incur a cost to acquire information about the fundamental before observing the stock price. In section 4 I present the empirical exercise and I conclude in section 5.

2. Model

The model consists of three periods, \( t \in \{0, 1, 2\} \), with three types of continuum agents: firms, informed speculators (one for each firm) and noise traders. Stocks for each firm are traded in a secondary market. Each firm’s manager needs to decide whether to continue or abandon an investment project. The investment decision is taken to maximize firm value (there is no shareholder/manager agency problem).

2.1. Firms

The economy is populated by a continuum of firms. At period \( t = 0 \) each firm is uncertain about its own fundamental value \( \theta \), which determines its final profits (for instance, the firm may be uncertain about the viability of a project or future demand). \( \theta \) is unobservable and firms have a common prior \( \theta \sim N(\mu_\theta, \sigma_\theta^2) \).

At \( t = 1 \) firms observe their own stock price \( q \) and decide whether to invest \((d = i)\) or not \((d = n)\). If a firm decides to invest it pays a fixed investment cost \( c > 0 \). In period \( t = 2 \) payoffs are realized for each firm according to

\[
\Pi^d = \begin{cases} 
\theta - c & d = i \\
0 & d = n 
\end{cases}
\]  

(1)

2.2. Financial Markets

For each stock there is one risk neutral informed speculator. He learns the firm’s fundamental value \( \theta \) at period \( t = 0 \). In this setting I am modeling the extreme case where the speculator is perfectly informed and the firm is not. This simplifying assumption allows tractability. I conjecture that similar results
would hold if the speculator has some private information about the firm fundamental that is orthogonal to the firm’s information. This would generate some learning from prices. At date 1, conditional on their information, informed speculators submit a market order $X_I(\theta)$ to a Walrasian auctioneer.\footnote{This order is not observed by the firm.} I assume that speculators do not observe the price when they trade, and hence submit a market order, as in Kyle (1985). This captures the idea that speculators, when they trade, do not have the market information that the firm will have when making the investment decision (recall that the firm bases its investment decision on the price of the security). I impose no any additional constraints on the demands by the informed speculators, such as short selling constraints. That is, the informed speculators either have deep pockets or have access to financing to buy or sell as many shares as they find profit maximizing.

The Walrasian auctioneer also observes a noisy supply curve from uninformed traders and sets a price to clear the market. The noisy supply for each stock is exogenously given by $X_N(\tilde{z}, q)$, a continuous function of an exogenous supply shock $\tilde{z}$ and a price $q$. The supply curve $X_N(\tilde{z}, q)$ is strictly decreasing in $\tilde{z}$ and increasing in $q$, so that supply is upward sloping in price. The supply shock $\tilde{z} \in \mathbb{R}$ is independent of other shocks in the economy, and $\tilde{z} \sim N(0, \sigma^2_z)$.

The usual interpretation of noisy supply is that there are agents who trade for exogenous reasons, such as liquidity or hedging needs. They are usually referred to as “noise traders”. In this setting, the presence of noise traders guarantees that prices will not be fully revealing, as there can be different prices for the same fundamental value.

To solve the model in closed form, I assume that $X_N(\tilde{z}, q)$ takes the following functional form: $X_N(\tilde{z}, q) = \epsilon q - \tilde{z}$. The parameter $\epsilon$ captures the elasticity of supply with respect to the price. It can be interpreted as the liquidity of the market: when $\epsilon$ is high, supply is very elastic with respect to the price, and large shifts in informed demand are easily absorbed in the price without having much of a price impact. This notion of liquidity is similar to that in Kyle (1985), where liquidity is considered high when the informed trader has a low price impact. These basic features, i.e., that supply is increasing in price and has a noisy component, are standard in the literature. It is also common in the literature to assume particular functional forms to obtain tractability. The specific functional form assumed here is close to Goldstein et al. (2012). The equilibrium price is given by the market clearing condition $\epsilon q - \tilde{z} = X_I(\theta)$.

In the last period $t = 2$, the informed speculators and noise traders earn a share of firms’ profits proportional to their stock ownership. The model timeline for each firm is depicted in Figure 1.
2.3. Equilibrium

I now turn to the definition of equilibrium in this economy.

**Definition 1. Perfect Bayesian Nash Equilibrium** An equilibrium with imperfect competition among informed speculators and learning from prices is defined as follows: (i) Each informed speculator chooses a trading strategy \( X_I(\theta) \) that maximizes expected profits subject to the market clearing condition \( X_I(\theta) = X_N(\hat{z}, q) \) and the investment strategy by the firm. (ii) Each firm chooses an investment rule to maximize expected payoffs given the observed stock price \( q \), prior beliefs about their own fundamental value and beliefs about the informed speculator trading strategy. (iii) Each player’s belief about the other players’ strategies is correct in equilibrium.

In other words, an equilibrium is a fixed point in strategies where each firm sets a best response (investment rule) to market prices, given prior beliefs and the informed speculator trading strategy, and speculators set their optimal demands recognizing the price impact of their trades and the firms’ reaction.

### 3. Solving the Model

In this section, I explain the main steps that are required to solve the model. Using the market clearing condition, I start by solving the optimal investment rule by the firm for a given stock price. I then characterize the optimization problem of the informed speculator for the given investment rule. Finally, given the investment rule by the firm and the trading rule by the informed speculator, I calculate the fixed point.

#### 3.1. Firms

After observing the stock price, the firm’s posterior distribution on its fundamental value is

\[
\xi(\theta \mid q) = \frac{\varphi(eq - X_I(\theta))\xi(\theta)}{\int_{-\infty}^{\infty} \varphi(eq - X_I(\theta))\xi(\theta)d\theta} \tag{2}
\]

where \( \varphi() \) is the density function of the normal distribution with mean 0 and variance \( \sigma_z^2 \) and \( \xi() \) is the density function of a normal distribution with mean \( \mu_\theta \) and variance \( \sigma_\theta^2 \).

Profit maximization implies that a firm with stock price \( q \) will invest if the expected profit under the posterior is nonnegative, \( \int_{-\infty}^{\infty} \theta \xi(\theta \mid q)d\theta \geq c \). In this setting, the firm’s decision is a cutoff rule, such that for any \( q \geq \bar{q} \) the firm will invest \((d = i)\), where \( \int_{-\infty}^{\infty} \theta \xi(\theta \mid \bar{q})d\theta = c \), and will not invest \((d = n)\) if \( q < \bar{q} \).
Lemma 1. If firm managers conjecture a linear demand function by the informed speculators of the form
\[ X_I(\theta) = a + b\theta, \]
then the cutoff price function \( \bar{q} \) is given by:
\[
\bar{q}_l = \frac{1}{\epsilon} \left[ a + cb - \frac{1}{b}(\mu_{\theta} - c)\frac{\sigma_z^2}{\sigma_{\theta}^2} \right] \tag{3}
\]

Proof in Appendix C.

Lemma 1 refers to the functional form of the cutoff price when managers believe that informed speculators’ trades are linear in the fundamental. More precisely, \( \bar{q}_l \) in Lemma 1 is the firms’ best response to linear demands by the informed speculators. The cutoff price is set as an optimal weighting between the prior information of the manager and the price signal. The fraction \( \sigma_z^2/\sigma_{\theta}^2 \) represents the ratio of the precision of the stock price to the precision of the manager’s prior information. When the managers’ precision is large relative to the precision of the price signal, \( \sigma_z^2/\sigma_{\theta}^2 \to \infty \), the cutoff price is \( \bar{q} \to -\infty \) (always invest) if \( \mu_{\theta} > c \) and \( \bar{q} \to \infty \) (never invest) if \( \mu_{\theta} < c \). In this case, the firm’s investment decision is independent of the stock price, as the manager’s decision is based exclusively on whether the ex-ante expected profits of the project are positive or negative. When the ratio of precisions between the signal and the prior is finite, managers set a finite \( \bar{q}_l \), in which case the investment decision depends on the observed stock price.

The cutoff rule also depends on the conjectured trading strategy of the informed speculators, i.e. the parameters \( a \) and \( b \). For example, if firms believe that informed speculators set their demands independently of the fundamental, e.g. \( b = 0 \), managers understand that the price signal contains no idiosyncratic information that would be useful to infer the fundamental, and rely only on their prior to make the investment decision.

3.2. Informed Speculators

The risk neutral informed speculators maximize the expected profits of their trading strategies,
\[
\max_{X_I(\theta)} E[X_I(\theta)(\Pi^d - q) \mid \theta],
\]
subject to the market clearing condition \( X_I(\theta) = X_N(\tilde{z}, q) \) and the firm’s investment rule described above. Since each informed speculator internalizes his market power, the optimization problem is transformed to
\[
\max_{X_I(\theta)} X_I(\theta)E[\Pi^d \mid \theta] - \frac{X_I(\theta)^2}{\epsilon} \tag{4}
\]

\(^8\)Lemma 1 is intended primarily to help build up intuition for the model mechanism. In section 3.2 I solve the model numerically, in which case speculators demands are not linear and Lemma 1 does not hold.
The first term in (4) is expected total earnings given the investment profits. The trader is perfectly informed about the value of the firm’s project, so his expectation is taken with respect to whether the firm will invest or not. The second term in (4) is the cost of the trading strategy.

For a firm with fundamental value \( \theta \), the probability that the stock price \( q \) is above the threshold \( \overline{q} \) is

\[
Pr(q \geq \overline{q} | \theta) = \epsilon \int_{\overline{q}}^{\infty} \varphi(\epsilon q - X_I(\theta))dq = \Phi \left[ \frac{1}{\sigma_z} (X_I(\theta) - \epsilon \overline{q}) \right],
\]

where \( \Phi \) is the cumulative distribution of the standard normal.

**Definition 2.** Let \( \psi(q, X_I(\theta)) \) be defined as the probability that the stock price \( q \) of a firm with fundamental \( \theta \) is above the firm’s cutoff rule: \( \psi(q, X_I(\theta)) \equiv Pr(q \geq \overline{q} | \theta) \).

The informed speculator’s optimization problem becomes

\[
\max_{X_I(\theta)} X_I(\theta) \left[ \psi(q, X_I(\theta))(\theta - c) \right] - \frac{X_I(\theta)^2}{\epsilon}
\]

(5)

To summarize, expected trading profits depend on the probability that the firm undertakes the project. When the informed speculator trades, he always incurs the trading cost, which is quadratic the number of shares he demands, while the expected revenue is proportional to the likelihood that the firm undertakes the project. This maximization captures the idea that an informed trader does not want to buy shares in a firm with a good investment project if the investment is likely to be cancelled. Similarly, it is not optimal for a trader to short sell a firm with a negative NPV project if the firm is not likely to invest.

To learn more about the impact of strategic trading and firm learning, below I also consider the following alternatives to the benchmark model:

**Alternative #1 - Perfect information:** Firms learn their true fundamental before making the investment decision. In this case, only firms with profitable projects (good fundamentals) will invest, i.e. \( \theta \geq c \). Since firms with bad projects \( \theta < c \) don’t invest, speculators don’t trade these companies, while buying \( X_I^1(\theta) = \frac{1}{2}(\theta - c) \) shares for firms with good fundamentals. At time zero, the expected price is zero for bad firms and \( E_0q^1 = \frac{1}{2}(\theta - c) \) for firms with positive NPV. Here the expectation \( E_0 \) is taken over the noise shock. In this case the firm’s stock price is half its expected profit. The results follow from market power, as the informed speculator recognizes that every unit he demands of the stock will increase the price by a factor of \( \frac{2}{\epsilon} \).

**Alternative #2 - No learning from prices:** Firms don’t use stock prices \( q \) to update their beliefs about fundamentals.\(^9\) If the ex-ante expectation of the investment return is greater than the investment

\(^9\)For instance, this would be the outcome if the firm is required to make its investment decision at the same time that the
cost, i.e. \( \mu_\theta > c \), all firms invest. In this case, the informed speculator demand is \( X^2_I(\theta) = \frac{\epsilon}{2}(\theta - c) \). Here, informed speculators take a long position on firms with positive net present value and short positions on firms with negative present value.

**Alternative #3 - Speculators do not internalize firms’ updating:** In this alternative setting, informed speculators make their trading decision assuming that firms invest without updating their prior. If \( \mu_\theta > c \), informed speculators’ demands are \( X^3_I(\theta) = \frac{\epsilon}{2}(\theta - c) \). From Lemma 1, firms set their cutoff price as

\[
\bar{q}^3 = -(\mu_\theta - c) \frac{2\sigma^2}{\epsilon^2\sigma^2_\theta}
\]

which is monotonically increasing in \( \theta \), indicating that firms with better fundamentals are more likely to invest than firms with bad fundamentals. In this alternative model, firms use the stock price as a signal and therefore make a better and more informed investment decisions. The result arises almost by construction, because some market participant (the informed speculator) is endowed with perfect information which makes the price a good signal to improve the firm’s decision. However, in what follows, I show that this learning channel is limited when the informed speculator internalizes the firm learning process.

I now turn to the solution of the benchmark model, when firms learn from stock prices and informed speculators are strategic in that they internalize both the price effect and the firm’s updating process. Proposition 1 presents the equilibrium.

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fundamental value is revealed to the informed speculator. In this case, the firm cannot use prices to update beliefs at the time of investment.
Proposition 1. There exists an equilibrium with strategic informed traders and firms. The equilibrium in strategies can be approximated around $\theta = c$ as:

(i) If the expected NPV of the investment project is non-negative under the firm’s prior, $\mu_\theta \geq c$, then the equilibrium demands by speculators and firms’ cutoff rule are:

- Informed speculators’ demand: $X_i^*(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma)$
- Firms’ cutoff rule: $q^* = (\mu_\theta - c) \frac{2\sigma^2 z^2}{\epsilon^2 \sigma^2 \Phi(\gamma)}$

where $\Phi()$ is the cumulative distribution of the standard normal and $\gamma = \frac{2(\mu_\theta - c)\sigma z}{\epsilon \sigma^2 \theta}$.

(ii) If the expected NPV of the investment project is negative under the firm’s prior, $\mu_\theta < c$, then informed speculators do not trade in equilibrium, $X_i^*(\theta) = 0$, and firms never invest, $q^* \rightarrow \infty$.

Proof in Appendix C.

Proposition 1 refers to the informed speculators demands’ and the firm strategy approximated around $\theta = c$. While a closed form solution for the entire space of $\theta$ is not available, the approximated solution provides the relevant economic intuition because it refers to the investment decision and the trading behavior for the marginal firm. More precisely, the approximated solution allows me to compare equilibrium strategies between firms with fundamentals slightly above and below the investment cost. While the comparative statics below are carried out with the linear approximation, exact numerical solutions are presented later to establish the general validity of the results.\footnote{From here onwards, all the analytical results in the benchmark model make use of this approximation unless stated otherwise.}

Case (i) in Proposition 1 refers to the equilibrium when the ex-ante expectation of the firm’s profit is non-negative. Here, speculators’ demands and the cutoff price are scaled by a factor of $\Phi(\gamma)$ and $\frac{1}{\Phi(\gamma)}$ respectively, relative to Alternative #3 above, in which speculators do not internalize the firm learning. From here onwards I will refer to $\gamma$ as the precision of managers’ information, which is inversely proportional to $\sigma^2_\theta$ (managers’ uncertainty about fundamentals).

Under Alternative #3, the cutoff price is strictly decreasing in the managers’ precision, $\frac{dq_3}{d\gamma} = -\frac{\sigma_z}{\epsilon} < 0$.

As discussed earlier, in this standard signal extraction problem, managers rely less on the stock price the more confident they are on their prior. In the benchmark model, when traders internalize the firm’s updating process, the cutoff price varies with the managers’ precision as follows:

$$\frac{dq^*}{d\gamma} = \frac{-\Phi(\gamma) + \gamma \Phi'(\gamma) \sigma_z}{\Phi(\gamma)^2} \frac{\sigma_z}{\epsilon} \tag{8}$$
In this case the cutoff price is also strictly decreasing in managers’ precision \((\frac{d\gamma}{d\gamma} < 0, \text{ proof in Appendix C})\). However the second term in the numerator in (8) has the opposite sign compared to a standard signal extraction problem. In particular, if managers have a more precise prior, the are likely to rely less on the stock price (given by the factor \(-\frac{1}{\Phi(\gamma)}\)). However, this effect is dampened by the factor \(\frac{\gamma \Phi'(\gamma)}{\Phi(\gamma)}\), since the manager understands that in this scenario stock prices will have more private information, as informed speculators trade more in absolute terms \((X_I(\theta) \sim \Phi(\gamma))\) when \(\gamma\) is higher. On the contrary, when \(\gamma\) is low (less precise prior), the firm manager wants to rely more on the stock price to make the investment decision by increasing \(\pi^*\), but he understands that when \(\gamma\) is low, informed speculators reduce their demands in absolute terms, which in turn makes the price signal less informative, dampening the learning channel. I expand on this discussion in the next section.

Finally, case (ii) in Proposition 1 refers to the equilibrium when the ex-ante expectation of the firm’s profit is negative. When \(\mu_\theta < c\), informed traders don’t trade and stock prices are determined solely by noise traders. Since firm managers understand this, they ignore the stock price, making the investment decision based exclusively on their prior information, which results in the investment project being canceled. While this result also holds when traders don’t internalize the firms’ updating process (Alternative #3), it is surprising that even when all agents understand the feedback from prices to investment, stock prices cannot promote better firm decisions by overcoming the information gap between traders and firms.\(^{11}\) Note that in this case, speculators cannot profit on their information despite having perfect knowledge of the fundamental. In this case, the informed trader would be better off taking over the firm as a private equity investor whenever the true \(\theta > c\), since he could then use his information to make efficient decisions for the firm.

The results indicate that the extent of information revelation through prices is sensitive to the ex-ante likelihood of the firm undertaking the project. Dow et al. (2011) has a similar result when studying information production in financial markets. In their model, a continuum of atomistic speculators pay a cost of acquiring information as long as others are also paying this cost. This in turn depends on whether the firm is likely to undertake the project in the first place. The less likely a firm is to invest, the less incentive traders have to produce information about the project. In my model, some speculators are endowed with perfect information and incur trading costs (i.e. the price effect of their own trades). While the model in Dow et al. (2011) studies complementarities between traders and information acquisition, I abstract from such concerns by assuming one informed speculator per firm. However, this simplification

\(^{11}\)The key assumption for this result is that the informed trader has no direct communication with the firm.
allows me to expand to a continuous space of firms’ fundamentals instead of the discrete setting in Dow et al. (2011) with only two possible valuations for the investment project (i.e. high and low). With this extension, my model is suitable for analyzing the interactions between the quality of ex-ante managerial information and trading behavior, stock price informativeness and investment efficiency. In what follows I present a detailed analysis of these interactions.

3.3. Informed Trading

Above, I showed that when informed traders do not trade \( X_I(\theta) = 0 \), the stock price depends on noise trading alone, which makes the stock price uninformative about the fundamental. Larger absolute informed speculator demands reflect the presence of informed trading in the stock, which leads to more informative prices. In other words, price informativeness refers to the amount of information the speculator reveals through the stock price, which in turns allows managers to learn about the fundamental value. To be precise, price informativeness is proportional to informed speculators’ trading volume.

**Definition 3.** Let the informed trading volume \( V_I(\theta) \) in a stock with fundamental \( \theta \) be defined as the absolute value of informed speculators’ demands: \( V_I(\theta) \equiv |X_I(\theta)| \).

**Corollary 1.** Informed trading volume decreases in managers’ uncertainty about the fundamental \( \frac{\partial V_I(\theta)}{\partial \sigma_\theta} < 0 \), for firms with fundamental value close to the investment cost \( \theta \) in the neighborhood of \( c \).

Corollary 1 indicates that informed trading volume is lower, and hence price informativeness is lower, when firm managers are less informed ex-ante about the fundamental. This result is exclusive to the benchmark strategic model, as in all three of the alternative models, informed trading volume is independent of the precision of the firms’ prior. Figure 2 presents equilibrium demands by informed speculators for different levels of managerial uncertainty \( \sigma_\theta \), for the case \( \mu_\theta = 1.05, c = \epsilon = \sigma_z = 1 \). The equilibrium demands in Proposition 1 are a linear approximation around \( \theta = c \). Figure 2 displays exact numerical solutions for the given model parameters. Consistent with Corollary 1, informed speculator demands decrease in absolute value for larger values of managerial uncertainty.

There is one important distinction to be made. Price informativeness in the model is not the same as prices being unbiased. Prices are unbiased if they reflect the correct expected value of the firm. Take the case when \( \mu_\theta < c \). According to Proposition 1, firms never invest and speculators do not trade in equilibrium. The expected price at \( t = 0 \) is zero for any firm, independently of the fundamental. Hence, expected prices are unbiased as they reflect the fact that the firm is not investing. However, in this
situation, prices are not informative, since they are not useful to the firm. In general, when the precision of managers’ prior information falls, prices informativeness falls, even though expected prices correctly reflect the real value of the firm (taking into account the investment decision).

According to Figure 2, equilibrium demands in the benchmark model are convex in $\theta$ for $\sigma_\theta > 0$. This result can be rationalized as follows: When speculators have positive information about a firm’s prospects, every share they buy of that firm increases the price of the stock, signaling to the firm that it should continue the project, which is the value-maximizing decision from the point of view of the speculators. Thus, in this case the incentives of the speculators and the firm are perfectly aligned. Meanwhile, when speculators have negative information about the firm’s fundamental ($\theta < c$), their inclination would be to short sell firm shares. However, speculators realize that every additional unit borrowed lowers the share price, making the firm more likely to cancel the investment project, which in turn reduces the payoff of the short position. As a result, the informed speculators reduce their short position when they have adverse information. This asymmetry in trading by informed speculators with positive or negative news about a firm’s investment outlook was first studied by Edmans et al. (2011) in a setting with a discrete distribution of firms’ payoff. While this is certainly an interesting result, asymmetric trading results from higher order terms in the solution. The first order effect, namely the reason why informed speculators optimally reduce their trading volume for firms with low quality information ex-ante, is due to the fact that low quality information increases the likelihood that firms won’t invest and speculators lose money whenever they trade and the firm does not invest.

3.4. Investment

I now consider the real side of the economy. Corollary 2 presents the ex-ante probability that a firm with fundamental $\theta$ will invest.

**Corollary 2.** If $\mu_\theta > c$, for firms with fundamental value close to the investment cost ($\theta$ in the neighborhood of $c$), the probability of undertaking the project in the benchmark model is

$$
\psi^*(\theta) = \Phi \left[ \frac{\epsilon(\theta - c)}{2\sigma_z} \Phi(\gamma) + \frac{2(\mu_\theta - c)\sigma_z}{\epsilon\sigma_\theta^2} \frac{1}{\Phi(\gamma)} \right]
$$

where $\Phi()$ is the cumulative distribution of the standard normal and $\gamma = \frac{2(\mu_\theta - c)\sigma_z}{\epsilon\sigma_\theta^2}$. Proof in Appendix C.

The probability of investment is monotonically increasing in $\theta$. Similar to Alternative #3, firms with better projects are more likely to invest than firms with bad projects, suggesting that learning from prices
improves the firms’ decision. However, in the benchmark model, the slope of the investment decision with respect to \( \theta \) is scaled by a factor of \( \Phi(\gamma) \). This indicates that the amount of information revealed through the stock price depends on \( \gamma \). In particular, the slope around \( \theta = c \) measures the efficiency of the investment decision, or how well firms distinguish between good and bad investment projects. Figure 3 depicts the probability of investment for the three alternative models and the benchmark strategic model. When the firm does not learn from prices (Alternative #2), the firm decides solely according to its prior. When \( \mu_\theta > c \), all firms invest and there is no distinction between different types of projects. The investment probability is one for all values of \( \theta \), and the slope around \( \theta = c \) is zero. Under perfect information (Alternative #1), the probability of investment is one for \( \theta \geq c \) and zero otherwise. In this case, firms perfectly differentiate between positive and negative NPV projects. The slope is undefined around \( \theta = c \), but one can think of it as infinity. For intermediate cases, a steeper slope around \( \theta = c \) indicates that managers are making better investment decisions. The main take away from this figure is that the slope around \( \theta = c \) rather than the level of the probability of investment (\( \psi(c) \)) measures investment efficiency in the model.

**Definition 4.** Investment efficiency is defined as the slope of the probability of investment with respect to \( \theta \) around the point \( \theta = c \):

\[
\frac{\partial \psi(\theta)}{\partial \theta} \bigg|_{\theta=c}
\]

**Corollary 3.** If \( \mu_\theta > c \), the investment decision is less efficient with strategic traders than in the non-strategic alternative, i.e.

\[
\frac{\partial \psi^3(\theta)}{\partial \theta} \bigg|_{\theta=c} > \frac{\partial \psi^\ast(\theta)}{\partial \theta} \bigg|_{\theta=c}.
\]

Proof in Appendix C.

Corollary 3 refers to the fact that the slope of \( \psi(\theta) \) around \( \theta = c \) is greater in Alternative #3 than in the model with strategic behavior, as shown in Figure 3 for a particular set of parameters. More precisely, learning from prices improves investment efficiency in both models, but this improvement is smaller in the strategic model.

In a standard signal extraction model, an uninformed manager is more likely to rely on the outside signal to learn about the fundamental. In such a case, one would expect that managers with less precise prior information rely more on the stock price to make their investment decision. In other words, the sensitivity of investment to the stock price should be higher for less informed managers. To study whether that intuition still holds in the benchmark model, I calculate the correlation between the expected stock price and the investment probability.

**Definition 5.** The correlation between the expected stock price and probability of investment is defined
as follows:

$$\text{Corr}(q, \psi) = \frac{1}{\epsilon} \int \left[ X_I(\theta) - \bar{X}_I \right] \left[ \psi(\theta) - \bar{\psi} \right] d\xi(\theta)$$

where \( \frac{1}{\epsilon} \bar{X}_I \) and \( \bar{\psi} \) are averages of the expected price and investment probability respectively, taken with respect to the space of fundamentals \( \theta \).

**Corollary 4.** The correlation between the expected stock price and the probability of investment is:

1. **In the benchmark model:**
   $$\text{Corr} (q^*, \psi^*) = \frac{1}{2} \Phi(\gamma) \phi\left( \frac{\gamma}{\Phi(\gamma)} \right) \left[ \sigma_\theta^2 + (\mu_\theta - c)^2 \right]$$

2. **In the non-strategic model:**
   $$\text{Corr} (q^*, \psi^*) = \frac{1}{2} \phi(\gamma) \left[ \sigma_\theta^2 + (\mu_\theta - c)^2 \right]$$

where \( \Phi() \) and \( \phi() \) are the cumulative and probability distribution functions of the standard normal respectively and \( \gamma = \frac{2(\mu_\theta - c)\sigma_z}{\epsilon \sigma_\theta^2} \). Proof in Appendix C.

Corollary 4 implies that, all else equal, the correlation between stock prices and investment is increasing in \( \sigma_\theta \) in both the benchmark model and the non-strategic alternative, as expected from the standard intuition discussed earlier. That is, managers with lower quality of information a priori are more likely to rely on their own stock price to make investment decisions. However, Corollary 4 implies that the correlation between investment and stock prices increases less with respect to managerial uncertainty \( (\sigma_\theta) \) in the strategic model than in the non-strategic alternative, i.e.

$$\frac{\partial \text{Corr}(q^*, \psi^*)}{\partial \sigma_\theta} < \frac{\partial \text{Corr}(q^3, \psi^3)}{\partial \sigma_\theta}.$$  \(^{12}\)

Two salient features of the equilibrium drive this result. First, informed trading volume is lower when managers are less informed about the fundamental. Second, less informed firms increase the cutoff price, but by less in the strategic case than in the non-strategic alternative, because they internalize that prices are less informative. In summary, in the strategic model managers rely less on the stock price to make the investment decision than when informed speculators fail to internalize the learning channel. Figure 4 presents the correlation between the stock price and the investment probability for different levels of managerial uncertainty, for the benchmark model and for alternative 3. \(^{13}\) The correlation is increasing in managerial uncertainty for both cases, but less so when traders internalize the firms’ learning from prices.

To summarize, the model has three main implications. (i) For lower quality of managers’ information a priori, there is less trading volume by informed speculators and lower price informativeness (Corollary 1). (ii) The investment decision is less efficient when traders internalize the fact that firms learn from

\(^{12}\)Another implication of Corollary 4 is that the correlation between the expected stock price and the probability of investment is smaller in the benchmark model when traders are strategic than in the non-strategic model: \( \text{Corr}(q^*, \psi^*) = \Phi(\gamma) \phi\left( \frac{\gamma}{\Phi(\gamma)} \right) \text{Corr}(q^3, \psi^3) \) and \( \Phi(\gamma) \phi\left( \frac{\gamma}{\Phi(\gamma)} \right) \leq 1 \)

\(^{13}\)The figure presents exact numerical solutions.
prices (Corollary 3). (iii) The correlation between expected stock price and investment is smaller when informed speculators behave strategically (Corollary 4).

4. Empirical Evidence

In this section I present empirical evidence on the connection between ex-ante managerial uncertainty about firms’ fundamentals and stock price informativeness. I also study how the correlation between investment and stock prices varies for different levels of managerial information. The empirical analysis that follows is based on a sample of U.S. public firms from 1990 to 2010. For each firm I construct two measures of managerial information and uncertainty, and one measure of stock price informativeness. These measures are described below.

4.1. Managerial Information and Uncertainty

When making corporate decisions, managers gather information about the outlook of their firms and the profitability of new products and projects. In the model outlined above, uncertainty about a firm’s fundamentals refers to the variance of the firm’s prior distribution of future profits. To measure firm uncertainty about fundamentals, one would need to know not only the firm’s point estimates of expected profits but the entire distribution. To my knowledge there are no surveys at the firm level with probability distributions on future earnings. For this reason, I rely on two proxies to measure managerial uncertainty about the outlook of their firm.

The first measure is based upon analysts’ earnings forecasts. At the firm level, surveys of analysts’ forecasts typically report first moments, e.g. expected earnings, profits or sales. As a proxy for firm uncertainty I instead use dispersion of analysts’ earnings forecasts from the Institutional Brokers Estimate System (IBES). Empirical evidence suggests that a large fraction of the information used by analysts comes from discussions with firm managers, which also suggests that analysts’ information is not news to the firm (Bailey et al. (2003)). Analysts collect information from each firm and issue their own forecast. The assumption is that managers with more precise information are more likely to convey such information to analysts covering the firm, and thus, one would expect less disagreement in the analysts’ forecasts. On the contrary, more uncertainty about fundamentals is likely to be reflected in more disagreement in the forecasts issued by the analysts covering a firm.\(^\text{14}\) The caveat, of course, is that consensus among

\(^{14}\text{For instance, analysts might be talking to different managers within a firm, and dispersion among analysts’ forecasts could thus reflect disagreement within the firm. Alternatively, disagreement among analysts could reflect precisely the fact }$
analysts need not imply a high degree of confidence in their point estimates. However, there is a large body of literature that has studied forecast dispersion, and on balance these studies confirm that forecast dispersion is a useful proxy for uncertainty.\footnote{Earlier papers using forecast dispersion to proxy for uncertainty include Bomberger and Frazer (1981), Lambros and Zarnowitz (1987) and Barron and Stuerke (1998). Using the Survey of Professional Forecasters (SPF), Lambros and Zarnowitz (1987) show a positive correlation between forecast dispersion and uncertainty, where uncertainty is proxied by the spread of the probability distribution of point forecasts. IBES distributes only point forecasts, but the SPF provides both point forecasts and the histogram of forecasts for GDP, unemployment, inflation, and other major macroeconomic variables. In recent papers, Avramov et al. (2009) and Guntay and Hackbarth (2010) study forecast dispersion as a measure of uncertainty about firms’ future earnings. Guntay and Hackbarth (2010) find that dispersion is positively associated with credit spreads, and it appears to proxy largely for future cash flow uncertainty.}

For each firm, I construct this proxy for uncertainty using all the forecasts issued by analysts within a fiscal year. Following Gilchrist et al. (2005), dispersion is defined as the logarithm of the fiscal year average of the monthly standard deviation of analysts' forecasts of earnings per share, times the number of shares, scaled by the book value of total assets. That is,

\[
DIS_{i,t} = \log \left( \frac{\sum_{j=1}^{12} N_{tj} SD_{tj}/12}{ASSETS_{i,t}} \right)
\]

(10)

where \( t \) and \( j \) denote year and month respectively. \( N_{tj} \) is the number of shares outstanding, and \( SD_{tj} \) is the standard deviation of the per-share earnings forecasts for all analysts making forecasts for month \( j \).

The second measure of managerial information quality is based on insider trading activity (Chen et al. (2007) and Foucault and Fresard (2013)). Managers should be more likely to trade their own stock and make a profit on these trades if they are more confident in their information. Although managers don’t always trade on information, the premise is that on average, managers with better information will trade more. To build this proxy for managerial information, I obtain corporate insiders’ trades from the Thomson Financial Insider Trading database. I measure the quality of managers’ information with the intensity of a firm’s insider trading activity, \( INSIDER_{it} \), calculated as the ratio of the firm’s shares traded by insiders in a year to the total number of firm shares traded. As in other studies I only include open market stock transactions initiated by the top five executives (CEO, CFO, COO, President and Chairman of the Board).\footnote{Foucault and Fresard (2013) and Peress (2010).} Since \( INSIDER_{it} \) is a measure of absolute insider trading activity, it captures managerial information but not the direction of such information, that is, whether the firm has a positive or a negative outlook. My second proxy for the firm’s uncertainty about fundamentals is \( 1 – INSIDER \). While insider trades may reveal managers’ firm-specific information not embodied in share prices, a potential drawback to this proxy for uncertainty is that the lack of insider trading might
simply indicate that market prices are close to insider’s beliefs about the fundamentals of a firm, rather than indicating low precision of managerial information. Nonetheless, we should expect better informed managers who are more confident about the quality of their information to trade more.

4.2. Price Informativeness

To measure the amount of firm specific information contained in stock prices I use price nonsynchronicity. Specifically, I measure price informativeness for a firm as the share of its daily stock return variation that is firm-specific, defined as $PI_{it} = 1 - R_{it}^2$, where $R_{it}^2$ is the $R^2$ from the regression in year $t$ of firm $i$’s daily returns on market and industry returns.\textsuperscript{17} The idea, first suggested by Roll (1988), is that trading on firm-specific information makes stock returns less correlated in the cross-section and thereby increases the fraction of total volatility due to idiosyncratic shocks.\textsuperscript{18} This measure is related to price informativeness in the model, in that increased informed trading volume should increase the idiosyncratic volatility of a firm’s stock price.

Firms are matched to their specific three digit SIC industry. I exclude firm-year observations with less than $10$ million book value of equity or with less than 30 days of trading activity in a year. I used CRSP data to measure stock returns and Compustat to measure book values. I exclude firms in financial industries (SIC code 6000-6999) and utility industries (SIC code 4000-4999). The final sample consists of an unbalanced panel with 5,607 firms and 33,610 firm-year observations of uncertainty and price informativeness between 1990 and 2010. I detail the construction of all the variables in Table 1 and Table 2 presents summary statistics. To reduce the effect of outliers all variables are winsorized at 1% in each tail. Finally, I scale all variables by their standard deviation so that the estimated coefficients are directly informative about the economic significance of the effects.

4.3. Empirical Methodology

To estimate the relationship between stock price informativeness and fundamental uncertainty, I consider the following baseline specification:

$$PI_{it} = \alpha_i + \delta_t + \beta {UNCER}_{it} + \gamma X_{it} + \epsilon_{it},$$  \hfill (11)

\textsuperscript{17}The market index and industry indices are value-weighted averages excluding the firm in question. This exclusion prevents spurious correlations between firm and industry returns in industries that contain few firms.

\textsuperscript{18}This measure has been used extensively in the literature studying feedback between prices and managerial decisions. See for example Durnev et al. (2004), Chen et al. (2007) and Foucault and Fresard (2013).
where the subscripts $i$ and $t$ represent respectively firm $i$ and year $t$. The dependent variable is price nonsynchronicity, my proxy for price informativeness. The explanatory variable $UNCER_{it}$ measures managers’ uncertainty about fundamentals as captured by one of the proxies discussed in subsection 4.1. The vector $X$ includes control variables such as firm size, measured as the natural logarithm of the book value of assets, level of cash flows and the number of analysts issuing forecasts for each firm. In addition, I account for time-invariant firm heterogeneity by including firm fixed effects ($\alpha_i$) and time-specific effects by including year fixed effects ($\delta_t$). The coefficient $\beta$ measures how managerial uncertainty about fundamentals is related to stock price informativeness over time and across firms.

Table 3 reveals that the coefficients on the dispersion between analysts’ earnings forecasts and insider trading are significantly negative in all specifications. While this does not establish causality, it suggests a robust negative correlation between uncertainty and price nonsynchronicity even after controlling for year and firm fixed effects. Of course this interpretation depends on the assumption that analysts’ earnings forecast dispersion and insider trading capture the quality of managers’ information. That is, to the extent that forecast dispersion and insider trading capture managers’ uncertainty about the firms’ fundamentals, prices seem to be less informative about firm-specific information when the precision of managers’ information is low.

In the model, the negative relationship between uncertainty and price informativeness is derived from two main assumptions. The first is that there are feedback effects from prices to firms’ decisions. The second is that both traders and firms are strategic. This provides a potential test exploiting a priori cross-sectional differences in the relationship between uncertainty and price informativeness. In particular, institutional investors are typically better informed than individual investors, and their trades are more likely to have price effects. This suggests that stocks with larger institutional ownership should exhibit a stronger negative correlation between uncertainty and price informativeness. I test this hypothesis by adding to the baseling regression the interaction between the measure of uncertainty and the percentage of shares held by institutional investors in any given stock ($UNCER_{it} \times INST_{it}$)

$$PI_{it} = \alpha_i + \delta_t + \beta_0UNCER_{it} \times INST_{it} + \beta_1UNCER_{it} + \beta_2INST_{it} + \gamma X_{it} + \epsilon_{it}. \quad (12)$$

Table 4 shows that the magnitude of the correlation between uncertainty and price informativeness is indeed greater for firms with a larger share of institutional ownership. The results are similar for both

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19Economies of scale imply that institutional investors can acquire information at a lower cost per share traded than individual investors.
proxies of firm fundamental uncertainty. Overall, the results suggest uncertainty and price informativeness are most strongly related for firms for which strategic behavior is most likely to be present. As shown in the model, such strategic behavior has important implications for how well markets reveal information, and for how firms use stock prices when making investment decisions.

4.4. Investment sensitivity to price

The model above suggested that there are limits to firms’ ability to use stock prices as a guide in making real investment decisions. This is because stock price informativeness is not exogenous with respect to the precision of the managers’ prior information. When a manager is less informed a priori, strategic informed traders optimally reduce their trading volume, making stock prices less informative about the fundamental. In turn, managers themselves end up relying less on the price signal relative to the alternative case when informed traders are non-strategic.

To test whether the quality of managerial information affects the sensitivity of investment to the stock price, I estimate a variant of a standard empirical investment equation as follows:

\[
I_{it} = \alpha_i + \delta_t + \beta_1 Q_{it-1} + \beta_2 Q_{it-1} \cdot UNCER_{it-1} + \gamma X_{it} + \epsilon_{it}
\]  

(13)

The dependent variable, \(I_{it}\), is the ratio of capital expenditures in that year to lagged fixed assets. Following other studies on the sensitivity of investment to stock price, I use Tobin’s average Q as a proxy for a firm’s market value. Average Q is defined as a firm’s stock price times the number of shares outstanding plus the book value of assets minus the book value of equity, divided by the book value of assets. The vector \(X\) includes control variables known to correlate with investment decisions such as cash flows and firm size. To test for the effect of the quality of managerial information on the relationship between investment and stock price, I interact lagged managerial uncertainty (\(UNCER_{it-1}\)) with Tobin’s Q. I also control for the direct effect of the quality of managerial information on investment. Following the specification in the previous section, I include firm fixed effects and year fixed effects.

Results are presented in Table 5. While the coefficient \(\beta_2\) is positive for both proxies of managerial uncertainty, these coefficients are not statistically significant, suggesting that there are no differences in the sensitivity of investment to stock prices for different levels of managerial information. Recall that the model did predict a higher correlation between stock prices and investment for less informed managers (as stock prices do contain some private information not possessed by managers), but this interaction is
limited in the benchmark model compared to the case when traders fail to internalize that firms learn from prices.

In columns (1) and (3) I control for stock price informativeness and its interaction with Q. As in previous studies (Chen et al. (2007), Bakke and Whited (2010) and Foucault and Fresard (2013)) I find that a firm’s investment is more sensitive to Tobin’s Q when its stock price is more informative. These results suggest that firm managers learn from the private information aggregated into the stock price when making investment decisions, but market prices does not seem to provide more guidance to managers with low quality of information.

5. Conclusions

In this paper I find limits to the ability of secondary markets to inform and guide firms’ real investment decisions. The economy is modeled as a strategic game between firms and informed speculators. Before making an investment choice, firms use stock prices to update their priors about their own fundamentals. Informed traders are strategic in that they internalize the firms’ inference problem. In this setting, I show that informed trading volume depends on the quality of managers’ prior information. In other words, the amount of private information that is aggregated into the stock price through the trading process is a function of managers’ initial information. Learning from prices is limited because prices are endogenously less informative for firms with low quality of managerial information, which are precisely those firms that would like to learn more from the stock market in the first place. In turn, real investment efficiency falls as the market signal is not as useful in helping managers differentiate between good and bad projects.

Using a sample of U.S. publicly traded companies, I document a positive correlation between the quality of managers’ information and stock price informativeness. I show that less informed managers do not rely more on the stock price to make investment decisions. Collectively, the evidence is suggestive of limits to the ability of firms to learn relevant information from stock prices.

The model presented here may have implications for firms’ decision about whether to be financed through public or private equity. More specifically, depending on the information gap between traders and the firm’s managers, the firm might benefit from an IPO or might be better off being held privately. In the same way, the quality of managers’ information should determine the choice of outside speculators to either become private or public equity investors. When managers have low quality information, speculators’ trading profits are limited, despite knowing the true value of the fundamental. In this case, the speculator
might be better off being a private equity investor in the firm, which would allow it to participate directly in the investment decision. On the contrary, when managers’ prior information is good, speculators’ trading profits are potentially large, and they can fully benefit from their information by trading on secondary markets. A further analysis of the links between managerial information, outsider information and the optimal form of equity finance is left for future work.

References


A. Appendix: Tables

Table 1
Variable definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - R^2$</td>
<td>Price informativeness, defined as one minus $R^2$ from regressing a firm’s daily returns on market and industry indices over year $t$</td>
<td>CRSP</td>
</tr>
<tr>
<td>$SIZE$</td>
<td>Logarithm of the book value of assets</td>
<td>Compustat</td>
</tr>
<tr>
<td>$INSIDER$</td>
<td>Ratio of firm’s shares traded by insiders in a given year to the total number of shares traded (in percentage terms). Insider traders refers to open market stock transactions by the top five executives (CEO, CFO, COO, President and Chairman of the board)</td>
<td>Thompson Financial Insider Trading Database and CRSP</td>
</tr>
<tr>
<td>$INST$</td>
<td>Percentage of shares held by institutional investors</td>
<td>Thompson Financial</td>
</tr>
<tr>
<td>$ANALYST$</td>
<td>Number of analysts issuing forecasts for each stock</td>
<td>IBES</td>
</tr>
<tr>
<td>$DIS$</td>
<td>Analyst forecast dispersion, defined as the natural logarithm of the standard deviation of analysts’ forecasts of earnings per share, times the number of shares, scaled by the book value of total assets as in Gilchrist et al. (2005)</td>
<td>IBES and Compustat</td>
</tr>
<tr>
<td>$CF$</td>
<td>Cash flow, defined as net income before extraordinary items + depreciation and amortization expenses + R&amp;D expenses scaled by assets</td>
<td>Compustat</td>
</tr>
<tr>
<td>$Q$</td>
<td>Average $Q$, defined as $[\text{Book value of assets - book value of equity + market value of equity}] / \text{book value of assets}$</td>
<td>Compustat</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment rate, defined as the ratio of capital expenditures to lagged fixed assets</td>
<td>Compustat</td>
</tr>
</tbody>
</table>

Table 2
Summary statistics
This table reports the summary statistics of the main variables used in the analysis. For each variable I present its mean, standard deviation, 5th, 25th, 50th, 75th and 95th percentile. All variables are defined in Table 1. The sample period is from 1990 to 2010.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number of observations</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
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<tr>
<td>$1 - R^2$</td>
<td>34819</td>
<td>0.82</td>
<td>0.18</td>
<td>0.43</td>
<td>0.72</td>
<td>0.89</td>
<td>0.97</td>
<td>1.00</td>
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<td>34819</td>
<td>7.67</td>
<td>2.61</td>
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<td>7.30</td>
<td>9.37</td>
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<td>12.69</td>
<td>0.01</td>
<td>0.08</td>
<td>0.43</td>
<td>3.16</td>
<td>34.04</td>
</tr>
<tr>
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<td>0.52</td>
<td>0.26</td>
<td>0.09</td>
<td>0.31</td>
<td>0.53</td>
<td>0.73</td>
<td>0.93</td>
</tr>
<tr>
<td>$DIS$</td>
<td>34819</td>
<td>-3.05</td>
<td>1.12</td>
<td>-4.89</td>
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<td>-3.07</td>
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<td>-1.15</td>
</tr>
<tr>
<td>$CF$</td>
<td>34819</td>
<td>1627</td>
<td>2502</td>
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<td>17.02</td>
<td>141</td>
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</tr>
<tr>
<td>$Q$</td>
<td>35051</td>
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<td>2.16</td>
<td>0.93</td>
<td>1.14</td>
<td>1.63</td>
<td>6.31</td>
<td>8.31</td>
</tr>
<tr>
<td>$I$</td>
<td>24155</td>
<td>0.29</td>
<td>0.42</td>
<td>0.49</td>
<td>0.11</td>
<td>0.19</td>
<td>0.34</td>
<td>0.79</td>
</tr>
</tbody>
</table>
Table 3
Price informativeness and managerial information

Definitions of all variables are listed in Table 1. The dependent variable is Price Informativeness. Managers’ uncertainty is proxied by the dispersion of analysts’ earnings forecasts ($DIS$) and insider trading activity $1 - INSIDER$. T-statistics are in parentheses. ***/**/* indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Measure of Uncertainty</th>
<th>DIS</th>
<th>1 − INSIDER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty $UNCER_{it}$</td>
<td>-0.10***</td>
<td>-0.18***</td>
</tr>
<tr>
<td></td>
<td>(17.90)</td>
<td>(26.30)</td>
</tr>
<tr>
<td>Size $SIZE_{it}$</td>
<td>-0.41***</td>
<td>-0.45***</td>
</tr>
<tr>
<td></td>
<td>(55.13)</td>
<td>(27.87)</td>
</tr>
<tr>
<td>Cash flow $CF_{it}$</td>
<td>0.25***</td>
<td>0.34***</td>
</tr>
<tr>
<td></td>
<td>(32.97)</td>
<td>(18.91)</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.14</td>
<td>0.52</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>24753</td>
<td>24921</td>
</tr>
</tbody>
</table>

Table 4
Price informativeness and managerial information: Interaction with institutional ownership

Definitions of all variables are listed in Table 1. The dependent variable is Price Informativeness. Managers’ uncertainty is proxied by the dispersion of analysts’ earnings forecasts ($DIS$) and insider trading activity $1 − INSIDER$. SIZE and CF coefficients are omitted. T-statistics are in parentheses. ***/**/* indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Measure of Uncertainty</th>
<th>DIS</th>
<th>1 − INSIDER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty $UNCER_{it}$</td>
<td>-0.02***</td>
<td>-0.02***</td>
</tr>
<tr>
<td></td>
<td>(3.40)</td>
<td>(5.09)</td>
</tr>
<tr>
<td>Interaction $UNCER_{it} × INST_{it}$</td>
<td>-0.05***</td>
<td>-0.02***</td>
</tr>
<tr>
<td></td>
<td>(11.95)</td>
<td>(4.36)</td>
</tr>
<tr>
<td>$INST_{it}$</td>
<td>-0.22***</td>
<td>-0.22***</td>
</tr>
<tr>
<td></td>
<td>(28.30)</td>
<td>(27.38)</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.73</td>
<td>0.74</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>24753</td>
<td>24921</td>
</tr>
</tbody>
</table>
Table 5
Managerial information and investment sensitivity to price
Definitions of all variables are listed in Table 1. The dependent variable is investment. Managers’ uncertainty is proxied by the dispersion of analysts’ earnings forecasts (DIS) and insider trading activity $1 - \text{INSIDER}$. T-statistics are in parentheses. ***/**/ indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Dependent variable: $I$</th>
<th>DIS</th>
<th>1 – INSIDER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.215***</td>
<td>0.225***</td>
</tr>
<tr>
<td></td>
<td>(8.11)</td>
<td>(8.45)</td>
</tr>
<tr>
<td>$Q \times PI$</td>
<td>0.038***</td>
<td>0.041***</td>
</tr>
<tr>
<td></td>
<td>(2.88)</td>
<td>(3.24)</td>
</tr>
<tr>
<td>$Q \times UNCER$</td>
<td>0.012</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>$PI$</td>
<td>-0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$UNCER$</td>
<td>-0.023*</td>
<td>-0.025**</td>
</tr>
<tr>
<td></td>
<td>(1.84)</td>
<td>(2.03)</td>
</tr>
</tbody>
</table>

Controls: Yes
Firm fixed effects: Yes
Year fixed effects: Yes
$R^2$: 0.39
Number of Observations: 19009
B. Appendix: Figures

Figure 1. Timeline for each firm. Informed speculator (IS), Noise traders (NS).

Figure 2. Equilibrium demands by informed speculators The figure shows informed speculators' demands in the model with strategic behavior and learning from prices for different levels of managerial uncertainty (i.e. different $\sigma_\theta$). The figure presents numerical solutions with model parameters as follows: $c = 1$, $\mu_\theta = 1.05$, $\sigma_z = 1$ and $\epsilon = 1$. 
Figure 3. **Probability of investment.** Probability of the firm undertaking the project under perfect information (alternative #1), no learning from prices (alternative #2), no strategic behavior (alternative #3) and strategic behavior. The parameters are $c = 1$, $\mu_\theta = 1.05$, $\sigma_\theta = 1$, $\sigma_z = 1$, $\epsilon = 1$, $c = 1$ and $\epsilon = 1$.

Figure 4. **Correlation between the stock price and the probability of investment.** Correlation between price and investment for different levels of managerial uncertainty ($\sigma_\theta$) for the strategic and non-strategic (alternative #3) models. The parameters are $c = 1$, $\mu_\theta = 1.05$, $\sigma_z = 1$ and $\epsilon = 1$. 
C. Appendix: Proofs

**Proof of Lemma 1:** If the firm manager conjectures a linear demand by the informed speculator of the form \( X_I(\theta) = a + b\theta \), the posterior on the fundamental is:

\[
\xi(\theta | q) = \frac{\varphi(\epsilon q - a - b\theta)\xi(\theta)}{\int_{-\infty}^{\infty} \varphi(\epsilon q - a - b\theta)\xi(\theta)d\theta}.
\]

where \( \varphi() \) is density function of the normal distribution with mean 0 and variance \( \sigma^2_z \) and \( \xi() \) is the density function of a normal distribution with mean \( \mu_\theta \) and variance \( \sigma^2_\theta \). Under the posterior, the cutoff rule is given by

\[
\int_{-\infty}^{\infty} \theta \varphi(\epsilon q - a - b\theta)\xi(\theta)d\theta = c
\]

Using the properties of the normal distribution I solve for the cutoff price, \( q_1 = \frac{1}{\epsilon} \left[ a + cb - \frac{1}{2} \left( \mu_\theta - c \right) \sigma^2_z \right] \).

**Proof of Proposition 1:** The method of this proof is to iterate among best responses to find the fixed point in strategies.

**Step 1a.** Assume a linear function for the informed speculators’ demand \( X_I^0(\theta) = \frac{\epsilon(\theta - c)}{2} \). From Lemma 1, the firm’s best response (cutoff price) is

\[
\bar{q}_1 = -(\mu_\theta - c) \frac{2\sigma^2_z}{\epsilon^2\sigma^2_\theta}
\]

**Step 1b.** Using \( \bar{q}_1 \), I find the optimal decision rule of the informed speculator. This is the solution to the first order condition from profit maximization (5):

\[
(\theta - c) \left[ \Phi \left( \frac{X_I(\theta) - \epsilon \bar{q}_1}{\sigma_z} \right) + \frac{1}{\sigma_z} X_I(\theta) \Phi' \left( \frac{X_I(\theta) - \epsilon \bar{q}_1}{\sigma_z} \right) \right] - \frac{2}{\epsilon} X_I(\theta) = 0
\]

For the reasons described in the main text, in this paper I am interested in the solution around \( \theta = c \). Linearizing the FOC around \( \theta = c \), I can guess and verify that the demand function of the informed speculators is of the form \( X_I(\theta) = k(\theta - c) \). The first order approximation of the FOC is:

\[
(\theta - c) \Phi \left( -\frac{\epsilon \bar{q}_1}{\sigma_z} \right) - \frac{2k(\theta - c)}{\epsilon} = 0
\]

Solving for \( k \), \( k = \frac{\epsilon \Phi(\gamma)}{2} \), where \( \gamma = \frac{2(\mu_\theta - c)\sigma_z}{\epsilon\sigma^2_\theta} \). Informed speculators’ demands are:
\[ X_1^1(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma) \]

**Step 2a.** Using the linear demands above \( X_1^1(\theta) \), I solve for the firm’s cutoff rule.

\[ \bar{q}^2 = - (\mu_\theta - c) \frac{2\sigma^2_z}{\epsilon^2 \sigma^2_\theta} \frac{1}{\Phi(\gamma)} \]

**Step 2b.** The iteration continues by assuming \( \bar{q}^2 \) to find the optimal decision rule of the informed speculator. Following the linearization in step 1b, the informed speculators’ demands are:

\[ X_1^1(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma) \frac{1}{\Phi(\gamma)} \]

Continuing the iteration procedure, in the \( n \)-th iteration the firm’s cutoff rule and the speculators’ demands are:

\[ \bar{q}^n = - (\mu_\theta - c) \frac{2\sigma^2_z}{\epsilon^2 \sigma^2_\theta} \frac{1}{f_{n-1}(\gamma)} \]

\[ X_1^n(\theta) = \frac{\epsilon(\theta - c)}{2} f^n(\gamma) \]

where \( f^n(\gamma) \) is a continued fraction of cumulative normal distributions of the form:

\[ f^n(\gamma) = \Phi \left( \frac{\gamma}{\Phi \left( \frac{\gamma}{\Phi \left( \frac{\gamma}{\cdots} \right)} \right)} \right) \quad (16) \]

**Lemma 2.** If \( \gamma \geq 0 \), the infinite continued fraction \( f^{(\infty)}(\gamma) \) converges to \( \Phi(\gamma) \). That is, \( \lim_{n \to \infty} f^n(\gamma) = \Phi(\gamma) \). If \( \gamma < 0 \), \( \lim_{n \to \infty} f^n(\gamma) = 0 \).

Using Lemma 2, the fixed point in strategies has the following form:

- If the expected NPV of the investment project is non-negative under the firm’s prior, \( \mu_\theta \geq c \) then
the equilibrium demands by speculators and firms’ cutoff rule are:

\[ X^*_f(\theta) = \frac{\epsilon(\theta - c)}{2} \Phi(\gamma) \]
\[ \bar{q}^* = -\left(\mu_\theta - c\right) \frac{2\sigma_z^2}{\epsilon^2 \sigma_\beta^2} \frac{1}{\Phi(\gamma)} \]

- If the expected NPV of the investment project is negative under the firm’s prior, \( \mu_\theta < c \), then, informed speculators don’t trade in equilibrium \( X^*_f(\theta) = 0 \) and firms never invest, \( \bar{q}^* \to \infty \).

**Proof that** \( \frac{d\bar{q}^*}{d\gamma} < 0 \): From equation (8) it is sufficient to prove that \( g(\gamma) \equiv -\Phi(\gamma) + \gamma \Phi'(\gamma) < 0 \) for \( \gamma \geq 0 \). First, note that \( g() \) is monotonically decreasing in \( \gamma \), \( g'(\gamma) = \Phi''(\gamma) < 0 \). Also \( g(0) = -\frac{1}{2} \). Hence \( g(\gamma) < 0 \).

**Proof of Corollary 2**: Replace the equilibrium demands by the informed speculators and firms’ cutoff rule in the probability of investment: \( Pr(q \geq \bar{q} \mid \theta) = \Phi \left[ \frac{1}{\sigma_z} (X^*_f(\theta) - \epsilon \bar{q}^*) \right] \).

**Proof of Corollary 3**: Investment efficiency when informed speculators internalize the firm’s updating process is:

\[ \left. \frac{\partial \psi^*(\theta)}{\partial \theta} \right|_{\theta=c} = \frac{\epsilon}{2\sigma_z} \Phi(\gamma) \Phi' \left( \frac{\gamma}{\Phi(\gamma)} \right) \] (17)

Investment efficiency when informed speculators don’t internalize the firm’s updating process is:

\[ \left. \frac{\partial \hat{\psi}(\theta)}{\partial \theta} \right|_{\theta=c} = \frac{\epsilon}{2\sigma_z} \Phi'(\gamma) \] (18)

The \( g(\gamma) \) can be defined as the ratio between the investment efficiency measure in the non-strategic case and the strategic case:

\[ g(\gamma) \equiv \left. \frac{\partial \psi^{bs}(\theta)}{\partial \theta} \right|_{\theta=c} / \left. \frac{\partial \psi^*(\theta)}{\partial \theta} \right|_{\theta=c} \] (19)

This ratio is strictly decreasing in \( \gamma \) if \( \gamma > 0 \), that is

\[ g'(\gamma) = -\gamma \Phi'(\gamma) \Phi(\gamma) + \gamma^2 \Phi'(\gamma)^2 < 0, \text{ if } \gamma > 0 \]

Also, \( \lim_{\gamma \to \infty} g(\gamma) = 1 \). Combining these two results, for any finite level of precision of managerial information (i.e. \( \gamma \) finite) and \( \mu_\theta > c \), the investment decision is less efficient with strategic traders than in the non-strategic benchmark.
Proof of Corollary 4: The first order approximation of the probability of investment for a firm with fundamental $\theta$ around $\theta = c$ can be written as: $\psi^3(\theta) = \Phi(\gamma) + \phi(\gamma)(\theta - c)$ for the alternative case #3 and $\psi^*(\theta) = \Phi\left(\frac{\gamma}{\Phi(\gamma)}\right) + \phi\left(\frac{\gamma}{\Phi(\gamma)}\right)(\theta - c)$ for the benchmark model.

In the non-strategic model (Alternative #3), the correlation between the probability of investment and stock prices, following Definition 5, is:

$$\text{Corr}(q^3, \psi^3) = \int_{-\infty}^{\infty} \frac{\phi(\gamma)}{2} (\theta - c)^2 d\xi(\theta)$$

Integrating above I obtain $\text{Corr}(q^3, \psi^3) = \frac{\phi(\gamma)}{2} \left[ \sigma^2_{\theta} + (\mu_{\theta} - c)^2 \right]$. Similarly, for the strategic model I use the linear approximation of the investment probability and integrate to calculate the correlation between prices and investment.