Measuring the Impact of Debt-Financed Public Investment

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Abstract

While debt-financed productive public investment raises a country’s debt ratios in the short run, it can also generate higher growth, revenues, and exports, leading over time to lower debt ratios. This paper develops a framework to assess whether countries meet the conditions for realizing the net benefits over the costs of public investment debt financing. While it is possible to achieve debt sustainability with an appropriate mix of concessional and non-concessional financing, this is a necessary but not sufficient condition. It is also important to ensure the operational viability of public investment projects by having in place adequate project management: (i) project screening and appraisal, (ii) a clear connection between capital and recurrent expenditures once the projects are launched, and (iii) safeguards for appropriate project implementation and facilities operations. To illustrate the strength of these results, the paper carries out three measurement exercises: (a) a simulation of the degree to which the ratio of optimal public investment responds to changes in key parameters related to project management in a general equilibrium model; (b) application of the public investment management (PIMa) index to benchmark a country’s public investment management capacity; and (c) presentation of the results of the Investment, Savings, and Macroeconomic Vulnerabilities tool aimed at tracking country choices in public finance and the impact of public projects on private investments.

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Measuring the Impact of Debt-Financed Public Investment\textsuperscript{1}

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\textsuperscript{1} This paper is a product of Economic Policy and Debt unit (PRMED) in the Economic Policy, Debt and Trade (PRMET) Department. It is part of a larger effort by the World Bank to shed better light on this interaction between debt-financed public investments, reforms designed to improve public investment management capacity, and debt sustainability.

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I. Introduction

Recent years have seen a surge in non-concessional borrowing for public investment by post-HIPC\(^3\) countries, in particular for infrastructure investment projects. Examples include new non-concessional borrowing by Ghana, Ethiopia, Tanzania and Zambia. Ghana was the forerunner in 2007, issuing a $750 million Eurobond, and signing a $292 million loan agreement with the China Ex-Im Bank. The proceeds of these non-concessional loans were initially assigned entirely for energy and transport sector investment projects, with the China Ex-Im Bank loan allocated specifically to the hydroelectric power plant at Bui. At end-2011, Ghana and the China Development Bank negotiated a $3 billion “Master Financial Agreement” to finance major infrastructure projects. Ethiopia followed these footsteps, signing three non-concessional loan agreements with China, Egypt and France for electricity transmission and generation, totaling just over $1.1 billion for the period 2009-10, as well as $833 million borrowed from the China Ex-Im Bank and from the China Development Bank for projects in industrial capacity building, as well as in maritime and railway transport. In July 2012, Tanzania announced a US$1.25 billion agreement between the Tanzanian and Chinese governments to finance construction of a gas pipeline from Mtwara to Dar es Salaam.\(^4\) Under this agreement, Chinese contractors will deliver the pipeline as a turnkey project to the Tanzania Petroleum Development Corporation (TPDC). Zambia borrowed $1.25 billion on non-concessional terms since the conclusion of its IMF program in June 2011, and issued a $750 million sovereign bond in September 2012. These debts have been incurred largely to finance infrastructure investment in priority sectors, such as energy and transport, identified in these countries’ National Development Plans. The rationale for undertaking new debt-financed public investments was that these projects were needed to unlock growth opportunities for which there was limited traditional (donor) financing. As a result, the new financing plans in these countries now blend concessional and non-concessional debt.\(^5\)

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\(^3\) The ‘High Debt Poor Countries’ (HIPC) debt relief initiative.

\(^4\) This deal was announced at the Fifth Forum on China-Africa Cooperation (FOCAC) held in Beijing from July 19 to the 20\(^{th}\) 2012.

\(^5\) It is important, nevertheless, to place this increase in non-concessional borrowing for public investment in perspective. On average, non-concessional borrowing still accounts for a small, albeit rising share of external financing by countries that received debt relief. In 2011 non-concessional debt accounted for just under 11 percent of the overall borrowing by these countries, up from 9 percent in 2010, with the remaining almost-90 percent in the hands of official bilateral and multilateral creditors. What has changed is that, because of the non-concessionality of these financial flows, closer attention is being given to the financial viability of the public investment projects they are financing. This attention is important because despite the high expected social-economic rates of return from public investments, particularly in infrastructure, the projected revenues from these investments are not always able to cover the project costs, given existing structure of tariffs and regulations.
The scale of this increase in debt-financed public investments by developing countries has raised three important questions:

- **What will be the growth impact of these investments?** The debt sustainability impact of this new borrowing to finance public infrastructure investment depends critically on its impact on growth. A number of empirical studies find a positive impact of public investment on growth both through a direct impact on economic activity and through spillover effects on private investment. Buffie et al. (2012) find that a sustained increase in debt-financed investment by 1 percent of GDP can raise real GDP growth in developing countries with good policy implementation capacity by about 0.5 percentage points. Gupta et al. (2011) find that the unadjusted output elasticity of public capital is only 0.25. These findings are consistent with the results reported by Perotti (2005) and Zandi (2008), who find that fiscal multipliers for investment spending are higher than for other types of public spending and tax cuts. Also, a study by the World Bank (2007) concludes that there are positive growth effects of public spending in general, and that of infrastructure, education, and health spending in particular.

- **What are the risks entailed by this shift toward a blend of concessional and non-concessional borrowing?** The works by Buffie et al. (2012) and by Andrle et al. (2012) suggest that caution should be used when developing countries are considering the financing options for public investment. When an economy can borrow concessionally to cover only part of an investment surge, difficult fiscal and private sector adjustments are unavoidable in the short to medium term, especially when the investment surge is front-loaded and the structural conditions of the economy are weak. While allowing for external commercial borrowing by less developed countries may mitigate some of these adjustment costs, the presence of weak structural and policy conditions, as well as unexpected exogenous shocks, can lead to debt sustainability problems. These external debt sustainability problems are magnified when considering domestic non-concessional borrowing. Since domestic commercial borrowing does not provide additional external resources, it requires even more drastic fiscal adjustments and worsens the crowding out

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8 The authors present an alternative measure of efficiency-adjusted capital for which the output elasticity would be around 0.25 (IMF, Working Paper No. 217, 2011).
9 Bilateral and multilateral agencies have traditionally been the main sources of financing for public investment projects in developing countries. With the support from its shareholders, these agencies provided debt relief to several developing countries over the last years with a view to ensure their improved debt sustainability and give them the fiscal space needed to carry-out new investments. These multilateral financing sources are increasingly strained, however. Some agencies are reaching their lending limits for individual countries, and are unable to meet all the demand from these countries to finance investments designed to close their infrastructure gaps. The increase in non-concessional financing flows to developing countries reflects this fact, as well as two other developments. The policy of induced low interest rates and quantitative easing in high income countries that has prompted a search for yield by global investors, seeking to open or guarantee access to natural resources. The cross-border bank lending tends to be cheaper than financing from international capital markets, with information asymmetries playing an important role in terms of access and costs. These banks have a closer relationship with borrowers and therefore have an advantage in monitoring creditworthiness. High risk borrowers gain access to financing with projects backed by well-defined revenue streams, usually based on natural resource revenue flows.
effect on the private sector. In this paper we show, however, that debt-financed public investment can be consistent with debt sustainability, provided that two conditions are satisfied: the net present value of all government debt must equal zero over all future periods (i.e., Ponzi-games are ruled out); and the net present value of primary deficits plus the value of initial debt must equal zero. To satisfy this second condition, public expenditures and investments need to follow an active rule that responds inversely to the stock of outstanding public debt, otherwise, debt would follow an explosive path, increasing at a rate higher than real GDP growth. For the realization of this condition, the government must eventually generate a surplus in its fiscal accounts to offset the prior increased expenditures and restore debt - and economic - sustainability. During this transition between carrying out these investments and the eventual generation of an offsetting budget surplus, there is the very real risk that the public assets will be severely depleted, which may postpone the return to a debt sustainability path.

- How efficient is public investment spending in these countries? Assessing the efficiency of a country’s public investment spending is important because one cannot take the potential impact of public investment on growth for granted, given connections between cost-effective asset creation ex post and the conditional quality of project proposal screening, appraising and budgeting ex ante. This link is particularly important in the context of poor governance and rudimentary public investment systems. It is in this context that it is important to ensure proper project budgeting and long term integration of capital and recurrent budgeting (i.e., sufficient funds are provided for effective operation and maintenance of the assets generated by these upfront investments). Proper project budgeting can only be ensured in turn if the revenue streams (tax or tariff revenues) are secured or productive social assets are created in the most cost-effective manner.

It is against the backdrop of the issues raised by this new borrowing that this paper develops a framework aimed at shedding better light on this interaction between public investments, debt sustainability, and reforms designed to improve public investment management capacity. To attain this general goal, the paper presents and illustrates with examples a dynamic general equilibrium framework, a cross-country index of Public Investment Management (PIMa) and a tool of Public Investment, Saving and Macroeconomic Vulnerability (PRISM). Section II outlines the dynamic general equilibrium model to specify the determinants of optimal public investment-to-GDP ratio and the conditions for the compatibility between optimal public investment policy and debt sustainability. Section III illustrates the strengths of this framework by carrying out three measurement exercises: (a) simulating the degree to which the ratio of optimal public investment responds to changes in key parameters of the model; (b) applying the PIMa index to benchmark a country’s public investment management capacity; and (c) presenting the results of the PRISM tool aimed at tracking country choices in public finance and their impact on private investments. Section IV closes the paper with some concluding observations and the proposal of future extensions.
II. Modeling the Impact of Debt-Financed Public Investment: A General Equilibrium Model

This section outlines an optimal dynamic general equilibrium model designed with two purposes in mind: (i) to illustrate the impact that changes in certain parameters related to public investment management capacity have on the optimal public investment-to-GDP ratio; and (ii) to demonstrate the conditions required for the compatibility between an optimal public investment policy and debt sustainability. The framework extends Marrero (2008, 2010) to capture the possibility of the government issuing debt to finance public investment projects (Bohn, 1995, 1998). This framework allows us to simultaneously characterize the conditions required for an optimal public investment policy and for debt sustainability. Moreover, it allows direct attention to two features that are relevant in project management in developing country settings: (i) the depreciation rate of public capital, which tends to be higher due to delays in maintenance; and (ii) the low elasticity of intertemporal substitution in consumption and low discount rate, since there are strong political demands to use a share of revenues from resource windfalls to meet the population’s more immediate consumption needs.

The economy is populated by a continuum of infinitely-lived, identical households and firms, with their sizes constant and normalized to be unity, and a government that uses income taxes and debt issues to finance public investment and other non-productive public expenditure concepts. Capital markets are perfect, labor is supplied inelastically and normalized to 1 and zero population growth is assumed. The model restricts attention to Cobb-Douglas technology and constant elasticity of substitutions (CES) utility because these functional forms are needed for the existence of steady-state equilibrium. In what follows, we drop the time subscript $t$ when it is unambiguous, and all variables are defined in per capita terms.

Firms and Technology

Each firm produces the single, non-storable consumption good in the economy, $y$, according to the following Cobb–Douglas technology: 

$$f(k, l, z, g) = A_0 k^{\alpha} l^{\theta} z^\varphi g^\theta,$$

where $k$ is private capital stock, $l$ is labor, $g$ is per capita public capital, and $z$ is an index of knowledge available to each firm; both, $g$ and $z$ augment the productive capacity of labor. $A_0$ is a technological scale factor, and $\alpha$, $\theta$, and $\varphi$ are the elasticities of output with respect to $g$, $k$, $l$ and $z$, respectively. The function $f(\cdot)$ is increasing, strictly concave and twice continuously differentiable, and all factors are essential in the production process and satisfy Inada conditions.

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10 Optimality is understood here in a policy-constrained Ramsey sense.
11 Similar technologies were assumed by Cassou and Lansing (1998) and Marrero and Novales (2005, 2007).
12 Inada conditions (named after Japanese economist Ken-Ichi Inada) are assumptions about the shape of a production function that guarantee the stability of an economic growth path in a neoclassical growth model.
Because investment decisions are made by households, the firm’s problem is static; firms demand \( k \) and \( l \), whereas \( g \) and \( z \) are taken as exogenous variables. Each firm pays the competitively-determined wage, \( w \), on the labor it hires and the rate \( r \) on the capital it rents. Furthermore, since \( f(\cdot) \) shows constant returns to scale in private factors - \( k \) and \( l \) -, profits are zero and optimally leads to the usual marginal productivity conditions:

\[
\begin{align*}
    r &= \alpha f(\cdot)/k \\
    w &= (1-\alpha) f(\cdot).
\end{align*}
\]

(2) (3)

Following Romer (1986), the average capital stock across firms is taken as a proxy for the knowledge index available to each single firm. While firms decide on \( k \), the other two variables \( g \) and \( z \) are outside of their control. Since firms are assumed to be identical, the average capital in the economy is equal to \( k \) at equilibrium (i.e., \( z=k \)), and overall per capita production is given by:

\[
y = F(k, g) = f(k, k, g) = A_k k^{\alpha+\varphi} g^{\varphi}.
\]

(4)

Since each firm neglects its own contribution on the aggregate capital stock, notice that \( \partial F(\cdot)/\partial k \) and \( \partial f(\cdot)/\partial k \) are not equal: \( \partial f(\cdot)/\partial k = \alpha y/k \) while \( \partial F(\cdot)/\partial k = (\alpha + \varphi) y/k \), i.e., the social return to accumulated capital is higher than the private return.

**Households**

There exist a large number of identical infinitely lived households that allocate resources between consumption, \( c \), and investment in physical capital, \( i \). Every period consumption is valued according to a Constant Elasticity of Substitutions (CES) utility function:

\[
\sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-1/\sigma} - 1}{1-1/\sigma} \right], \quad \sigma > 0, \sigma \neq 1,
\]

(5)

\[
\sum_{t=0}^{\infty} \beta^t \ln(c_t), \quad \sigma = 1,
\]

where \( \beta \) is the discount factor, between zero and one, and \( 1/\sigma \) is the constant elasticity of intertemporal substitution (EIS), also between 0 and 1. Private capital accumulates over time according to the following expression:

\[
k_{t+1} = (1-\delta)k_t + i_t; \quad t = 0,1,\ldots,
\]

(6)

where \( \delta \) is a linear depreciation rate of private capital, between zero and one. Because \( \delta<1 \), this specification allows for long-lived private capital, which is a relevant and realistic element in the model.

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13 The Cobb-Douglas for technology and the CES for utility are functional forms required for the existence of a balance growth path in this economy.
The budget constraint faced by the household is

\[ c_t + k_{t+1} + (b_{t+1} / R_t) \leq w_t (1 - \tau_t) + k_t \left[ 1 - \delta + r_t (1 - \tau_t) \right] + b_t, \]  

(7)

where \( \tau_t \) is a tax rate applied to total household income, \( b_t \) is the government indebtedness to the private sector (public debt), denominated in time \( t \)-goods and maturing at the beginning of the period \( t \), \( R_t \) is the gross rate of return on one-period bond held from \( t \) to \( t + 1 \) and interest earnings on bonds are assumed tax exempt [Ljungqvist and Sargent (2000)]. Households cannot accumulate an arbitrary large amount of assets (private and public debt) to ensure that the model precludes the existence of Ponzi schemes; hence, the following transversality condition must be satisfied:

\[
\lim_{T \to \infty} \left( \prod_{i=0}^{T-1} R_t \right) k_{T+1} = 0, \tag{8}
\]

\[
\lim_{T \to \infty} \left( \prod_{i=0}^{T-1} R_t \right) b_{T+1} = 0. \tag{9}
\]

The representative household maximizes (5) subject to (7), \( c_t \geq 0 \), \( k_{t+1} \geq 0 \), and the Non-Ponzi Game (NPG) condition. Corner solutions are avoided from this problem and restrictions hold with equality because of the special form of the utility function and the fact that \( c_t \) is a normal good. Optimal conditions are standard (for all \( t=0, 1, \ldots \)): the consumption-saving decision (the Ramsey-Keynes condition):

\[
(c_{t+1} / c_t)^{\sigma} = \beta \left[ 1 - \delta + r_{t+1} (1 - \tau_{t+1}) \right], \tag{10}
\]

and the non-arbitrage condition between \( k_{t+1} \) and \( b_{t+1} \) is:

\[
R_t = 1 - \delta + r_{t+1} (1 - \tau_{t+1}), \tag{11}
\]

The Public Sector

To finance total expenditures (productive and non-productive), the government can trade one-period bonds in addition to the collection of income taxes. The government decides the fraction \( x \) of output that is devoted to public investment (productive spending),

\[ i_{g_t} = x_t y_t, \tag{12}\]

Given \( i_{g_t} \), public capital accumulates according to:

\[ g_{t+1} = i_{g_t} + \left(1 - \delta_g \right) g_t, \tag{13}\]
with $\delta_g$ belonging to $[0,1]$ the depreciation rate of public capital might be different than that of private capital.\textsuperscript{14} It also claims a constant fraction, $\psi$, of output in the form of public consumption (i.e., non-productive spending),

$$c_{gt} = \psi y_t,$$ \hspace{1cm} (14)

that contributes neither to production, nor to consumer welfare. This assumption ensures that public expenditure represents a significant share of economic output as the economy grows.\textsuperscript{15} In addition to these (standard) public expenditure concepts, we consider an additional component of public expenditures, $q_t$, whose ratio to output responds to changes in the public debt-to-output ratio:

$$q_t / y_t = \pi_0 - \pi_1 (b_t / y_t).$$ \hspace{1cm} (15)

Parameter $\pi_0$ may be positive or negative and affect whether the level of primary deficit rises or falls with an increase in real output, while the parameter $\pi_1$ exclusively measures how strong the primary deficit (through expenditures or transfers) responds to changes in public debt levels. Next, the paper will show that $\pi_1 > 0$ is necessary for the fiscal policy (optimal or non-optimal) to be compatible with a sustainable public debt trajectory.\textsuperscript{16}

Given these elements, the public sector’s budget constraint is:

$$b_{t+1} = a_t y_t R_t + (1 - \pi_t) R_t b_t, \text{ with } a_t = x_t + \varphi + \pi_0 - \tau,$$ \hspace{1cm} (16)

where the primary fiscal deficit, $d_t$, is given by $d_t = a_t y_t - \pi_1 b_t$, which, as noted above, responds to the levels of output and public debt -- Bohn (1998) and Greiner (2008) assume a similar exogenous rule.

**Debt sustainability**

To study the sustainability properties of public debt, we rewrite the government budget constraint as a ratio of output, i.e., $\tilde{b}_t = b_t / y_t$ and $y_{t+1} / y_t = 1 + \gamma$ as the output growth rate (see the Appendix for details),

$$\tilde{b}_{t+1} = \frac{a_t R_t}{1 + \gamma_t} + \frac{(1 - \pi_t) R_t}{1 + \gamma_t} \tilde{b}_t.$$ \hspace{1cm} (17)

\textsuperscript{14} Ai and Cassou find support for this in the form, with an estimated $\delta_g$ of just over half of $\delta$.

\textsuperscript{15} Wasteful spending can be strongly correlated with the degree of inefficiency and corruption in the public sector.

\textsuperscript{16} Bohn (1998) and Greiner (2008) assume an active rule for the entire primary deficit, but we just restrict to public expenditure (or transfers from abroad if its magnitude is negative). Bohn (1998) proves that an estimated negative response of primary deficit to the debt-GDP ratio can be interpreted as a new test for the sustainability of the fiscal policy.
For fiscal policy to be sustainable, sustainability being defined as the absence of default risk, two conditions must be satisfied (Neck and Sturm, 2008). The first one is that the government cannot run Ponzi games in the long run (i.e., 8-9 must be satisfied); namely, the government cannot run a policy that uses the issuance of ever increasing new debt to repay old debt and to finance interest payments. Hence, the present discounted value of government debt, calculated over all future periods, must be zero. The NPG condition defined for $b_t$ is:

$$\lim_{t \to \infty} \left( \prod_{s=0}^{t-1} \left( \frac{R_i}{1 + \gamma_i} \right) \right) b_T = 0.$$  \hspace{1cm} (18)

The second condition for fiscal policy sustainability is:

$$\tilde{b}_0 - a_0 \sum_{i=0}^{\infty} (1 - \pi_i) \sum_{i=0}^{\infty} \left( \frac{R_i}{1 + \gamma_i} \right) = 0,$$  \hspace{1cm} (19)

that says that the present discounted value of primary deficits plus the value of initial debt must be zero. This condition implies that running substantial deficits over a long time is consistent with sustainability as long as these deficits can be compensated for by sufficiently high future surpluses. As we will show next, to be a steady-state equilibrium consistent with debt sustainability, $\pi_i > 0$ is required [Bohn (1998)].

**The competitive equilibrium**

For any period $t$, a fiscal policy is given by the vector $\mu_t = (x_t, \psi_t, q_t, b_t, \tau_t)$. A feasible fiscal policy, $\Pi$, is defined as a vector sequence $\{\mu_t\}_{t=0,1,2...}$, with $x_t, \psi_t \geq 0$, satisfying the budget constraint (17) and fiscal sustainability conditions (19)-(20).

**Definition 1.** A vector of sequences $\{c_t, c_{gt}, q_t, k_{t+1}, g_{t+1}, b_{t+1}, i_t, i_{gt}, y_t, r_t, w_t\}_{t=0,1,...}$ constitutes a $\Pi$-competitive equilibrium if, given a feasible policy $\Pi$, a price system $\{r_t, w_t\}_{t=0,1,...}$, and initial conditions $k_0, g_0, b_0 > 0$: i) $\{k_{t+1}\}_{t=0,1,...}$ solve the profit maximizing problem of the firms [i.e., (2)-(3) hold]; ii) $\{c_t, k_{t+1}\}_{t=0,1,...}$ solve the household problem [i.e., (10), (11), (7), non-negativity conditions and transversality (8)-(9) hold]; iii) technology constraints (4), (6), (13) are satisfied; iv) markets clear every period, i.e.,

$$y_t = c_t + c_{gt} + q_t + i_t + i_{gt}, \hspace{1cm} t = 0,1,...$$  \hspace{1cm} (20)

**Definition 2.** A Balanced Growth Path, BGP, (or steady-state equilibrium) is a competitive equilibrium allocation, such that $y_t, c_t, k_t$ and $g_t$ grow at constant rates, given by $\gamma$, and the output/capital ratio is constant.
II.1. The Ramsey problem and the optimal public investment policy

There are alternative competitive equilibriums implied by different policies. The following Ramsey problem chooses the competitive equilibrium allocation that maximizes household welfare, given competitive equilibrium (for households and firms) and resource conditions:

\[
\max_{\{c_t, k_t, g_t\}_t} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{t+1}^{1-\sigma} - 1}{1-\sigma} \right\}, \text{ subject to:} \quad (21)
\]

\[
c_t^{-\sigma} - \beta c_{t+1}^{-\sigma} \left[ 1 - \delta + f_k(t + 1)(1 - \tau_{t+1}) \right] = 0, \quad (22)
\]

\[
g_{t+1} - (\tau_t - \psi - \pi_0)F(t) - \left( 1 - \delta_g \right)g_t - b_{t+1}/R_t + b_t(1 - \pi_1) = 0, \quad (23)
\]

\[
c_t + k_{t+1} - (1 - \delta)k_t - (1 - \tau_t)F(t) + b_{t+1}/R_t - b_t = 0. \quad (24)
\]

The first constraint corresponds to the optimal household consumption-saving decision, the second combines the accumulation of public capital with the government budget constraint and the third is the budget constraint of the household.\(^{17}\) The Ramsey problem in its Lagrangian form is:\(^{18}\)

\[
\max_{\{c_t, k_t, g_t\}_t} = \sum_{t=0}^{\infty} \beta^t \left\{ c_{t+1}^{1-\sigma} - \frac{1}{1-\sigma} \left[ c_t^{-\sigma} - \beta c_{t+1}^{-\sigma} \left( 1 - \delta + f_k(t + 1)(1 - \tau_{t+1}) \right) \right] \right\}
\]

\[
- \lambda_1 \left[ g_{t+1} - (\tau_t - \psi - \pi_0)F(t) - \left( 1 - \delta_g \right)g_t - b_{t+1}/R_t + b_t(1 - \pi_1) \right]
\]

\[
- \lambda_2 \left[ c_t + k_{t+1} - (1 - \delta)k_t - (1 - \tau_t)F(t) + b_{t+1}/R_t - b_t \right] \}
\]

\[
(25)
\]

We follow a similar strategy as in Marrero (2008, 2010) to solve this problem and show next the optimal conditions.\(^{19}\) See the Appendix for details. First, we have the condition that equalizes the net rates of returns from public and private capital,

\[
A(1 - \theta)(g_{t+1})^{\theta}(1 - \psi - \pi_0) - \delta = A\theta(g_{t+1})^{\theta-1}(1 - \psi - \pi_0) - \delta_g, \quad (26)
\]

where \(g_{t+1} = \frac{g_{t+1}}{k_{t+1}}.\) It is easy to show that this equation has a unique and positive solution for \(g_{t+1}\) for all \(t.\) Thus, given \(g_0/k_0,\) the entire sequence of \(g/k\) (the optimal) can be easily characterized. Given constant paths for \(y\) and \(\pi_0,\) the \(g/k\) trajectory is constant as well and the

\(^{17}\) Notice that the resource constraint is not included since it is a result of combing the constraint of the household with that of the government.

\(^{18}\) See Chapter 12 in Ljungqvist and Sargent (2000) for a detailed description of similar Ramsey problems and associated Lagrangians.

\(^{19}\) Further details are available upon request.
The optimal public-investment

Although we mainly focus on public investment, we first characterize the optimal tax rate, $\tau^+$. Combining (26) and (27), we easily obtain,

$$\tau^+ = 1 + \frac{1-\theta}{\alpha} (1 - \psi - \pi_0) = 1 - \frac{\alpha + \psi}{\alpha} (1 - \psi - \pi_0),$$

which is constant for all $t$ (given constant levels of $\psi$ and $\pi_0$). Interestingly enough, having the government have the chance of issuing debt, the optimal income tax rate is limited to correct externalities in the economy: the learning-by-doing, which is a positive externality ($\phi$ affects negatively to $\tau^+$), and the fact that wasteful expenditure is a fraction of output ($\psi+\pi_0$ affects positively to $\tau^+$). It is clear that $\tau^+$ is lower than one, however it may be negative. For instance, if $\phi=0$, $\tau^+=\psi+\pi_0$, taxes are used to finance the part of non-productive public expenditure that depends on output, while public investment is entirely financed with public debt; however, the higher $\phi>0$, the larger is the necessity of issuing debt to finance not only public investment but also other public expenditure concepts. To impose positive tax rates (a realistic assumption), we should restrict our parameter set to

$$\frac{\psi+\pi_0}{1-\psi-\pi_0} > \frac{\phi}{\alpha}.$$  

20 The solution to this problem shows two desirable properties. First, the Ramsey solution achieves the first best allocation and, second, the Ramsey policy is time consistence (see Marrero, 2010, for details).
Next, we obtain the equation that determines the optimal public investment-to-output ratio. Using the government budget constraint, we obtain

\[-\frac{\delta}{R} (1 + \gamma) + (1 - \pi_1) \hat{b} = -(x + \psi + \pi_0 - \tau) \cdot A \hat{g}^\theta, \quad (30)\]

which can be substituted in (30) (rewrite those condition in terms of g) and obtain an expression for x:

\[\hat{g}(\gamma + \delta_g) - xA \hat{g}^\theta = 0. \quad (31)\]

Furthermore, we can use (51) and characterize,

\[A \hat{g}^\theta = \frac{(1-\gamma)^\sigma - \beta (1-\delta_g)}{\theta \beta (1-\psi - \pi_0)} \hat{g}, \quad (32)\]

which combined with (31) leads to a condition for the optimal public investment size, \(x^+\):

\[x^+ = \theta \beta (1-\psi - \pi_0) \frac{\gamma + \delta_g}{(1-\gamma)^\sigma - \beta (1-\delta_g)}. \quad (33)\]

This expression is the same as in Marrero (2008), hence a similar discussion applies. As commented above, we characterize this condition with independence to the debt-sustainability condition. First, since \((1+\gamma)^\sigma - \beta (1-\gamma) > 0\) (from the NPG) and \(\psi - \pi_0 < 1\), \(x^+\) is positive. Moreover, in principle, \(x^+\) might be above or below the threshold \(\beta \theta (1-\psi - \pi_0)\). However, it cannot be above \(\beta \theta (1-\psi - \pi_0)\) by much, since it can be proved (see Marrero, 2008) that \(x^+\) cannot exceed \(\theta (1-\psi - \pi_0)\) which is the ratio that maximizes the steady-state growth rate (hence, it is also lower than one). Second, notice that an explicit expression for the optimal stationary policy cannot be obtained since \(\gamma\) depends on economic fundamentals and policy parameters in a non-trivial manner. Hence, changes in any parameter show a direct impact on \(x^+\) and an indirect effect through growth. Indeed, the private capital depreciation rate can only affect the optimal policy through the indirect channel. Moreover, although we do not elaborate on this in this paper, a second indirect channel (correlated with the first one) is through the debt sustainability condition.

The results of the model can be stated in accordance with their effects among key parameters, such as the EIS, \(1/\sigma\), or the public capital depreciation rate, \(\delta_g\), on the optimal public investment ratio. The main findings of this sensitivity analysis are the following: (i) a lower IES (as well as a lower \(\beta\)) lowers the optimal public investment-to-GDP ratio -- a low EIS means that households prefer flat consumption trajectories to volatile ones, hence they have little incentives to substitute

\[\text{Futagami} \text{ et al. (1993)} \text{ pointed out that the optimal stationary public investment policy may differ substantially in an economy with transition dynamics, but they did not specify the condition for this. This condition contributes in this respect. The unique environment in which an explicit expression for the optimal public investment ratio can be determined is whenever } u(c)=\log(c) \text{ (i.e., } \sigma=1 \text{) and } \delta_g=1, \text{ where the optimal public investment-to-output ratio would be the standard } \beta \theta (1-\psi - \pi_0), \text{ which is the same than in } \text{Marrero and Novales (2007)} \text{ but for } \pi_0.\]
consumption intertemporally. Given other economic fundamentals, optimal public investment policy should not crowd-out current consumption and, as a consequence, the public investment-to-output ratio would be low. (ii) A higher depreciation rate for public capital, due for instance to uneven maintenance of public capital, further reduces the optimal rate of public investment to GDP, its impact coming from the direct and indirect channel commented above. (iii) \( \theta \) is positively related to the rate of return on public capital, hence it favors a higher \( x^* \). Finally, a higher \( \psi \) or \( \pi_0 \) implies that a larger proportion of output must be financed by distortionary taxes, which reduces the return to private investment.

II.2. Debt sustainability analysis

Along the BGP (long-run equilibrium), \( R_t, 1+\gamma_t \) and \( b_t \) converge to constant levels, said \( R^*, 1+\gamma^* \) and \( b^* \). The NPG-condition clearly implies that \( 1+\gamma^*<R^* \). Hence, (42) (see the Appendix) can be rewritten as:

\[
\bar{b}_0 - a_0 \sum_{t=0}^{\infty} \left[ \frac{(1-\pi_1)R^*}{1+\gamma^*} \right]^t = 0,
\]

which, for \( b_0>0 \) and \( a_0\neq0 \), requires that \( (1-\pi_1)R^*/(1+\gamma^*)<1 \) to be satisfied, hence,

\[
\pi_1 > \bar{\pi}_1 = 1 - \frac{1+\gamma^*}{R^*} \in (0,1). \tag{35}
\]

Using (10) and (11), the threshold \( \pi_1 \) can be rewritten as:

\[
\bar{\pi}_1 = 1 - \beta(1 + \gamma^*)^{1-\sigma} \in (0,1). \tag{36}
\]

This condition proves that the government needs to follow, at least in the long run, a public deficit rule that responds inversely with the level of public expenditure. Moreover, the degree of this reaction needs to be stronger for low-growing economies, with low value to future consumption (low \( \beta \)) and low preference to substitute consumption intertemporally (low \( 1/\sigma \)). The incidence of \( \delta_g \) (or \( \delta \)) is also negative, since higher depreciation rate harms growth. Once established this requirement for \( \pi_1 \), variables \( \bar{b}_0 \) and \( a_0 \) are related in the long run by:

\[
\bar{b}_0 \left( 1 - \frac{(1-\pi_1)R^*}{1+\gamma^*} \right) = a_0 = x + \psi + \pi_0 - \tau, \tag{37}
\]

Discussion and implications of debt sustainability

The paper shows that for debt sustainability to be consistent with optimal public policy, two conditions must be satisfied. First, the net present value of government debt, calculated over all

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22 Low EIS (i.e., \( 1/\sigma \) lower than 1), which might reflect the stronger preferences in developing countries to consumption now than in the future, would imply that most of the effect is direct, since the indirect effect through economic growth is small.

23 In an imperfect capital market framework, an alternative explanation to this result would be that households are liquidity constrained and the lower the EIS, the more difficult it is to intertemporally substitute consumption across periods.
future periods, must equal zero (i.e., the government is ruled out to play a Ponzi-game). Implied by the first condition, the second condition is that the net present value of primary deficits plus the value of initial debt must be zero. To satisfy this latter condition in this dynamic general equilibrium framework, at least one particular concept of public disbursement needs to follow an active rule responding inversely to the stock of outstanding public debt. Otherwise, debt would follow an explosive path, growing at a faster rate than real GDP, thus violating the NPG condition. Under these conditions, the government (alone or supported by outside assistance) must generate sooner or later sufficient fiscal surplus to restore debt - and economic - sustainability. However, during this adjustment, investments are likely to be severely depleted, which may delay the recovery process and the return to a sustainable path.

In this model, it is easy to realize that the requirement of debt sustainability (i.e., that part of public expenditure be counter-cyclical with public debt) can also be interpreted as a necessary condition to alleviate public debt burden by setting the interest rate on debt sufficiently below the one required by the market. The simplest route to provide this relief is through an external source (i.e., concessional credits). Financing plans in post-HIPC countries that blend concessional (i.e., below market interest rates) with non-concessional (i.e., market-determined interest rates) debt can be seen as a feasible and suitable strategy to finance future investment plans if the average debt burden is below that of the market and debt sustainability is compatible with an optimal public investment policy in a Ramsey sense.

Once the debt sustainability problem is resolved, the remaining issues to consider are the viability, maintenance and productivity of these public investment projects. Having to resort to non-concessional financial flows (to supplement concessional loans) may force governments to pay closer attention to the broader viability of the public investment projects they are implementing. Although the model outlined above is not stochastic, these ideas and implied results can be inferred from taking a closer look at certain key parameters of the model that could be related with this more responsible, credible and careful action: the public sector depreciation rate, the IES and the public capital elasticity. As shown above, the lower IES (captured by a lower \(\beta\)) and any fundamental change negatively affecting growth (for example, a higher \(\delta_g\) or a higher \(\delta\)) would increase \(\pi_1\), which can be interpreted as a higher effort required for accomplishing debt sustainability.

### III. Measuring the Impact of Debt-Financed Public Investment

Two policy implications can be drawn from the model outlined above on the long-run contribution of public investment to the level or growth rate of aggregate income and productivity. First, improvements in public investment management capacity influences growth by changing the degree to which public investment expenditure translates into factor accumulation.24 Savings are then perceived as a necessary but not a sufficient condition for

---

24 This is not to say that factor accumulation is the only determinant of growth. After accounting for physical and human capital accumulation, something else accounts for the bulk of cross-country growth differences. This “something else” accounts for the majority of cross-country differences in both the level of per capita gross domestic product (GDP) and the growth rate of per capita GDP. The term total factor productivity (TFP) is used to refer to this unexplained determinant of growth.
increased growth. As stressed in Stern (1991) management and organization are also key elements of growth in countries where capital is scarce. By improving the quality of project proposals, appraisals and budgeting, public investment management raises the returns on public capital by ensuring maintenance of existing assets and reducing waste. This link between capital investment and growth cannot be taken for granted, especially in the context of poor governance and rudimentary public investment systems. Second, when primary fiscal surpluses are allowed to substitute for private savings, this places fiscal policy in general, and policy decisions regarding the balance between capital and recurrent expenditures in particular, at the center of the discussion about the determinants of growth.25

III.1 Public Investment Management Capacity

To measure the degree to which the ratio of optimal public capital-to-GDP, \( x^+ \), responds to changes in the factors identified in the endogenous growth model outlined above, this section begins by carrying out a sensitivity analysis of a public capital accumulation equation, embedded in the general equilibrium model to capture insights on the effect of several key factors and parameters. The public capital accumulation equation is specified as follows (see Section II above and Marrero, 2008):

\[
x^+ = \theta \beta (1 - \psi)(\gamma + \delta_g/(1+\gamma)^{1/\sigma}) - \beta(1 - \delta_g),
\]

where \( x^+ \) is the optimal ratio of public capital to GDP, \( \beta \) is the discount factor of the representative household, \( \psi \) is the share of output devoted to public services, \( \delta_g \) is the depreciation rate of public capital, \( \gamma \) is the economic growth rate and \( 1/\sigma \) is the inter-temporal elasticity of substitution. Note that the total effect of all the factors on the ratio of public capital to GDP is the sum of the direct and indirect effects, with the strength of the indirect effects largely dependent on the depreciation rate of public capital and on household preferences between present and future consumption.

There are two main findings from a sensitivity analysis aimed at measuring the weight of factors related to the quality of public investment management on growth. A higher depreciation rate for public capital (\( \delta_g \)), due for instance to uneven maintenance of public capital, reduces the quality-adjusted public capital-to-GDP ratio, with negative effects on growth. Similarly, setting the elasticity of inter-temporal substitution at unity, to reflect the stronger preference in developing countries to consume now rather than in the future, magnifies the ‘congestion’ effect. These two factors have a bearing on the level of public investment, given the country’s institutional setting, on the impact of public investments on private capital accumulation, and, as a result, on a country’s long-term growth trajectory.

The importance of a country’s institutional setting is highlighted in the Public Investment Management (PIMa) index (see Figure 1). The PIMa index is drawn from the diagnostic framework by Rajaram et al. (2010) and its application in detailed country case studies. It establishes the relationship between countries (horizontal axis) and their ranking across the PIMa

cycle: project evaluation, implementation and facility operations, selection and budgeting, screening and appraisal (vertical axis). In doing so, it provides a benchmark against which one can assess the potential impact of public investment on growth given connections between cost-effective asset creation *ex post* and the conditional quality of project proposal screening, appraising and budgeting *ex ante*. The link between capital investment – asset creation – and growth cannot be taken for granted, however, especially in the context of poor governance and rudimentary public investment systems, as reflected in multiple gaps in all stages of the PIMa cycle. This fact underscores why it is also important to examine each country’s results beyond the headline number in the index and consider a country’s performance on various individual components of the index. While the PIMa indicators cover a broad range of components, what distinguishes the outcomes across a range of country studies is the degree of centralization of the project screening and appraisal responsibilities and the connection between capital and recurrent expenditures beyond the project construction period.

**Figure 1**: Public Investment Management (PIMa) - Selected Countries

Sources: Rajaram et al, et al. (2011) and World Bank country case studies.

A review of in-depth country studies across different country-settings and different levels of development (ranging from high income countries with more advanced public investment management systems to donor-dependent and resource rich countries) suggests that these countries lie between two extremes: one set of countries with full or near fully effective functioning PIMa’s at all stages and another set of countries where core PIMa functions are either missing or existing only de jure but not in practice. It is for the second group of countries with weak PIMas that an upstream ‘gate keeping’ stage, linked with financing over the entire project life, would be useful. Also, between these two extreme PIMa arrangements there are useful examples that underscore the importance of the other stages of public investment.

26 See also Dabla-Norris et al. (2011) for a different approach to creating cross-country PIM indexes.

27 See Rajaran et al (2010) and several World Bank country case studies on public investment management referred in the references.
management. Brazil performs well in project implementation and facilities management but lags behind in earlier stages related to project screening and budgeting – a sign that it is very difficult to remedy later in the project cycle any mistakes made at earlier stages. Belarus scores well in project selection and budgeting thanks to the recent establishment of independent review of project appraisal but lags in project implementation. 28, 29

These results are relevant because the quality of public investment management, as reflected in the adequacy of project selection and implementation, is statistically significant in explaining variations in economic growth [Gupta et al. (2011)]. Specifically, project selection and implementation, including competitive bidding by contractors and medium-term project frameworks with links to annual budgets, can affect the quality of public investment for the better and, as a result, their impact on growth can be supportive especially in lower income countries.

III.2 Public financial management and debt sustainability

Measuring the more straightforward impact of public investment efficiency on overall growth becomes more nuanced when measuring the implications of public investment financing on public financial management and debt sustainability. The works by Buffie et al. (2012) and by Andrle et al. (2012) suggest that when an economy can borrow concessationally to cover only part of an investment surge, difficult fiscal and private sector adjustments are unavoidable in the short to medium term, especially when the investment surge is front-loaded and the structural conditions of the economy (project selection and implementation capacity) are weak. While allowing for external commercial borrowing by less developed countries may mitigate some of these adjustment costs, the presence of weak structural and policy conditions, as well as unexpected exogenous shocks, call for caution. Similarly, domestic non-concessional borrowing exacerbates the problems usually found as a result of commercial borrowing in general because it does not provide additional external resources, and, as a result, requires even more drastic fiscal adjustments and worsens the crowding out effect on the private sector. 30

28 Public investment, particularly infrastructure, may also respond to political economy motives rather than simple economic efficiency considerations. Henisz and Zelner (2006) present evidence that interest group pressure and the structure of political institutions affects investments by state-owned electric utilities. Guasch et al. (2007) show that weak operational frameworks increase the likelihood of political interference and make the expropriation of sunk investments more likely, jeopardizing the realization of medium term returns.

29 This is important because the PIMa assessment offers a framework for synchronizing Public Investment Management and budgeting (see Appendix 1).

30 The most important effect appears to be that drastic fiscal adjustments lead to postponements in asset maintenance, reducing the infrastructure benefits derived from these investments.
To measure the impact on a country’s growth, this paper draws next on the findings of the PRISM (Public Investment, Savings and Macroeconomic Vulnerability tool) to track a country’s trends in public sector revenues, expenditures and capital formation, and how these decisions are being financed.31 Two examples illustrate how PRISM can shed some light on the trends in these variables and their implications. The first example comes from Ghana (Figure 2), where increased public sector revenues (including grants) have been accompanied by higher public sector consumption, with energy sector subsidies and public sector wages accounting for the most of these expenditure increases. Increased public sector wages have had both fiscal and labor market implications, with important consequences for the benefits of new debt-financed public investments for the private sector. By comparing the trends one can identify two problems: (i) public sector wages have increased as a share of GDP, crowding out other expenditure and leading to a public sector wage premium, and (ii) this premium has placed a barrier for the expansion of employment in the private formal sector, since higher public sector wages bid up private formal sector wages. Higher private sector wages have squeezed private sector profits, leading to lower levels of private investment outside the resource-rich sectors (oil, gas and mining). Lower investments have implied in turn lower increases in labor productivity, lower growth and lower job creation.

![Figure 2: Ghana-Debt Composition and Public Sector Expenditure Trends (2004-2011)](image)


31 The questions the data splits in PRISM seek to answer are: (i) whether that country is over-absorbing or under-absorbing relative to its GDP (Gross Domestic Product) and GDI (Gross Domestic Income), by how much and for how long? (ii) whether the country’s terms of trade are positive or negative, by how much and for how long? (iii) what are the country’s main external financing sources (e.g., remittances, FDI, external and domestic loans, and concessional and non-concessional debt); and (iv) to what degree is the country open to trade (goods and services) -- a factor that affects the volatility of GDP, GDI and GDE (Gross Domestic Expenditures) via the terms of trade channel).
The second example comes from Tanzania (Figure 3), where the share of public capital formation in GDP has been rising, while the share of public sector consumption has been kept in check.

### Figure 3: Tanzania-Debt Composition and Public Sector Expenditure Trends (2004-2011)

![Tanzania Debt Composition and Public Sector Expenditure Trends (2004-2011)](source)

The other dimension that is captured through the PRISM tool is the private sector response to public investments. This response is contingent on the discipline in managing public finances, and specifically the government’s commitment to harmonizing capital and recurrent expenditures. These private sector benefits range from the direct benefits that raise the productivity of human and physical capital (for example, roads that provide access to remote areas making private investment possible) to lower infrastructure costs that increase economies of scale, productivity, and growth. The PRISM tool helps track recent trends in the public and private sector’s gross fixed capital formation, and how these decisions are being financed.

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32 Statistically Gross Fixed Capital Formation (GFCF) measures the value of acquisitions of new or existing fixed assets by the business sector, governments and households (excluding their unincorporated enterprises) less disposals of fixed assets. GFCF is a component of the expenditure on GDP, and thus shows something about how much of the new value added in the economy is invested rather than consumed. GFCF is called "gross" because the measure does not make any adjustments to deduct the consumption of fixed capital (depreciation of fixed assets) from the investment figures. For the analysis of the development of the productive capital stock, it is important to measure the value of the acquisitions less disposals of fixed assets beyond replacement for obsolescence of existing assets due to normal wear and tear. “Net fixed investment” includes the depreciation of existing assets from the figures for new fixed investment, and is called net fixed capital formation. Gross Fixed Capital Formation (GFCF) is therefore not a measure of total investment, because only the value of net additions to fixed assets is measured, and all kinds of financial assets are excluded, as well as stocks of inventories and other operating costs (the latter included in intermediate consumption). If, for example, one examines a company balance sheet, it is easy to see that fixed assets are only one component of the total annual capital outlay. The most important exclusion from GFCF is land sales and purchases. The original reason, leaving aside complex valuation problems involved in estimating the value of land in a standard way, was that if a piece of land is sold, the total amount of land already in existence, is not regarded as being increased thereby; all that happens is that the ownership of the same land changes. Only the
The picture that emerges in the case of Tanzania is that improvements in public investment management capacity (as measured by the PIMa) have allowed the Tanzanian public sector to play a supportive role in the private sector’s development with little measurable crowding out effects. Overall gross fixed capital formation has increased alongside gross fixed capital formation by the private sector, while the public sector has accompanied this increase at a distance, ensuring the provision of infrastructure services (transport and energy), as measured by the trend improvement in gross fixed capital formation (Figure 4).

![Figure 4: Tanzania - Gross Fixed Capital Formation, total, private and public, 2000-2011 (% of GDP)](image)

Source: World Development Indicators.

Ghana provides a different example of the impact of public investment on the private sector -- one where gross fixed capital formation has been trending downward over the period 2004-2011 (Figure 5). The downward trend in gross fixed capital formation during this period appears to be the result of up and down shifts in fixed capital formation in the private and the public sectors. While public capital formation was trending up from 2006 to 2008, private capital formation was shifting downwards. In the aftermath of the Global Financial Crisis, public capital formation declined and stabilized at a lower level of around 7 percent of GDP. Meanwhile, private capital formation had a brief recovery in 2009 before declining in 2010 and 2011 bringing alongside overall capital formation down. The trends in public capital formation appear to reflect a ‘crowding out’ effect, whereby increased government expenditure on debt service obligations comes at the expense of the fiscal space for public investment, as shown in Figure 2 above.

The value of land improvement is included in the GFCF measure as a net addition to wealth. In special cases, such as land reclamation from the sea, a river or a lake (e.g. a polder), new land can indeed be created and sold where it did not exist before, adding to fixed assets. The GFCF measure always applies to the resident enterprises of a national territory, and thus if e.g. oil exploration occurs in the open seas, the associated new fixed investment is allocated to the national territory in which the relevant enterprises are resident.

The improvements in the provision of public services in energy and transport are captured by Tanzania’s improved ratings in the World Bank Doing Business Indicators on access to energy and trading across borders.
These findings are important because, while the general equilibrium model outlined above underscores the importance of sound fiscal management for debt sustainability, the impact of new borrowing to finance public investments on debt sustainability depends first and foremost on their impact on growth. A number of empirical studies find a positive impact of public investment on growth both through a direct impact on economic activity and through spillover effects on private investment. 34 A summary way to measure this impact is to calibrate the general equilibrium model with partial equilibrium estimates. For instance, Buffie et al. (2012) find that, assuming good policy implementation, an increase by 1 percentage of GDP in capital budgeting can raise real GDP growth by about 0.5 percentage points. 35 Gupta et al. (2011), on the other hand, estimate that the unadjusted output elasticity of public capital is reported at 0.25. 36

IV. Conclusions

This paper presents a combination of analytical tools, specifically a dynamic general equilibrium framework, a PIMa index, and a PRISM tool aimed at better assessing whether countries meet the necessary and sufficient conditions for optimizing the benefits of debt-financed public investments. The dynamic general equilibrium model is designed to illustrate the impact that changes in parameters related to public investment management (e.g., changes in public capital depreciation rates due to variations in the level of maintenance, and the share of output devoted to public services) on the optimal rate of public investment-to-GDP ratio. The other tools (the PIMa and the PRISM) are complementary in illustrating the impact of

36 The authors present an alternative measure of efficiency-adjusted capital for which the output elasticity would be around 0.15 (IMF, Working Paper No. 217, 2011).
three focal sets of interrelated issues: (i) the country’s public investment management capacity; (ii) the public financial management environment where these investments take place; and (iii) the dynamic impacts of public investment on private sector gross fixed capital formation.

The parameters captured by the general equilibrium model, the PIMa index, and the PRISM tool are important because they reflect the country’s institutional setting and the country’s long-term growth potential. This analysis is warranted given the scale of the investment effort being made by developing countries. Plans to increase debt financing of capital investments would need to be reviewed from a broader perspective. In particular, they need to satisfy two basic tests: what is needed to meet the public investment management challenge (e.g., absorption capacity), and what is the impact of these incremental investments on the private sector. To sharpen the understanding of these challenges and the growth impact of public investments, the paper attempts to unbundle the social and economic benefits from the cost reduction in infrastructure services induced by these investments and to measure the impact of public investment on the private sector’s fixed capital formation. The paper gauges the dynamic effects above and beyond a comparative static analysis to measure the increased impact of public investment on the private sector’s fixed capital formation. In doing so, future extensions attempt to extend the existing research in two directions:

- To use dynamic general equilibrium simulations to explore the relationship between public investment policies and debt sustainability.
- To use data from PIMa and PRISM to carry out an empirical analysis of the relationship between public investment productivity and the management and quality of the projects.

Three main conclusions emerge:

- **The simple dynamic general equilibrium model** illustrates how actions designed to reduce the depreciation rate for public capital, due for instance to better maintenance, affect both the overall rate of return on public capital and, therefore, the optimal level of public investment-to-output ratio. More importantly, this action can help to make compatible the implementation of an optimal public investment program with a debt sustainability policy.
- **The PIMa index** highlights the country’s position in the project management cycle, finding that the degree of centralization and quality of the project screening and appraisal responsibilities, as well as the connection between budgeting capital and recurrent expenditures once these projects are launched, emerge as the most important factors for the performance of the project.
- **The PRISM tool** underscores the parallel between the discipline in the public investment management and in the management of public finances. Countries committed and able to harmonize capital and recurrent expenditures tend to build more productive public capital assets. Countries that better manage public investments appear to complement better the private investment inductive to growth (no crowding out effect).
Appendix 1: The Public Investment Management (PIMa) and budgeting index

This Appendix sketches out the steps for synchronizing Public Investment Management (PIMa) and budgeting.

Appendix 2: Fiscal policy sustainability

Define $\tilde{b}_t = b_t / y_t$ and $y_{t+1}/y_t = 1 + \gamma_t$ as the output growth rate and rewrite the government budget constraint,

$$\tilde{b}_{t+1} = \frac{a_t R_t}{1 + \gamma_t} \tilde{b}_t + \frac{(1 - \pi_1) R_t}{1 + \gamma_t} ,$$

which is a first order difference equation with time varying terms. Given a particular level of $\tilde{b}_0$, the solution to this equation is:

$$\tilde{b}_t = \left(1 - \pi_1\right) \prod_{s=0}^{t} \frac{R_s}{1 + \gamma_s} \left[\tilde{b}_0 - a_0 \sum_{s=0}^{t} (1 - \pi_1)^s \prod_{i=0}^{s} \frac{R_i}{1 + \gamma_i} \right]$$

Multiplying both sides by $\prod_{s=0}^{t} \left(\frac{R_s}{1 + \gamma_s}\right)^{-1}$

$$\prod_{s=0}^{t} \left(\frac{R_s}{1 + \gamma_s}\right)^{-1} \tilde{b}_t = \left(1 - \pi_1\right) \left[\tilde{b}_0 - a_0 \sum_{s=0}^{t} (1 - \pi_1)^s \prod_{i=0}^{s} \frac{R_i}{1 + \gamma_i} \right]$$

For fiscal policy to be sustainable [Neck and Sturm (2008)], the present discounted value of government debt, calculated over all future periods, must be zero (i.e., the Government cannot run Ponzi games in the long-run). Taking limits in (40),

$$\lim_{t \to \infty} \prod_{s=0}^{t} \left(\frac{R_s}{1 + \gamma_s}\right)^{-1} \tilde{b}_t = \left(1 - \pi_1\right) \left[\tilde{b}_0 - a_0 \sum_{s=0}^{\infty} (1 - \pi_1)^s \prod_{i=0}^{s} \frac{R_i}{1 + \gamma_i} \right]$$

and using the NPG condition for $\tilde{b}_t$, $\lim_{t \to \infty} \prod_{s=0}^{t} \left(\frac{R_s}{1 + \gamma_s}\right)^{-1} \tilde{b}_t = 0$ easily obtain the second condition for fiscal policy sustainability,\(^{37}\)

$$\tilde{b}_0 - a_0 \sum_{s=0}^{\infty} (1 - \pi_1)^s \prod_{i=0}^{s} \frac{R_i}{1 + \gamma_i} = 0,$$

which says that the present value of primary deficits plus the value of initial debt must be zero.

\(^{37}\) It is easy to show that the transversality condition (9) expressed for $\tilde{b}_t$ is given by $\lim_{t \to \infty} \prod_{s=0}^{t} \left(\frac{R_s}{1 + \gamma_s}\right)^{-1} \frac{\tilde{b}_0 (1 + \gamma_s)}{R_t} = 0$. 
Appendix 3. Solving the Ramsey problem

To simplify notation, we define the following ratios: \( \lambda_{3t} = \frac{\lambda_3}{c_t} \), \( \lambda_{2t} = \frac{\lambda_2}{c_t} \), \( \lambda_{1t} = \frac{\lambda_1}{k_t+1} \) and \( \bar{u}_c(t) = \frac{\bar{u}_{c+1}}{c_t} \). We also use \( F(t) \) and \( f(t) \) to denote \( F(k_t, l_t, g_t) \) and \( f(k_t, l_t, g_t) \), respectively, and \( f_k(t), F_k(t), f_{kk}(t), F_{kk}(t) \) to denote first and second time derivatives of \( f(\cdot) \) and \( F(\cdot) \), respectively, with respect to \( k \) and so on of the indicated object, evaluated at a particular allocation. Optimal conditions of the Ramsey problem are:

\[
\tau_t: -\alpha \lambda_{3t-1} + \lambda_{2t} - \lambda_{3t} = 0; \quad t > 0 \tag{43}
\]

\[
: \lambda_{20} - \lambda_{30} = 0, \quad t = 0 \tag{44}
\]

\[
c_t: \quad 1 + \sigma \frac{\lambda_{1t}}{c_t} - \lambda_{3t} - \sigma \frac{\lambda_{1t-1}}{c_{t-1}} [1 - \delta + f_k(t)(1 - \tau_t)] = 0; \tag{45}
\]

\[
k_{t+1}: \beta \lambda_{1t+1} (1 - \tau_{t+1}) k_{t+1} \frac{\partial f_k(k_{t+1})}{\partial k} \bar{u}_c^{-1}(t) - \lambda_{3t+1} + \\
: + \beta \lambda_{2t+1} (\tau_{t+1} - \psi - \pi_0) F_k(t + 1) \bar{u}_c^{-1}(t) + \\
: + \beta \lambda_{3t+1} [1 - \delta + (1 - \tau_{t+1}) F_k(t + 1)] = 0 \tag{46}
\]

\[
g_{t+1}: \beta \lambda_{1t} \bar{u}_c^{-1}(t)(1 - \tau_{t+1}) k_{t+1} \frac{\partial f_k(k_{t+1})}{\partial g} \bar{u}_c^{-1}(t) - \lambda_{2t} + \\
: + \beta \lambda_{2t+1} (\tau_{t+1} - \psi - \pi_0) F_g(t + 1) + [1 - \delta_g] + \\
: + \beta \lambda_{3t+1} (1 - \tau_{t+1}) F_g(t + 1) = 0; \tag{51}
\]

\[
b_{t+1}: \frac{1}{R_t} c_t^{-\sigma} (\lambda_{2t} - \lambda_{3t}) - \beta c_{t+1}^{-\sigma} (\lambda_{2t+1} - \lambda_{3t+1})(1 - \pi_1) = 0; \tag{47}
\]

\[
\lambda_{1t}: \frac{c_t^{-\sigma} - \beta c_{t+1}^{-\sigma} (1 - \delta + f_k(t + 1)(1 - \tau_{t+1}))}{c_t} = 0; \tag{48}
\]

\[
\lambda_{2t}: \frac{g_{t+1} - (\tau_t - \psi - \pi_0) F(t) - (1 - \delta_g) g_t - b_{t+1} / R_t + b_t (1 - \pi_1)}{c_t} = 0; \tag{49}
\]

\[
\lambda_{3t}: [c_t + k_{t+1} - (1 - \delta) k_t - (1 - \tau_t) F(t) + b_{t+1} / R_t - b_t] = 0. \tag{50}
\]

Following arguments in Marrero (2010), we have \( \lambda_{3t} = \lambda_{2t} = 1 \) and \( \lambda_{1t} = 0 \) for all \( t \). Hence, conditions (45) and (46) can be significantly simplified and the following set of conditions summarizes the Ramsey allocation:

\[
k_{t+1}: F_k(t + 1)(1 - \psi - \pi_0) + 1 - \delta = \bar{u}_c(t)/\beta; \tag{51}
\]

\[
g_{t+1}: F_g(t + 1)(1 - \psi - \pi_0) + 1 - \delta_g = \bar{u}_c(t)/\beta; \tag{52}
\]
\[ \lambda_{1t} : c_t^\sigma - \beta c_{t+1}^\sigma (1 - \delta + f_b(t + 1)(1 - \tau_{t+1})) = 0; \]  
\[ \lambda_{2t} : g_{t+1} - (\tau_t - \psi - \pi_0)F(t) - (1 - \delta_g)g_t - b_{t+1}/R_t + b_t(1 - \pi_1) = 0; \]  
\[ \lambda_{3t} : [c_t + k_{t+1} - (1 - \delta)k_t - (1 - \tau_t)F(t) + b_{t+1}/R_t - b_t] = 0. \]
References


