

The Long-Run Economic Costs of AIDS: A Model with an Application to South Africa

Clive Bell, Shantayanan Devarajan, and Hans Gersbach

Primarily a disease of young adults, AIDS imposes economic costs that could be devastatingly high in the long run by undermining the transmission of human capital—the main driver of long-run economic growth—across generations. AIDS makes it harder for victims' children to obtain an education and deprives them of the love, nurturing, and life skills that parents provide. These children will in turn find it difficult to educate their children, and so on. An overlapping generations model is used to show that an otherwise growing economy could decline to a low-level subsistence equilibrium if hit with an AIDS-type increase in premature adult mortality. Calibrating the model for South Africa, where the HIV prevalence rate is over 20 percent, simulations reveal that the economy could shrink to half its current size in about four generations in the absence of intervention. Programs to combat the disease and to support needy families could avert such a collapse, but they imply a fiscal burden of about 4 percent of GDP.

While the costs of AIDS in terms of human suffering are undeniably large, estimates of the associated macroeconomic costs have tended to be more modest, whether their basis be an explicitly formulated economic–demographic model or cross-country regression analysis. Most earlier studies of the former kind that focus on Africa—the continent where the epidemic has hit the hardest—put the annual loss of GDP at about 1 percent.¹ These estimates stem from a particular view of how the economy functions: the AIDS-induced increase in mortality, even if it reduces labor supply, also reduces the pressure of population on existing land and capital, thereby raising the productivity of labor. Even if there is an accompanying decline

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1. See, for example, Arndt and Lewis (2000); Cuddington (1993); Cuddington and Hancock (1994); Kambou, Devarajan, and Over (1992); and Over (1992).

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in aggregate savings and investment (from the reallocation of expenditures toward medical care, for instance), the net impact on the growth of GDP per capita turns out to be small. Econometric investigations based on country panel data yield the same result. Bloom and Mahal (1997), for example, found no effect on GDP at all; on returning to the question later with new data, Bloom and others (2001) managed to extract a small adverse effect.

This article argues that the long-run economic costs of AIDS are almost certain to be much higher—and possibly devastating. In doing so, it joins company with some other authors (Corrigan, Glomm, and Méndez 2004, 2005; Ferreira and Pessoa 2003), who recently and independently have pursued an approach based on an overlapping generations framework. This approach involves a very different view of how the economy functions over the long run, one that emphasizes the importance of human capital and its transmission across generations. The accumulation of human capital—that is, the stock of knowledge and abilities embodied in the population—is the force that generates economic growth over the long run. The mechanism that drives the process is the transmission of knowledge and abilities from one generation to the next.

The implications of this model are particularly relevant to Africa, the continent with the lowest level of human capital and the highest prevalence of the disease. In many African countries, AIDS presents a formidable hurdle to long-run economic growth. The application of the model to South Africa, that Sub-Saharan outlier with relatively high levels of income and human capital (and HIV prevalence), reveals that in the absence of specific interventions, a decline from middle-income status is possible in the long run.

The argument establishing how AIDS can severely retard economic growth is made in three steps.² First, AIDS destroys existing human capital in a selective way, striking primarily young adults. Some years after they have been infected, it reduces their productivity by making them sick and weak. It then kills them in their prime, destroying the human capital formed in them through child-rearing, formal education, and learning on the job.

Second, AIDS weakens or even wrecks the mechanisms that create human capital in the next generation. The quality of child-rearing depends heavily on the parents' human capital. If one or both parents die before their offspring reach adulthood, the transmission of knowledge and potential productive capacity across the two generations will be weakened. At the same time, the loss of income due to disability and early death reduces the lifetime resources available to the family, which can lead to the children spending much less time, if any, at school. The chance that the children will contract the disease in adulthood also makes investment in their education less attractive, even when both parents themselves remain uninfected. The weakening of these transmission processes

2. The argument here is confined to those factors that are the most important. For a longer list of the epidemic's economic effects and related discussion, see, for example, Bell, Devarajan, and Gersbach (2004) and Corrigan, Glomm, and Méndez (2005).

is insidious: its effects are felt only over the longer run, as the poor education of children today translates into low productivity of adults a generation hence.

Third, as the children of AIDS victims become adults with little education and limited knowledge received from their parents, they are less able to invest in their own children's education, and a vicious cycle ensues. If nothing is done, the outbreak of the disease can eventually precipitate a collapse of economic productivity. Early in the epidemic, the damage may appear to be slight, but as the transmission of capacities and potential from one generation to the next is progressively weakened and the failure to accumulate human capital becomes more pronounced, the economy will begin to slow down, with the growing threat of a collapse to follow.

The argument has two important implications for economic policy. The first is fiscal. By killing off mainly young adults, AIDS also seriously weakens the tax base and thus reduces the resources available to meet the demands for public expenditures, including those aimed at accumulating human capital, such as education and health services not related to AIDS. As a result, the state's finances will come under increasing pressure, exacerbated by the growing expenditures on treating the sick and caring for orphans.

The other effect is an increase in inequality. If orphaned children are not given the care and education enjoyed by those whose parents remain uninfected, the weakening of the intergenerational transmission mechanism will express itself in increasing inequality among the next generation of adults and the families they form. Social customs of adoption and fostering, however well established, may not be able to cope with the scale of the problem, thereby shifting the onus onto the government, which is likely to experience increasing fiscal difficulties and thus to lack the resources to assume this additional burden.

The policy objective, therefore, is to avoid such a collapse. The instruments available for this purpose are (a) spending on measures to contain the disease and treat the infected, (b) aiding orphans, in the form of income support or subsidies contingent on school attendance, and (c) taxes to finance the expenditure program. The central policy problem is to find the right balance among these interventions to ensure economic growth over the long run without excessive inequality.

This article relates to recent contributions to the literature as follows. Those that adopt an overlapping generations framework have chosen somewhat different points of emphasis. In the model here, higher mortality risk undermines the formation of human capital through three channels. First, if one or both parents die early, their children will have less productive capacity because less human capital is transmitted. Second, the loss of income caused by early death in a family reduces schooling. Third, the chance that the children will be infected as adults makes investment in their education less attractive. Corrigan, Glomm, and Méndez (2004, 2005) consider only the first two channels, but they allow for effects on the accumulation of physical capital, which are absent in the model

here. Thus, this article complements theirs in the task of establishing how AIDS might influence the course of per capita income. As for the possible magnitude of these effects, Corrigan, Glomm, and Méndez (2004) calibrate their model to Sub-Saharan economies and find that for infection rates³ of around 15–20 percent, the growth rate of per capita income drops about 30–40 percent. Ferreira and Pessoa (2003) concentrate on the reduced returns to investment in schooling in a setting free of uncertainty, and estimate that the time devoted to it can decline by up to a half.

Young (2005) adopts a quite different perspective on how the AIDS epidemic impinges on the South African economy. He embeds a Beckerian household model, with endogenous participation, fertility, and education decisions, in a Solovian constant-savings-rate macroeconomic framework. In estimating the behavioral equations and simulating the evolution of the South African economy, two competing effects are emphasized. On the one hand, the epidemic is likely to have a negative impact on orphans' accumulation of human capital. On the other hand, high prevalence rates lower fertility. Young finds that even with the most pessimistic assumptions regarding educational attainment, the fertility effect dominates and future per capita consumption possibilities are enhanced. Although more channels through which the epidemic may harm human capital accumulation are considered here, fertility is exogenous. Sensitivity tests are conducted, however, and these reveal that changes in the level of fertility have only minor effects on the growth of productivity. Bruhns (2005) develops a closely related theoretical model in which households choose the level of fertility and applies it to Kenya. Her conclusions are broadly similar to the ones arrived at here.

Some econometric studies look at aspects of the link between AIDS and human capital. McDonald and Roberts (2004) estimate an augmented Solow model that incorporates both health and education capital. They employ a panel of 112 countries over a longer timespan than that of Bloom and others (2001) and conclude that the macroeconomic effects of HIV/AIDS have been substantial, especially in Africa, where the average marginal impact on income per capita of a 1 percent increase in the HIV prevalence rate is estimated to be –0.59 percent. Hamoudi and Birdsall (2004) provide indirect econometric evidence that AIDS reduces schooling in Africa. Using data from Demographic and Health Surveys conducted in 23 Sub-Saharan African countries and employing two specifications, they settle on the estimate that a reduction in life expectancy at birth of 10 years is associated with a fall of 0.6 years in the average schooling attained by that cohort. Given that life expectancy at birth in most countries in Southern and East Africa fell by at least 10 years over 1985–2000 (Dorrington and Schneider 2001) and that average schooling among the population aged 25–49 years was in the modest range of 3–6 years, this is a significant and disturbing finding. Although their measure of mortality differs from the one used here, their finding supports the general approach adopted here. Other

3. The term is theirs, but a close reading strongly suggests that they mean prevalence rates.

recent microeconomic work suggests that orphans indeed suffer various setbacks. Gertler, Levine, and Martinez' (2003) study of Indonesian children, for example, shows that orphans are less healthy, less likely to go to school, and overall less prepared for life. Case, Paxson, and Ableidinger (2002) found for a group of African countries that the schooling of orphans depends heavily on how closely related they are to the head of the adopting household.

In section I of this article, we tackle the question of how AIDS impinges on the economy conceptually by extending the model of Bell and Gersbach (2001) to deal with disease-ridden environments, in which premature adult mortality is increased by the outbreak of an epidemic. Parents have preferences over current consumption and the level of human capital attained by their children. The decision about how much to invest in education is influenced by premature adult mortality in two ways: the family's lifetime income depends on the adults' health status and the expected payoff depends on the level of premature mortality among the children when they reach adulthood.

In section II, we apply the model to South Africa. The choice of South Africa as a test bed is a natural one on several grounds. First, the very nature of the model demands that the available economic and demographic series be long and fairly reliable if the base for calibration is to be solid. Second, South Africa is a middle-income country that has experienced substantial growth over much of the past half century. A collapse of the kind analyzed in section I, were it to occur, would therefore mean that there is a long way to fall. Third, the epidemic has progressed rapidly in South Africa, from a prevalence rate among the population aged 15–49 years of about 1 percent in 1990 to just over 20 percent in 2003 (UNAIDS 2004).

Finally, in section III, we examine policies to avert the long-run economic decline caused by AIDS. Interventions in the spheres of health and education are examined. Finding the right balance between these two sets of measures is the central policy problem, and the results in this section attempt to illuminate how the balance should be struck. In any case, the sheer magnitude of the problem indicates that additional public spending of the order of 3–4 percent of GDP may be needed to contain the epidemic and ward off its worst effects.

I. THE MODEL

There are two periods of life, childhood and adulthood. On becoming adults, individuals form families and have children. When the children are very young, they can neither work nor attend school. Since investment in education is assumed to be the only form of investment, the family's full income is wholly consumed in this phase. Only after this phase is over, do the adults learn whether they will die prematurely—and thus leave their children as half or full orphans. Early in each generation of adults, therefore, all nuclear families fall into one of the following four categories: (a) both parents survive into old age, (b) the father

dies prematurely, (c) the mother dies prematurely, and (d) both parents die prematurely. These states are denoted by $s_t \in S_t := \{1, 2, 3, 4\}$. The subjective probability that a family formed at the start of period t lands in category s_t is denoted by $\pi_t(s_t)$.⁴ Once their states have been revealed, families make their allocative decisions accordingly, and the formation of human capital takes place. What follows is a terse account of the main elements; the details are set out in appendix A1.

Human capital is formed by a combination of child-rearing, whose quality depends on the parents' combined human capital, $\Lambda_t(s_t)$, and the child's formal education, e_t , expressed as a fraction of school-going years. A child so reared in period t attains the following level of human capital in period $t + 1$:

$$(1) \quad \lambda_{t+1} = \begin{cases} z(s_t)f(e_t)A_t(s_t) + 1, & s_t = 1, 2, 3 \\ \xi, & s_t = 4 \end{cases}$$

where $z(s_t)$ represents the strength with which capacity is transmitted across generations, $f(\cdot)$ can be thought of as the "educational technology," and the presence of the 1 in the upper branch grants this basic (normalized) level of human capital to wholly uneducated adults. $\xi (\leq 1)$ is the level of human capital attained by full orphans who grow up without care or education.

Let an individual's output be proportional to his or her level of human capital, an assumption that is certainly plausible over the very long run. Then a household with n_t children that finds itself in state s_t will have a well-defined level of full income, which the adults can allocate between consumption and investment in the children's education. The latter pays off in the form of each child's human capital on reaching adulthood. The (surviving) parents' optimal level of such investment, $e_t^0[A_t(s_t), s_t, \kappa_{t+1}^e]$, depends on the level of full income, the relative price of education, the strength of their altruism toward their children, and the expected level of premature adult mortality in period $t + 1$,

$$\kappa_{t+1}^e \equiv \frac{[1 + \pi_{t+1}(1) - \pi_{t+1}(4)]}{2},$$

as they subjectively estimate it in period t . Substituting e_t^0 into equation 1 yields

$$(2) \quad \lambda_{t+1} = \begin{cases} z(s_t)f(e_t^0(A_t(s_t), s_t, \kappa_{t+1}^e))A_t(s_t) + 1, & s_t = 1, 2, 3 \\ \xi, & s_t = 4 \end{cases}$$

Equation (2) describes a random dynamical system. Note that each child in any given family state s_t attains the same λ_{t+1} in adulthood with certainty, but he or she can wind up in any of the states $s_{t+1} \in \{1, 2, 3, 4\}$ after reaching adulthood and forming a family in period $t + 1$, and the succeeding branches proliferate in the future. The attendant threat of growing inequality will occupy an important

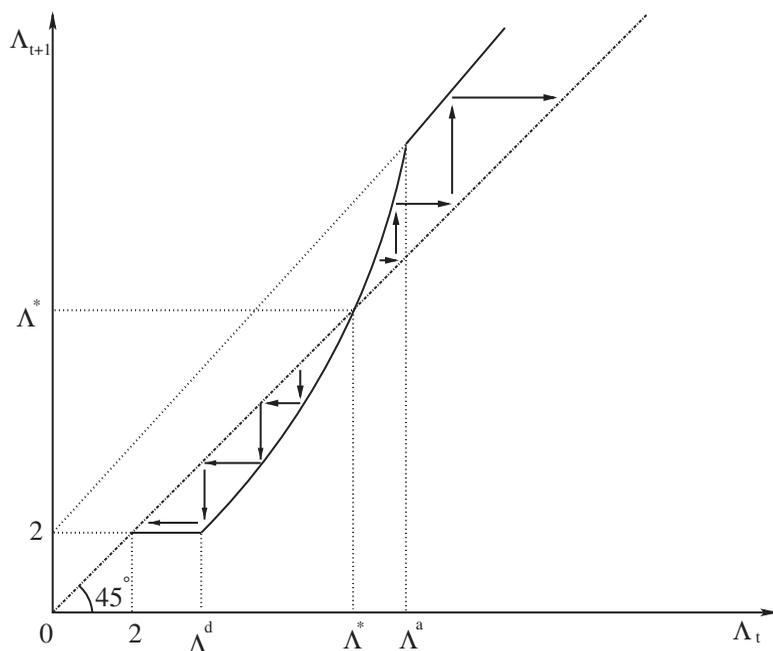
4. The population is assumed to be large enough that this is also the fraction of all families in that state after all premature adult deaths have occurred. Observe that these probabilities change over the course of the epidemic (table 2).

place in the analysis of policy interventions, but there is no space to go into the dynamical properties of the system in any detail here. What follows is aimed only at clarifying certain of their qualitative features.

It suffices, for this particular purpose, to look at what happens when there is no premature adult mortality [$\pi_{t+1}(1) = 1$ for all t], so that the only state that is ever observed is $s_t = 1$. To derive the typical dynamics, it is assumed that altruism is not operative when the adults are uneducated, that is, $e_t^0 = 0$ when λ_t is sufficiently close to 1. It can then be shown that the system has at least two stationary states with respect to human capital if $z(1)f(1)2\lambda^a + 1 \geq \lambda^a$, where λ^a is the lowest level of an adult's human capital such that a two-parent household chooses full education for the children in such an environment (Bell and Gersbach 2001).

The resulting phase diagram is illustrated in figure 1, where $\Lambda^d (> 2)$ denotes the smallest endowment of the adults' human capital such that they begin to send their children to school and $\Lambda^a (= 2\lambda^a)$ denotes the corresponding endowment at which children finally enjoy full-time schooling. As depicted, the system has just two stationary states with respect to human capital. One is the state of economic backwardness, defined as $\Lambda = 2$. This stable state is a poverty trap, wherein all generations are at the lowest level of human capital. The other is an unstable state ($\Lambda_t = \Lambda^* \forall t$), in which the parents' human capital is such that they

FIGURE 1. Phase Diagram Without Premature Adult Mortality



choose a positive level of education for their children, who then attain $A^*/2$ in adulthood. To be precise, and recalling equation (2), A^* satisfies

$$\frac{A^*(1)}{2} = z(1)f(e_t^0(A^*(1), 1, 1))A^*(1) + 1$$

where $\pi_t(1) = 1$ for all t . Observe that, starting from any $A > A^*$, unbounded growth is possible if and only if $2z(1)f(1) \geq 1$, and that the growth rate then approaches $2z(1)f(1) - 1$ asymptotically.

These results reveal that the intrusion of premature adult mortality may affect the system's dynamics not only by changing the probabilities of the states but also by increasing the values of A^d , A^* and A^a for states 1, 2, and 3, respectively, and thereby increasing the range of human capital levels within which a progressive decline into backwardness will set in. This turn of events is now examined in more detail.

Disease, Increasing Inequality, and Economic Collapse

The process by which the outbreak of an epidemic like AIDS may lead to economic collapse can be described as follows. At the start of period $t = 0$, a society of homogeneous, two-parent families, each with adult human capital endowment $2\lambda_0$, is suddenly assailed by a fatal disease. While the children are still young, all adults learn whether they are infected with the disease, and the survivors then choose the consumption–education bundle $(c_0^0(s_0), e_0^0(s_0))$ for $s_0 = 1, 2, 3$. How does the outbreak affect the subsequent development of the society? Children who are left as unsupported orphans ($s_0 = 4$) fall at once into the poverty trap. Even if both parents survive but have been such orphans in childhood, they cannot afford to send their children to school (as assumed above), and their succeeding lineage remains there. To discover what happens to the rest, the critical value function $\lambda^*(s, \kappa)$ is introduced for $s \in \{1, 2, 3\}$, which is defined for stationary fertility and mortality, $n_t = n, \forall t$ and $\kappa_t = \kappa, \forall t$. In this setting, it is natural to assume perfect foresight, namely $\kappa_{t+1}^e = \kappa_t = \kappa \forall t$.

$$(3) \quad \lambda^*(s, \kappa) = z(s)f(e^0(A^*(s), s, \kappa))A^*(s) + 1$$

where $A^*(1) = 2\lambda^*(1)$, $A^*(2) = A^*(3) = \lambda^*(2) = \lambda^*(3)$, and κ is a sufficient statistic of premature adult mortality in the stationary state, in which, by definition, all expectations are realized. $\lambda^*(s, \kappa)$ is the stationary-state level of human capital associated with a particular state s , that is, in any pair of generations, parent or parents and offspring share the same state. Equation (3) states that if adults with human capital λ^* find themselves in family state s and the mortality environment κ , they will make choices for their children such that the latter will attain the same level of human capital on reaching adulthood.

The critical value function has two key properties, which are established in Bell, Devarajan, and Gersbach (2003):

1. $\partial\lambda^*(s, \kappa)/\partial\kappa < 0$, $s = 1, 2, 3$
2. $\lambda^*(1, \kappa) \leq \lambda^*(2, \kappa) = \lambda^*(3, \kappa)$

The first property implies that a permanent increase in premature adult mortality may cause a group that was earlier enjoying self-sustaining growth to fall into the poverty trap. The second property implies that single-parent families generally need higher individual levels of human capital than two-parent ones to escape the trap, in which case an increase in premature adult mortality also increases the share falling into the poverty trap by increasing the proportion of one-parent families.

In the long run, if nothing is done to support full orphans and the children of needy, one-parent households, the share of uneducated families will grow until, in the limit, the whole population is in a state of economic backwardness. Not only do some adults meet an early death but the whole society descends progressively into the poverty trap. Two questions arise. First, what are the chances that the AIDS epidemic will so increase the level of premature adult mortality as to precipitate a collapse? Second, what arrangements for support and insurance are there to prevent such a collapse? These questions are addressed with reference to South Africa in the next section.

II. AN APPLICATION TO SOUTH AFRICA

This section falls into two parts. In the first, we cover the results of the calibration rather than the procedure itself, the details of which can be found in Bell, Devarajan, and Gersbach (2003). The robustness of the calibration is examined using a sensitivity analysis of the critical value function. In the second part, we develop three benchmark simulations of the model so calibrated.

Calibration and Sensitivity Analysis

Beginning with the fundamental difference equation (1), the parameters $z(s)$, the functional form $f(e)$, and the boundary value of λ are needed. In view of the highly nonlinear nature of the system and the limited information available, the form $f(e) = e$ is chosen. Since the unit time period of the model is a generation, with two overlapping generations, it is defensible to set the span of each at 30 years.

Inspection of the series for South African GDP reveals that the period from 1960 to 1975 was one of fairly steady and appreciable growth. This early subperiod is viewed as plausible initial basis for assessing how the post-apartheid

economy ought to be able to perform over the long haul. Denoting calendar years by the subscript k and ignoring child labor, GDP in year k is

$$(4) \quad Y_k = \alpha L_k \lambda_k$$

where L_k and λ_k denote the size of the labor force and the average level of efficiency in that year, respectively (table 1), and the parameter α is the productivity of a unit of human capital. Since the labor force series begins in 1965, that year is the starting point for the calibration procedure. The series for e_t is quinquennial and takes the form of the average years of schooling among the population aged 25 years and older—for example, 4.06 years in 1960. Defining full schooling as 10 years (ages 6–15 years inclusive) yields an average value of e for those born between 1905 and 1935 of 0.406, which is denoted by e_{60}^B .

Employing equation (1) recursively, together with the relation between a family's earnings and its endowment of human capital and the series in table 1, yields the estimates $z = 0.818$, $\alpha = 3,419$, and $\lambda_{65} = 2.696$. The final step is to shift the starting point to 1960. As pointed out in the introduction, the AIDS prevalence rate rose from about 1 percent in 1990 to just over 20 percent a decade later. This is a strong argument for choosing 1990 as the date of the outbreak of the epidemic in South Africa, and hence 1960 as the starting point in the chosen 30-year framework. The interpolation from table 1 implies that λ grew at an annual rate of 0.58 percent between 1965 and 1990; thus $\lambda_{60} = \lambda_{65}/(1.0058)^5 = 2.620$.

Two comments on these estimates are in order. First, the parameter α has the dimension of 1995 U.S. dollars per efficiency unit of labor per year. According to these estimates, therefore, a two-parent household in 1960 with two economically active adults and all the children attending school full-time would have had a family income of $\alpha \lambda_{60}$ or \$17,915. In the event of a complete collapse that left the entire population uneducated, the family's income would be just \$6,840

TABLE 1. GDP, Schooling, and Labor Force, 1960–95

Year	Y_k (1995 U.S. Dollar)	e_k^B	L_k	Y_k/L_k
1960	49.2×10^9	0.406	Not available	Not available
1965	68.4×10^9	0.410	7.42×10^6	9,220
1970	90.6×10^9	0.447	8.24×10^6	10,990
1975	113.0×10^9	0.453	9.25×10^6	12,230
1980	127.4×10^9	0.461	10.34×10^6	12,320
1985	132.4×10^9	0.495	11.93×10^6	11,100
1990	144.7×10^9	0.500	13.58×10^6	10,650
1995	151.0×10^9	Not available	15.29×10^6	9,880
2000	172.1×10^9	Not available	16.98×10^6	10,130

Source: World Bank 2002; Barro and Lee 1996.

TABLE 2. Family State Probabilities Corresponding to Premature Adult Mortality Rates

Year	$\pi(1)$	$\pi(2)$	$\pi(3)$	$\pi(4)$
${}^{20}q_{20}$				
1990 ($D = 0$)	0.855	0.101	0.039	0.005
2010 ($D = 1$)	0.294	0.165	0.347	0.194
${}^{30}q_{20}$				
1990 ($D = 0$)	0.793	0.164	0.080	0.018
2010 ($D = 1$)	0.112	0.180	0.272	0.436

Note: ${}_s q_x$ denotes the probability that an individual will reach the age of $s + x$ years, conditioned on reaching the age of x years. State probabilities do not correspond to the actual years shown but to the steady states associated with each disease environment ($D = 0, 1$).

Source: Authors' computations based on Dorrington and others (2001).

in the absence of child labor. Second, the estimate of z yields the value of the intergenerational growth factor when children attend school full-time, namely $2z = 1.636$. This corresponds to an annual growth rate of productivity of about 1.64 percent over the long run, which seems rather modest in light of the East Asian experience, but quite in keeping with South Africa's recent performance.

The form of social organization has thus far remained conveniently in the background, but now that preferences must be specified, a definite choice is unavoidable. For much of the period in question, South Africa was quite rural, so one can make the case that there was widespread pooling of orphaned children, with all surviving parents caring for all children. This arrangement is a salient feature of the benchmark cases to be analyzed below. Let preferences over current consumption and the children's attained level of human capital on reaching adulthood be logarithmic:

$$(5) \quad EU_t = 2[b \ln c_t(0) + n_t \left(\frac{\kappa_{t+1}^e}{\kappa_t} \right) \ln \lambda_{t+1}]$$

where the state $s_t = 0$ denotes pooling, and a representative pair of surviving adults cares for n_t/κ_t children, all of whom are valued and treated identically. Given that the calibration is anchored to 1960, both κ_{60} and households' expectations in 1960 concerning the level of κ_{90} are needed. The realized value of κ_{90} was 0.86. The great reductions in mortality in those three decades benefited children far more than adults, however, so that it is defensible to set the expected value of κ_{90} at the actual value of κ_{60} . Finally, it is assumed that in 1960 a representative couple, unaware of and untouched by AIDS in any way, chose the average years of schooling attained by the generation born between 1935 and 1965. This yields the value $b = 33.45$.

To complete the array of economic parameters, estimates of β , the fraction of an adult's consumption to which each child has a claim, and γ , a child's human capital when employed as a child laborer, are needed. Setting β at 0.5 seems

unobjectionable. A much lower value of γ is called for: $\gamma = 0.2$ yields a maximal level of annual earnings from a child's labor of $\alpha\gamma = \$685$, which may be on the high side, but this is balanced by the fact that no direct costs of schooling have been included.

Turning to the demographic components of the model, the population roughly doubled between 1960 and 1990, so that in keeping with the assumptions in section I and the generation span of 30 years, each mother had, on average, four surviving children over that period. Whether AIDS will affect fertility in the future is unclear (some evidence points to a modest decline), but what is certain is that AIDS has already contributed to a marked rise in mortality among children under the age of 5 years (Dorrington and others 2001). Since there is also some evidence that fertility had started to fall by the early 1990s (World Bank 2002), it is assumed that each mother will have three surviving children from 1990 onward.

The overriding concern in calibrating the model demographically, therefore, is with premature mortality among adults. The benchmark case is that where there is no epidemic ($D = 0$), which, in view of the low prevalence rate in 1990, is taken to be the age-specific mortality profile for that year, as set out in Dorrington and others (2001). The second reference case is that where the epidemic has reached maturity ($D = 1$) in the absence of any effective measures to combat it. The corresponding profile is assumed to be Dorrington and others' forecast for 2010.

The next step is to calculate the corresponding state probabilities $\pi_t(s_t)$, which requires an assumption about the incidence of the disease among couples. The probability of transmission within a union appears to be of the order of 10 percent a year under the conditions prevailing in East Africa (Marseille, Hofmann, and Kahn 2002), which, when cumulated over the median course of the disease from infection to death of a decade, implies that the probability of the event that both partners become infected, conditional on one of them getting infected outside the relationship, is about 0.65. Given the uncertainties involved, a less concentrated pattern of mortality within families has been assumed, namely that the incidence is independently and identically distributed. The resulting state probabilities are set out in table 2, where their values correspond not to the actual years shown but rather to the steady states associated with each disease environment ($D = 0, 1$).

The appalling dimensions—social, economic, and psychological—of the epidemic in its mature phase are plain. In the absence of AIDS, 85 percent of all children would grow up enjoying the care, company, and support of both natural parents, and fewer than 1 percent would suffer the misfortune of becoming full orphans (table 2). If the epidemic is left to run unchecked, it will leave almost 20 percent of the generation born from 2010 onward full orphans, about 50 percent will lose one parent in childhood, and a mere 30 percent or so will reach adulthood without experiencing the death of one or both parents. The epidemic will also reverse the usual pattern of excess mortality among fathers—from

about twice as high as among mothers to a third to a half lower. Given the mother's special role in securing the young child's healthy development, it can be argued that this reversal imparts additional force to the shock.

The final step is to undertake some sensitivity analysis. Since the decisive factor in the system's dynamics is how λ_t lies in relation to the critical, steady-state values $\lambda^*(s, \kappa, n, z)$, an appropriate way of investigating the robustness of the calibration procedure is to examine the sensitivity of $\lambda^*(\cdot)$ to variations in the parameter values estimated or derived above. The values of z , α , and λ_{60} are estimated jointly, so one cannot be varied without modifying the others. Two types of sensitivity analysis can be performed. First, the three parameters can be varied within this straitjacket. Second, taking α and λ_{60} as given, z (as well as κ and n) can be varied in such a way that the whole configuration is actually more optimistic than the one that emerged from the calibration (for example, by setting z in excess of 0.818). The second approach is chosen because it evaluates the robustness of the findings over a much wider domain and allows the parameters to take new values after 1990 (as already indicated for κ and n).

Table 3 sets out the values of $\lambda^*(\cdot)$ for a variety of plausible parameter values. In keeping with the above discussion of fertility and mortality, the choices are $n = 3$ and $n = 4$, with $\kappa = 0.860$ and $\kappa = 0.338$, which correspond to $D = 0$ and $D = 1$, respectively. The intermediate value $\kappa = 0.6$ represents a less dramatic, or waning, epidemic. In addition to the calibrated value $z = 0.818$, somewhat more optimistic values can be considered, namely 0.9 and 1.0, as well as the possibility that the future value of z may be reduced by the higher dependency ratio that will attend higher premature adult mortality (say, $z = 0.7$). Beginning with the calibrated values $n = 4$, $\kappa = 0.860$, and $z = 0.818$, equation (3) yields $\lambda^*(s, \cdot) = 2.06$, 2.10, and 4.33 for $s = 0$, 1, and 2, respectively, the first two of which lie comfortably below λ_{60} . Since the fraction of one-parent households under nuclear family arrangements was a modest 14 percent (see table 2), it can be assumed that the implicit burden of supporting them and full orphans was both tolerable and actually taken up. It follows that regardless of the family arrangements actually in force, the South African economy had already been launched on a path toward steady-state growth before the epidemic broke out in the early 1990s.

The reduction in fertility from $n = 4$ to $n = 3$ after 1990 has only a very slight effect on λ^* under both family arrangements. The fall in n implies a smaller weight on the term for altruism toward children, but this is just outweighed by the correspondingly smaller claims that fewer children make on the family's resources—whether they are raised under pooling or within a nuclear family. Indeed, this effect is small in all the parameter constellations in table 3, which leads to the conclusion that plausible changes in fertility do not play an important role in determining the qualitative nature of the system's dynamics.

The other striking feature of table 3, by contrast, is the sensitivity of $\lambda^*(s, \cdot)$ to κ . In all variations for $\kappa = 0.338$ (that is, $D = 1$), $\lambda^*(s, \cdot) > \lambda_{90} = 3.14$, which

TABLE 3. Critical Values of λ^* (s, κ, z, n)

κ	0.860				0.600				0.338			
	0.7	0.818	0.9	1.0	0.7	0.818	0.9	1.0	0.7	0.818	0.9	1.0
$n = 3$												
$s = 0$	2.58	2.14	1.90	1.67	3.74	3.11	2.77	2.43	6.71	5.60	5.00	4.40
$s = 1$	2.61	2.18	1.94	1.71	3.84	3.23	2.91	2.58	6.94	5.90	5.33	4.76
$s = 2$	5.30	4.44	3.97	3.50	7.74	6.53	5.87	5.22	13.92	11.82	10.69	9.56
κ	0.860				0.600				0.338			
	0.7	0.818	0.9	1.0	0.7	0.818	0.9	1.0	0.7	0.818	0.9	1.0
$n = 4$												
$s = 0$	2.51	2.06	1.81	1.56	3.66	3.01	2.66	2.31	6.60	5.45	4.83	4.21
$s = 1$	2.55	2.10	1.86	1.62	3.79	3.17	2.84	2.51	6.91	5.85	5.27	4.70
$s = 2$	5.22	4.33	3.85	3.37	7.66	6.43	5.76	5.10	13.58	11.73	10.59	9.44

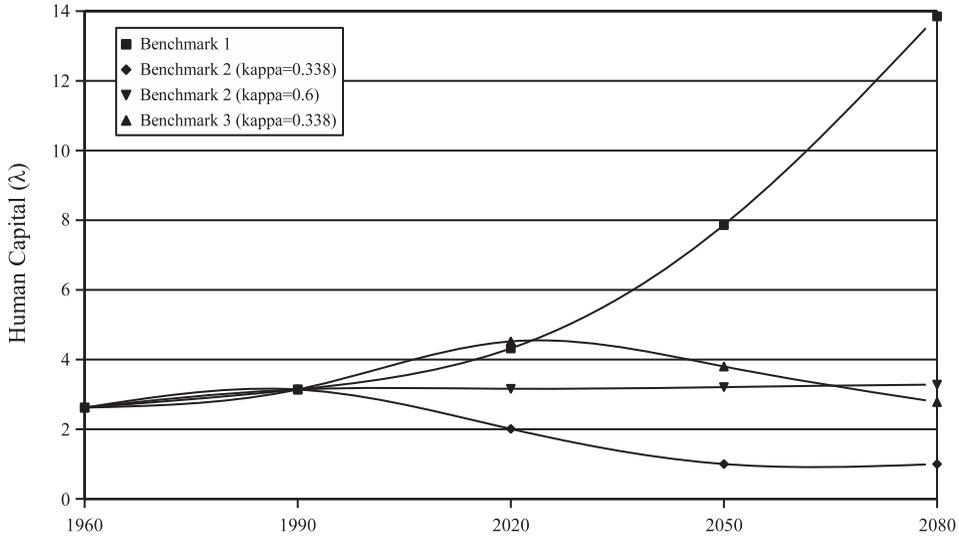
points to a progressive economic collapse in the face of an undiminished continuation of the epidemic and in the absence of any countervailing intervention. If $\kappa = 0.6$ and $n = 3$, this fate is avoided under both family arrangements (assuming, as above, that needy families will be supported) when z takes the value 0.9 or higher. When z takes the calibrated value 0.818, however, the pooling arrangement only barely escapes the trap, whereas the two-parent nuclear family ($s = 1$) barely slips into it. Summing up, these results suggest that even allowing for some uncertainty about the calibrated values of z and λ_{60} and about the estimated value of κ in the steady state corresponding to $D = 1$, as well as the behavior of fertility, the current course of the epidemic poses a very real threat to the long-term growth of the South African economy.

Simulations

Three simulations of the course of the economy for the period after 1990 form the set of benchmarks.

BENCHMARK 1: POOLING, NO AIDS. The corresponding trajectory of the variable λ_t , about which all else revolves, is plotted in figure 2. As noted above, the key feature of this story is that steady-state growth is ultimately attained. Starting from the modest level of 0.5 in 1960, education becomes virtually full-time in the generation born from 2020 onward, by which point, income per head is two-thirds higher than in 1960, with another increase of 80 percent in the next generation. The burden of child-dependency is limited throughout: 0.65 adopted children per couple in addition to the four of their own before 1990 and 0.49 in addition to the three of their own thereafter. This is the relatively happy counterfactual into which AIDS intrudes at $t = 0$ (1990).

FIGURE 2. Comparison of Benchmarks ($z = 0.818$)



The results are summarized in table 4, which provides a compact summary of all three benchmarks that relate to the values of the parameters calibrated above. From 1990 onward, a representative family under pooling comprises two surviving adults and 3.49 children in the absence of AIDS and two surviving adults and 8.87 children in its presence.

BENCHMARK 2: POOLING, AIDS, AND NO INTERVENTION. If, following the full onset of the AIDS epidemic, premature adult mortality remains at the level that

TABLE 4. Three Growth Paths for the South African Economy

No AIDS				AIDS ^a					
				$\kappa_{120}^e = \kappa_{90}$			$\kappa_{150}^e = \kappa_{90}$		
Year	λ	e	$y(0)$	λ	e	$y(0)$	λ	e	$y(0)$
1960	2.62	0.50	19,500	2.62	0.50	19,500	2.62	0.50	19,500
1990	3.14	0.64	22,340	3.14	0.20	26,370	3.14	0.68	23,400
2020	4.32	0.97	29,590	2.01	0.00	17,770	4.52	0.39	34,690
2050	7.86	1.00	53,720	1.00	0.00	12,900	3.80	0.29	30,280
2080	13.85	1.00	94,720	1.00	0.00	12,900	2.78	0.14	24,250

Note: All results are based on $_{30}q_{20}$.

^a $\kappa_{120}^e = \kappa_{90}$: in 1990, households formed expectations about adult mortality in 2020, when their children will have reached adulthood, that reflected the actual course of the epidemic over the period 1990–2020, as set out in table 3; $\kappa_{150}^e = \kappa_{90}$ is analogously defined when such expectations are revised starting only in 2020.

yields the steady-state probabilities in table 2, the consequences of doing nothing will be nothing short of disastrous, as seen in figure 2. Within a few generations, the epidemic sets in train a complete collapse of both the economy and, almost surely, the social institution of pooling. The extremely high level of premature mortality among adults leaves the community relatively impoverished from the start and with an intolerable burden of dependency: each surviving couple has to care for almost two adopted children for each one of their own. Education is correspondingly neglected, with unrelieved child labor ($e = 0$) for the generation born starting in 2020. The descent into backwardness ($\lambda = 1$) is complete by 2050, when family income is a little less than two-thirds its level in 1960, and there are almost twice as many children for each couple to care for. The results are summarized in table 4.

It might be argued that both variants with AIDS in table 4 constitute unduly pessimistic estimates of the conditions prevailing in 1990–2020 and beyond in terms of the level of mortality and the growth of long-term productivity. The sensitivity analysis in section II covers this possibility, but those findings are expanded on here.

Suppose, for example, that from 1990 onward, κ were to fall, not to 0.338 as above, but less precipitously, to 0.6, say. Since $\lambda^*(0, \kappa = 0.6) < \lambda_{90}$ (table 3), no collapse follows; but the system teters on the brink, with virtual stagnation thereafter (figure 2). Turning to the growth of productivity over the long run, prudent economic management and social integration after 1990 ought to yield an improvement over $z = 0.818$. Suppose, then, that $z = 1$, which corresponds to a doubling of λ every generation (or 2.31 percent a year) under full-time schooling. If κ continues at 0.338, however, the collapse that ensues is scarcely less dramatic than that when $z = 0.818$ (figure 2).

BENCHMARK 3: POOLING, AIDS, AND DELAYED EXPECTATIONS. The second variant in table 4 reflects the possibility that households will take some time to revise their expectations. Suppose this revision does not occur until the very start of the next generation, when the childhood experience of parental death will be vivid in the minds of the next cohort of young adults: their firm expectations are $\kappa_{150}^e = \kappa_{90}$. Suppose, further, that these expectations are realized and that this scale of mortality persists into the future. The happy—but false—expectations about future mortality that are formed in 1990, coupled with what is assumed to be the generous altruism of full pooling, induce adults to invest heavily in the children’s education, despite the sharp reductions in available resources caused by the outbreak of the epidemic. Yet, although the adults in the generation starting out in 2020 are every bit as well endowed with human capital as they would have been in the absence of the epidemic, their expectations concerning their children’s future are so bleak as to induce them to roll back investment in schooling to levels not seen since the mid-20th century. The result is to send the entire system into a progressive decline.

As reported in table 4, income per capita in benchmark 3 peaks in the period starting in 2020, and two generations later, the fresh cohort of adults will be scarcely more productive than their forebears in 1960. Only a revival of optimism about the future and the resumption of low levels of premature adult mortality to confirm it will stave off a complete collapse.⁵ Note that a collapse is possible even when the mortality shock affects only one 30-year generation, depending upon how and when expectations are formed.

III. POLICY OPTIONS

All policies are assumed to be financed by lump-sum taxes. Furthermore, the government chooses the level of public expenditure not to optimize a classically specified intertemporal welfare function over an infinite horizon—a problem that is almost impossible to solve in the framework—but to restore steady-state growth and then maintain it. The policy program takes the form of a sequence of taxes and expenditures that achieves this objective, if it is at all feasible.

Health Policy

Health policy takes the form of spending on measures to combat the disease. For some diseases, treatment may result in a complete cure. There is no such prospect for the victims of AIDS; but the treatment of opportunistic infections in the later stages and the use of antiretroviral therapies can prolong life and maintain productivity. In the present overlapping generations setting, therefore, treatment may be thought of as reducing premature adult mortality in the probabilistic sense.

It remains to establish the relationship between the state probabilities and spending on combating the disease. This is accomplished by choosing a functional form for the relationship between the probability of premature death among adults, q , and the level of expenditures on combating the disease, η , and then making the simplifying assumption that the incidence of the disease is independently and identically distributed. For simplicity, and erring on the side of optimism, it is also assumed that such aggregate expenditures produce a pure public good, so that

$$(6) \quad q(D = 1) = q(\eta; D = 1)$$

where $q(\eta; D = 1)$ is to be interpreted as the efficiency frontier of the set of all measures that can be undertaken to reduce q in the presence of the disease.

Very little is known about the exact shape of the function $q(\cdot)$, but $q(0; D = 1)$ should yield the estimates in table 2. A second, plausible, condition

5. The fact that false expectations can be helpful in overcoming shocks raises delicate questions about the value of transparency in public policy in this context. They are avoided here.

is that arbitrarily large spending on combating the epidemic should lead to the restoration of the status quo ante, that is, $q(\infty; D = 1) = q(D = 0)$. For reasons that will become clear shortly, it is desirable to choose a functional form that not only possesses an asymptote but also allows sufficient curvature over some relevant interval of η , so that the natural choice falls on the logistic:

$$(7) \quad q(\eta; D = 1) = d - \frac{1}{a + ce^{-b\eta}}.$$

Hence,

$$(8) \quad q(0; D = 1) = d - \frac{1}{a + c}$$

and

$$(9) \quad q(\infty; D = 1) = d - \frac{1}{a} = q(D = 0).$$

The full estimation of the function $q(\cdot)$ is described in appendix A2. The procedure yields the values of the parameters a , b , c , and d for men, women, and both combined, which are set out in table A2-1 for two values of the cost of saving a disability-adjusted life year. The associated functions $q(\eta; D = 1)$ are convex to the origin and have relatively strong curvature over the interval $\eta \in (300, 700)$ (figure A2-1). The said values depend on the annual cost (K) of a course of generic drugs. Marseille, Hoffmann, and Kahn (2002) set K at \$395. Early in 2006, however, the annual cost of a course of generic drugs was about \$200, so some might regard the first estimate as too conservative in terms of the cost-effectiveness of treatment, as opposed to prevention—even though it bears stating that neither estimate makes any allowance for the other components of highly active antiretroviral therapy and the threat that drug-resistant strains will proliferate when the full regime is not rigorously followed. The subsections that follow begin with the results based on the calibrated values of the parameters and the health-cost factor $K = 395$. The robustness of these findings to changes in all these parameters are then examined.

Policy Option 1: Spending on Health Under Pooling

The results of spending on health under pooling are qualitatively striking (table 5). The optimal level of spending on combating the epidemic immediately upon outbreak in 1990 ($t = 0$) is \$963, which is about 4.5 percent of GDP, rising to \$1,029, or 3.6 percent of GDP, in 2020, when productivity is 30 percent higher. Fiscally speaking, this is a tall order and a very substantial long-run burden, especially in view of the fact that the additional taxes are assumed to be raised in lump-sum form.⁶ If this program is politically feasible, it will eventually yield

6. Under the distortionary tax systems that rule in practice, the marginal cost of a unit of public revenue can range between 1.3 and 1.7 or higher still.

TABLE 5. Policy Option 1: Spending on Combating the Disease under Pooling

Year (1960)	λ (2.62)	e (0.50)	η (0)	κ (0.860)	n/κ (4.65)	$y(0)$ (19,503)
<i>K</i> = 395						
1990	3.14	0.60	963	0.849	3.53	22,445
2020	4.10	0.87	1,029	0.852	3.52	28,365
2050	6.83	1.00	1,029	0.852	3.52	46,725
2080	12.18	1.00	1,029	0.852	3.52	83,269
<i>K</i> = 200						
1990	3.14	0.62	714	0.854	3.51	22,412
2020	4.16	0.90	755	0.855	3.51	28,718
2050	7.12	1.00	755	0.855	3.51	48,713
2080	12.65	1.00	755	0.855	3.51	86,521

Note: η is the per household level of spending on combating the disease; $y(0)$ is the level of income accruing to each pair of surviving adults and the children in their care.

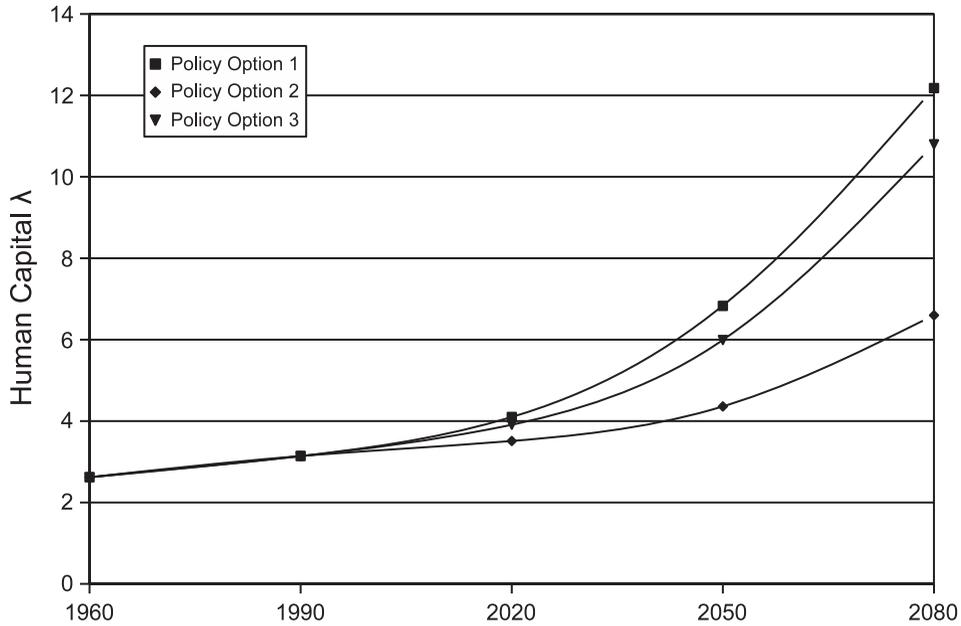
steady-state growth, with full and universal education attained in 2050. With (optimal) spending at this level, premature mortality among adults would be scarcely higher than in the complete absence of the disease.

A comparison with benchmark 1 reveals that the costs of dealing with AIDS in terms of lost output are modest at first but become quite large by 2080, when productivity is about 88 percent of its benchmark level, even with the optimal package of interventions under the favorable conditions of the case considered here (table 4). The long-run rate of growth is unaffected by AIDS under this policy program; for once full-time schooling is reached, the growth rate depends only on $z(0, \kappa)$, which is assumed to be constant at $z(0, 0.86) = 0.818$. Taking a somewhat broader view, therefore, the outcome is encouraging, in that the general character of benchmark 1 is still attainable (figure 3), including a relatively low level of premature adult mortality. Thus, the maintained assumption that pooling will survive the shock is arguably validated.

Given this rather encouraging qualitative finding, it is still natural to ask whether lower costs of generic drugs will yield significant quantitative gains. When $K = 200$, the optimal values of η in 1990 and 2020 and thereafter are substantially lower at \$714 and \$755, respectively, but the corresponding values of κ still rise, to 0.854 and 0.855, respectively. With a much lower fiscal burden and a slight improvement in premature adult mortality, λ increases a little more rapidly than when $K = 395$, so that its level in 2080 is almost 4 percent higher.

Policy Option 2: Nuclear Families, Lump-Sum Subsidies

The results under policy option 1 are predicated on the assumption that the government acts at once to nip the epidemic in the bud. In fact, the epidemic had assumed alarming proportions by 2000, with many children already left as

FIGURE 3. Policy Options 1, 2, and 3 ($K = 395$)

orphans and even more destined to become orphans, thus calling into question the whole system of pooling. If this social institution does break down, leaving tightly defined nuclear families to emerge instead, then the government will face the challenging task not only of averting a collapse, but also of preserving equality within each generation. To make both possible, additional assumptions are needed about the formation of human capital when children are left as half or full orphans. Under the assumption that $z(1) = z(2)/2$, that is, single parents can do just as well as couples in raising their children if they have the income, it is possible to preserve equality of educational outcomes among all children with at least one living parent by subsidizing one-parent families so as to induce them to choose the same level of education that two-parent families choose. By hypothesis, no family takes in full orphans, so that these children must be cared for in orphanages. It is assumed that these institutions, when properly staffed and run, substitute perfectly for parents, at least where the formation of human capital is concerned. The operating rule is that each full orphan also enjoys the same level of consumption as a child in a single-parent household.

When the family structure is nuclear, a good policy program to overcome the shock caused by AIDS must ensure a substantial tax base, not only in the present but also in the next generation. The instruments available for this purpose are taxes on two-parent households, spending on combating the disease, the size of the subsidy to single-parent households, and the proportions of half and full orphans to be supported. They are chosen subject to the above restrictions

designed to preserve equality, if at all possible, and to the government's budget constraint.

Given the complexity of using full-scale forward induction, a somewhat simpler approach is chosen. The aim is to maximize the expected size of the tax base in the next period, where all parties hold the firm expectation that there will be a continuation of the level of premature adult mortality (and hence of η) prevailing in the present. That is, stationary expectations are assumed, which permit the maximization problem to be written so that it effectively contains no variables or parameters pertaining to the future. In particular, families' decisions about education depend on κ_{t+1}^e ; but under stationary expectations, $\kappa_{t+1}^e = \kappa_t$. The (bounded) rationality of these expectations is secured by imposing the condition that η does not fall from one period to the next, for this will rule out policy programs under which the value of investments in education will be reduced ex post by failures to take adequate measures against the disease in the next period. It should be emphasized that if it is possible to stave off a collapse of the economy through a policy program derived on the basis of stationary expectations so formulated, then it certainly will be possible to do even better by using the full apparatus of forward induction. Since all adults possess at least one unit of human capital, the tax base is defined, for present purposes, as the excess of the aggregate level of human capital over the aggregate level when all adults have but one unit.

The optimum sequence⁷ yields a continuation of growth with complete equality—all orphans receive the support needed to bring them up to par with the children of two-parent households in each and every period (table 6). Growth is distinctly sluggish, however, which points to a collapse that would be somewhat narrowly averted. The (uniform) years of schooling rise noticeably more slowly across succeeding generations than under pooling, with full-time schooling achieved only in 2080, when the level of productivity is only slightly more than double its value in 1990. Spending on combating the disease is also higher in absolute terms throughout and, combined with the transfers required to support needy children, $g(2)$, this yields a much heavier fiscal burden than under pooling. Two-parent households pay a little over 20 percent of their income in the form of a lump-sum tax (τ) to finance this program in 1990 and receive very little relief until rapid growth begins from 2080 onward, when one-parent families need less support.

The differences between policy options 1 and 2 call for some explanation. Under pooling, which ensures equality, the objective is to maximize the (uniform) level of productive efficiency (λ) in the next generation, whereas with nuclear families, it is the size of the future tax base that matters when the government has to undertake the task of replacing the institution of pooling with subsidies and orphanages. In the latter arrangement, it may be worthwhile to trade off educational attainment to secure more surviving adults at

7. The optimization problem is set out in full in Bell, Devarajan, and Gersbach (2003).

TABLE 6. Policy Option 2: Nuclear Families, Lump-Sum Subsidies

Year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
<i>K</i> = 395										
1990	3.14	0.49	0.49	1,101	2,174	4,223	1.0	1.0	0.854	22,536
2020	3.51	0.58	0.58	1,127	2,470	4,607	1.0	1.0	0.854	24,887
2050	4.36	0.79	0.79	1,179	3,143	5,466	1.0	1.0	0.854	30,228
2080	6.60	1.00	1.00	1,179	2,214	4,707	1.0	1.0	0.854	45,136
<i>K</i> = 200										
1990	3.14	0.50	0.50	796	2,294	3,862	1.0	1.0	0.86	22,506
2020	3.59	0.62	0.62	814	2,654	4,309	1.0	1.0	0.86	25,327
2050	4.62	0.86	0.86	850	3,491	5,335	1.0	1.0	0.86	31,892
2080	7.53	1.00	1.00	850	1,030	3,027	1.0	1.0	0.86	51,493

Note: $g(2)$ is the income transfer to each one-parent family that receives such support; τ is the level of the special lump-sum tax on each two-parent family; $\delta(s)$, $s = 2,4$ is the fraction of all children in family state s receiving public support; $y(1)$ is the level of gross income accruing to a two-parent family.

the later date. That is exactly what has happened here: the absolute level of η is 14 percent higher than under pooling in both 1990 and 2050, despite the fact that productivity under pooling is 57 percent higher in the latter period. The other contributing factor arises from the fact that raising children in orphanages draws some adults out of the production of the aggregate private good—a cost that does not arise (by assumption) under pooling. The upshot is that families have less disposable income than under pooling, so that their children receive fewer years of schooling and growth is much slower. As under pooling, the long-run rate of growth is unaffected by AIDS in this fairly good sequence; but the traverse to steady-state growth is a painfully long one.

How much less painful would this trek be when $K = 200$? As under pooling, the optimal levels of η are just over 25 percent lower than when $K = 395$, and κ edges up further, almost to what its level would be in the absence of the epidemic. The absolute tax burden on two-parent families is also somewhat lighter: 8.6 percent lower in 1990, 7.5 percent in 2020, 2.6 percent in 2050, and almost 36 percent lower in 2080, when single-parent families need much less income support to be induced to choose full-time schooling. The effects on the accumulation of human capital are small at first, but by 2080, λ is 14 percent higher than when $K = 395$. Since full equality in terms of human capital within each generation emerges as part of the optimal program, this faster pace requires that one-parent families need more generous support up to 2080, and $g(2)$ is correspondingly more generous—5.5 percent higher in 1990, 7.5 percent in 2020, and 11 percent in 2050.

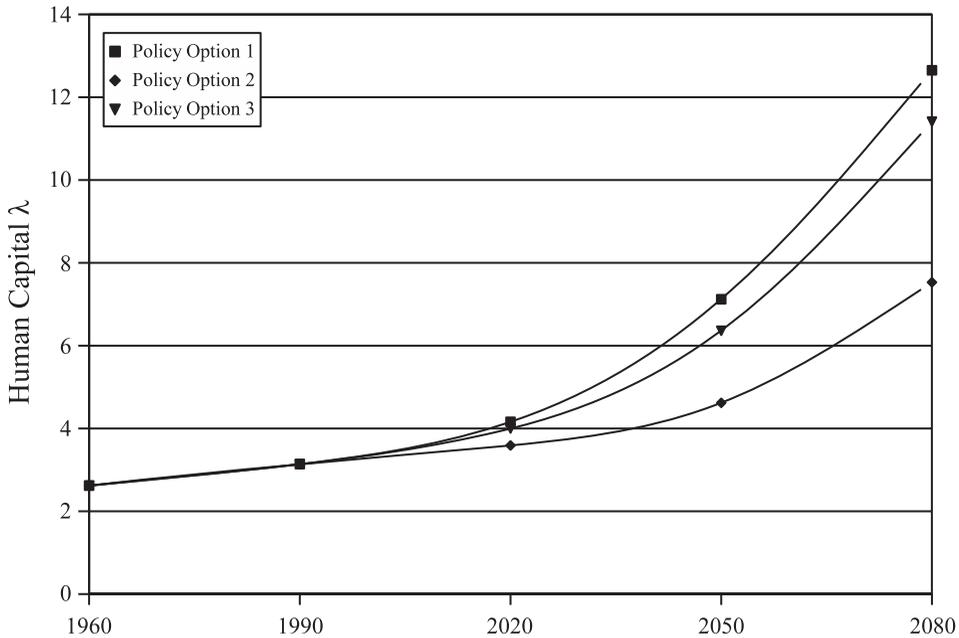
Policy Option 3: Nuclear Families, School Attendance Subsidies

The results for this option are qualitatively similar to those under policy option 2, but growth is considerably more rapid (table 7; figures 3 and 4). Given the efficiency of school attendance subsidies relative to lump-sum transfers (and hence the lower taxes on two-parent households), one would expect a swifter attainment of full-time schooling in this variant, and this is indeed the case here. The precise reasoning runs as follows. Choose the optimal levels of taxes on two-parent households and spending on health under policy option 2. This program will yield the same demographic structure, the same level of education among such families, and the same total tax revenues. The outlays under policy option 3 needed to induce the same level of education among the children of one-parent households, however, will be smaller than under policy option 2. These children will also have a lower level of consumption, a standard to which full orphans are tethered. It follows that there will be an excess of total revenue over expenditures. Let η be held constant, so as to keep the demographic structure unchanged, and let the taxes on two-parent households be reduced slightly, which will induce a small rise in $e(1)$. By continuity, there will still be enough funds to finance the additional subsidies to half and full orphans that will be needed to preserve equality in education, and hence in human capital in the next generation of adults. It follows that policy option 3 strictly dominates option 2 in all periods from $t = 0$ onward.

Full education is reached in 2050, as is the case under pooling, although productivity is 12 percent lower, because of the accumulated effects of lower attainments in the two preceding generations. Spending on measures to combat the disease is a little higher than under pooling at first but a little less from 2020 onward. It is about 13 percent lower than its counterpart under policy option 2 throughout, so that more premature deaths are implicitly accepted, although the differences in κ_t are small. A measure of the comparative efficiency of conditional educational subsidies is that satisfactory growth is

TABLE 7. Policy Option 3: Nuclear Families, School Attendance Subsidies

Year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
<i>K</i> = 395										
1990	3.14	0.57	0.57	973	280	1,886	1.0	1.0	0.850	22,337
2020	3.91	0.78	0.78	1,022	289	2,101	1.0	1.0	0.852	27,220
2050	5.99	1.00	1.00	1,022	197	2,310	1.0	1.0	0.852	40,960
2080	10.80	1.00	1.00	1,022	0	2,642	1.0	1.0	0.852	73,839
<i>K</i> = 200										
1990	3.14	0.58	0.58	716	287	1,502	1.0	1.0	0.86	22,342
2020	4.00	0.82	0.82	748	295	1,713	1.0	1.0	0.86	27,723
2050	6.36	1.00	1.00	748	0	1,702	1.0	1.0	0.86	43,515
2080	11.41	1.00	1.00	748	0	2,256	1.0	1.0	0.86	78,019

FIGURE 4. Policy Options 1, 2, and 3 ($K = 200$)

achieved with amounts paid to one-parent households that are barely a tenth of the lump-sum transfers made under policy option 2.

The tax burden on two-parent households is correspondingly lighter: the absolute payment per household is a little less than one-half of that under policy option 2 in 1990, rising to 56 percent in 2080. The difference in productivities is very large in 2080: namely $2z(1)$ to one, or 1.636, which implies a much lower relative tax burden. The latter falls from about 8.6 percent of income in 1990 to 3.6 percent in 2080 under policy option 3, and from 19.3 percent to 10.4 percent under policy option 2. Observe that although the payment of school attendance subsidies ends from 2080 onward, τ is higher than in 2050. The reason is that the raising and caring for full orphans require the time and effort of adults specifically employed for this purpose, the costs of which rise with λ .

An Optimistic Variant

This analysis of alternative policies concludes with brighter assumptions about the long-term rate of growth of productivity and the costs of antiretroviral drugs. The results for $z = 1$ and $K = 200$, perhaps the most plausible of optimistic constellations, are reported in tables 8–10. Table 8 sets out the first benchmark ($D = 0$). In the absence of the epidemic, the increase in z from 0.818 to 1 makes for large gains indeed over three generations. Full schooling is achieved in 2020, and λ in 2080 is almost 75 percent higher than in the reference case. Turning to policy interventions in the face of the epidemic, the optimal value of η is not

TABLE 8. An Optimistic Variant ($z = 1, K = 200$): The First Benchmark ($D = 0$)

	λ	e	κ	n/κ	$y(0)$
1960	2.62	0.50	0.86	4.65	19,503
1990	3.14	0.68	0.86	3.49	22,257
2020	5.26	1.00	0.86	3.49	35,967
2050	11.52	1.00	0.86	3.49	78,771
2080	24.04	1.00	0.86	3.49	164,380

TABLE 9. Policy Option 1: Spending on Combating the Disease under Pooling ($z = 1, K = 200$)

Year	λ	e	η	κ	n/κ	$y(0)$
1990	3.14	0.65	714	0.85	3.51	22,334
2020	5.07	1.00	714	0.85	3.51	34,690
2050	11.15	1.00	714	0.85	3.51	76,218
2080	23.29	1.00	714	0.85	3.51	159,274

TABLE 10. Policy Options 2 and 3: Nuclear Families, Lump-Sum and School Attendance Subsidies ($z = 1, K = 200$)

Year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
Policy option 2										
1990	3.14	0.54	0.54	796	2,294	3,863	1.0	1.0	0.86	22,439
2020	4.37	0.83	0.83	842	3,287	5,087	1.0	1.0	0.86	30,248
2050	8.25	1.00	1.00	842	31	2,033	1.0	1.0	0.86	56,400
2080	11.50	1.00	1.00	842	0	2,979	1.0	1.0	0.86	119,637
Policy option 3										
1990	3.14	0.62	0.62	716	287	1,513	1	1	0.85	22,276
2020	4.87	1.00	1.00	716	0	1,507	1	1	0.85	33,319
2050	10.75	1.00	1.00	716	0	2,164	1	1	0.85	73,476
2080	22.49	1.00	1.00	716	0	3,477	1	1	0.85	153,790

affected under pooling, and λ in 2080 is a mere 3 percent smaller than the corresponding benchmark value. With nuclear families, the pace is also distinctly quicker, even under policy option 2. A more efficient educational technology will do much to ease the task of maintaining growth and welfare in the face of the epidemic, but it will not necessarily stave off a collapse in the absence of any other intervention (table 3).

CONCLUSIONS

The AIDS epidemic will peak far in advance of the economic damage it will ultimately cause. In southern Africa, where prevalence rates among people aged 15–49 years are already 20 percent and higher, the worst is still to come. The scale of that damage, in terms of accumulated losses in GDP per capita, will also be large even if the measures designed to combat the disease and to ensure the education of orphans are well chosen, and the fiscal means employed to finance them are highly efficient. Without such measures, and given a continuation of high levels of mortality, economic collapse is a very real danger.

The main reason for these gloomy findings lies in the peculiarly insidious and selective character of the disease. By killing mostly young adults, AIDS does more than destroy the human capital embodied in them; it deprives their children of the very things they need to become economically productive adults—their parents' loving care, knowledge, and capacity to finance education. This weakening of the mechanism through which human capital is transmitted and accumulated across generations becomes apparent only after a long lag, and it is progressively cumulative in its effects. Therein lies the source of the difference between the findings in this article and those of many previous studies, which have focused either on the role of quasi-fixed factors over the medium run or on the historical record to date.

What are the lessons for public policy? Where the prevalence rate is still low, as in much of Asia, Eastern Europe, the Middle East, and Latin America, it is of the utmost importance to contain the disease at once: for the economic system as well as for individuals, an ounce of prevention is worth more than a pound of cure.

Where the epidemic is more advanced, combating the disease and its economic effects successfully will require a large and determined fiscal effort, the correct design of which is a complicated matter. Intuitively, the question is: What combination of measures should be adopted to promote the formation of human capital and good health when the threat of a collapse looms? These measures are partly complementary. Maintaining good health means that the human capital embodied in individuals during childhood and training will survive and pay off into old age, not only for them but also for their children. When public funds are very scarce, however, some tradeoffs will be unavoidable, requiring the concentration of resources on some programs or groups at the expense of others. The hope here is that the knowledge about what works in the fields of child rearing, education, the care of orphans, health, and so forth can reveal how to formulate combined programs of interventions that will ward off the threat of an economic collapse. The true social rate of return to such programs can be extremely high, whereas that derived from calculations based on standard (local) cost–benefit analysis may be quite modest. Fiscal policy in general, and policy in the social sectors in particular, must be formulated with a clear eye on its contribution to solving the

long-run economic problem posed by AIDS. For in the event of a collapse of productivity, little else will matter.

These points are vividly illustrated by the results for South Africa. In the absence of the epidemic, there would have been the prospect of modest, but accelerating growth of per capita income. An unabated continuation of the epidemic could bring about a progressive collapse. With the right interventions, this fate can be averted, although the costs are high, even under favorable social arrangements for the care of orphans. If those arrangements break down, growth is likely to be rather sluggish. These conclusions must be regarded as preliminary, and various aspects of the analysis need further work and refinement. That much conceded, the sensitivity analysis nevertheless suggests that these findings are robust to changes in a variety of key assumptions and parameter values. And it would be unconscionable to err on the side of optimism.

APPENDIX A1. MICROFOUNDATION OF THE MODEL

This appendix presents the microfoundation of the dynamical system. It does not attempt to model individuals' sexual behavior, with all its gratifications and risks. While the said probabilities are therefore exogenous, their values in the application to South Africa are based on the epidemiological work of Dorrington and others (2001).

The first topic is the formation of human capital. Consider a family at the start of period t . Let λ_t^f be the father's endowment of human capital, λ_t^m the mother's, and $A_t(s_t)$ their total human capital when the family is revealed to be in state s_t . Then,

$$(A1.1) \quad A_t(1) = \lambda_t^f + \lambda_t^m, A_t(2) = \lambda_t^m, A_t(3) = \lambda_t^f, A_t(4) = 0.$$

Assuming that there is assortative mating, $\lambda_t^f = \lambda_t^m$, equation (A1.1) specializes to

$$(A1.2) \quad A_t(1) = 2\lambda_t, A_t(2) = A_t(3) = \lambda_t, A_t(4) = 0,$$

where the superscripts f and m may now be dropped without introducing ambiguity.

Human capital is assumed to be formed by a process of child-rearing combined with formal education. In the course of rearing their children, parents give them a certain capacity to build human capital for adulthood, a capacity that is itself increasing in the parents' own human capital. This gift will be of little use, however, unless it is complemented by at least some formal education, in the course of which the basic skills of reading, writing, and calculating can be learned. Let the proportion of childhood devoted to education be denoted by $e_t \in [0, 1]$, the residual being allocated to work, and for simplicity, let all the

children in a family be treated in the same way.⁸ The human capital attained by each of the children on reaching adulthood is given by

$$(A1.3) \quad \lambda_{t+1} = \begin{cases} z(s_t)f(e_t)A_t(s_t) + 1, & s_t = 1, 2, 3 \\ \xi, & s_t = 4 \end{cases}.$$

Beginning with the upper branch of equation (A1.3), the term $z_t(s_t)$ represents the strength with which capacity is transmitted across generations. For simplicity, the father's and mother's contributions are assumed to be perfect substitutes: $z(2) = z(3)$. It is also assumed that where transmitting this capacity is concerned, two parents can rear a child at least as well as one, but, in view of perfect substitutability, no better than twice as well as one. Hence, recalling equation (A1.2),

$$(A1.4) \quad z(2) = z(3) \geq z(1) \geq z(2)/2 = z(3)/2.$$

Thus, the upper branch of equation (A1.3) can be rewritten as

$$(A1.5) \quad \lambda_{t+1} = (3 - s_t)z(s_t)f(e_t)\lambda_t + 1, \quad s_t = 1, 2$$

with both types of single-parent families being identical in this respect. The function $f(\cdot)$ represents educational technology—translating time spent on education into learning. It is assumed to be strictly increasing and differentiable, with $f(0) = 0$. Observe that equation (A1.3) and $f(0) = 0$ imply that children who do not attend school at all attain, as adults, only some basic level of human capital, which has been normalized to unity.

According to the lower branch of equation (A1.3), there is a miserable outcome for full orphans who do not enjoy the good fortune to be adopted or placed in (good) institutional care. Deprived of love and care, and left to their own devices, they go through childhood uneducated, to attain human capital ξ (≤ 1) in adulthood.

The next step is to relate human capital to current output, which takes the form of an aggregate consumption good. Output is assumed to be proportional to inputs of labor measured in efficiency units. A natural normalization is that an adult who possesses human capital in the amount λ_t is endowed with λ_t efficiency units of labor, which he or she supplies completely inelastically. A child's efficiency will be somewhat lower than the parents', all other things being equal, on the grounds of age alone. To reflect these considerations, let a child supply $(1 - e_t)\gamma$ efficiency units of labor when the child works $1 - e_t$ units of time. It is

8. This analysis skips the fact that girls often receive less education than boys. The ensuing inequality in human capital introduces analytical and empirical difficulties whose importance, for the purpose of this article, does not seem to warrant specific treatment.

plausible to assume that $\gamma \in (0, \xi)$, that is, a full-time working child is at most as productive as an adult who happened to be an uneducated orphan. A family with n_t children therefore has a total income in state s_t ($s_t = 1, 2, 3$) of

$$(A1.6) \quad y_t(s_t) = \alpha[A_t(s_t) + n_t(1 - e_t)\gamma]$$

where the scalar $\alpha (> 0)$ denotes the productivity of human capital, measured in units of output per efficiency unit of labor input.

Household Behavior

All allocative decisions are assumed to lie in the parents' hands, as long as they are alive. Any bequests at death are ruled out, so that the whole of current income, as given by equation (A1.6), is consumed. Within the family, let the husband and wife enjoy equality as partners, and let each child obtain a fraction $\beta \in (0, 1)$ of an adult's consumption if at least one adult survives. Full orphans ($s_t = 4$) do not attend school and consume what they produce as child laborers.

Without any taxes or subsidies, the household's budget constraint may therefore be written as

$$(A1.7) \quad [(3 - s_t) + n_t\beta]c_t + \alpha n_t\gamma e_t \leq \alpha[(3 - s_t)\lambda_t + n_t\gamma], \quad s_t = 1, 2$$

where c_t is the level of each adult's consumption. The expression on the left hand side represents the costs of consumption and the opportunity costs of the children's schooling. The expression on the right hand side is the family's so-called full income⁹ in state $s_t = 1, 2, 3$. Observe that single-parent households not only have lower levels of full income than their otherwise identical two-parent counterparts, but they also face a higher relative price of education, defined as $\alpha n_t\gamma / [(3 - s_t) + n_t\beta]$.

Couples have children while they are young until some exogenously fixed number have survived infancy, a target that may vary from period to period. With n_t thus fixed,¹⁰ the adults wait until the state of the family becomes known, and the survivor then chooses some feasible pair (c_t, e_t) subject to condition (A1.7).

Parents are assumed to have preferences over their own current consumption and the human capital attained by their children in adulthood, taking into account the fact that investment in a child's education will be wholly wasted if that child dies prematurely in adulthood. Let mothers and fathers have identical preferences, and for two-parent households, let there be no joint aspect to the consumption of the pair (c_t, e_t) : each surviving adult derives (expected) utility

9. A household's full income is the scalar product of its endowment vector and the vector of market prices. Here, output is taken as the numéraire.

10. Although there is much evidence in favor of at least some replacement fertility, this is evidently a strong assumption. In the numerical application, however, variations in n_t turn out to have only weak effects on the system's dynamics (see table 4).

from the pair so chosen, and these utilities are then added up within the family. In effect, whereas c_t is a private good, the human capital of the children in adulthood is a public good within the marriage.

Since all the children attain λ_{t+1} , the only form of uncertainty is that surrounding the number who will not die prematurely as adults, which is denoted by the random variable a_{t+1} . Let preferences be separable, with representation

$$(A1.8) \quad EU_t(s_t) = (3 - s_t)[u(c_t) + (E_t a_{t+1})v(\lambda_{t+1})], \quad s_t = 1, 2$$

where the contribution $v(\lambda_{t+1})$ counts only when death does not come early, E_t is the expectation operator, and $E_t a_{t+1}$ is the expected number of children surviving into old age. The subutility functions $u(\cdot)$ and $v(\cdot)$ are assumed to be increasing, continuous, concave, and twice-differentiable. Denoting by κ_t^e , the parents' subjective probability that a child will survive to old age and recalling assumption 1 and that all children are treated identically yield

$$(A1.9) \quad E_t a_{t+1} v(\lambda_{t+1}) = n_t \kappa_{t+1}^e v(\lambda_{t+1})$$

where λ_{t+1} is given by equation (A1.3). A reduction in κ_{t+1}^e therefore effectively entails a weaker taste for children's education. It will be convenient in what follows to rewrite equation (A1.8) as

$$(A1.10) \quad EU_t(s_t) = (3 - s_t)\{u(c_t) + n_t \kappa_{t+1}^e \cdot v[z(s_t)f(e_t)A_t(s_t) + 1]\}, \quad s_t = 1, 2$$

since both types of single-parent families are identical. Hence, it suffices to examine the states $s_t = 1, 2$. A family in state $s_t (= 1, 2)$ in period t solves the following problem:

$$(A1.11) \quad \max_{[c_t(s_t), e_t(s_t)]} EU_t(s_t) \quad \text{s.t. (A1.7), } c_t \geq 0, e_t \in [0, 1].$$

Let $[c_t^0(s_t), e_t^0(s_t)]$ solve problem (A1.11), whose parameters are $(\alpha, \beta, \gamma, \kappa_{t+1}^e, \lambda_t, n_t)$. Using the envelope theorem yields

$$\frac{\partial EU_t(s_t)}{\partial \lambda_t} > 0, \frac{\partial EU_t(s_t)}{\partial \kappa_{t+1}^e} > 0.$$

Since current consumption is maximized by choosing $e_t = 0$, it follows that the parents' altruism toward their children must be sufficiently strong if they are to choose $e_t > 0$.

If both goods are noninferior, it follows at once that

$$\frac{\partial e_t^0(s_t)}{\partial A_t(s_t)} \geq 0, \quad \frac{\partial c_t^0(s_t)}{\partial A_t(s_t)} \geq 0.$$

Dynamics

There are no insurance arrangements in the above account, so that premature adult mortality in period t will affect not only the level but also the distribution of human capital in period $t + 1$. As noted above, full orphans will suffer low productivity in adulthood, as expressed in the lower branch of equation (A1.3). Such mortality also affects the distribution of families across states 1, 2, and 3 in period t and will thus affect the level and distribution of human capital in period $t + 1$ if $e_t^0(s_t)$ varies across states and with the severity of premature adult mortality, as it normally will when λ_t is not too large. These repercussions will then make themselves felt in future periods, even if premature adult mortality vanishes after period t .

To state all this formally, recall that the family chooses $e_t^0(s_t; \cdot)$ in light of its resources and expectations so as to solve problem (A1.11). Hence, equation (A1.3) may be written so as to make these influences explicit:

$$(A1.12) \quad \lambda_{t+1} = \begin{cases} z(s_t)f(e_t^0(A_t(s_t), s_t, \kappa_{t+1}^e))A_t(s_t) + 1, & s_t = 1, 2, 3 \\ \xi, & s_t = 4 \end{cases}$$

Equation (A1.12) describes a random dynamical system—random in the sense that each child in any given family state s_t can wind up in any of the states $s_{t+1} \in \{1, 2, 3, 4\}$ after reaching adulthood and forming a family in period $t + 1$.

APPENDIX A2. THE FUNCTION $q(\cdot)$

With four parameters to be estimated, two additional independent conditions beyond those in the text are required. One way of proceeding is to pose the question: What is the marginal effect of efficient spending on q in high- and low-prevalence environments? That is to say, estimates are needed of the derivatives of $q(\eta; D = 1)$ at $\eta = 0$ and some value of η that corresponds to heavy spending, when the scope for exploiting cheap interventions has been exhausted. To obtain such estimates, we used the estimated costs of preventing a case of AIDS or saving a disability-adjusted life year by various methods, as reported by Marseille, Hofmann, and Kahn (2002).

When the prevalence rate is high, the authors argue, the most cost-efficient form of intervention is to target prostitutes for the specific purpose of controlling sexually transmitted diseases and promoting the use of condoms. The associated cost per AIDS case averted in Kenya is given as \$8–12. It seems reasonable to infer that this cost recurs annually. Other preventive measures are less cost-effective by a factor of up to 10 or more. Marseille, Hofmann, and Kahn (2002) put the

average cost per disability-adjusted life year of a diverse bundle of such measures at \$12.50. For these measures, the assumption that η produces a pure public good is not far off the mark. Now, a reduction in q of 0.01 over a span of 30 years yields 0.3 disability-adjusted life years. Allowing for the fact that there is substitution among diseases, that is, if one does not succumb to AIDS, there is always the threat of something else, the expenditure of another \$12.50 when η is small will yield a net reduction in $q(D = 1)$ of about $(0.01) \cdot (1/0.3) \cdot [1 - q(D = 0)] = 0.028$. Recalling that η is defined with reference to a population of adults whose measure has been normalized to unity and rounding up to \$15, we have

$$(A2.1) \quad \frac{dq(0; D = 1)}{d\eta} = -\frac{cb}{(a + c)^2} = -\frac{0.028}{15}.$$

Following the purposive and determined implementation of the full battery of preventive measures, the remaining intervention is to treat the infected. There is now neither a cure nor the prospect of one for perhaps decades to come. Opportunistic infections can, of course, be treated in the later stages of the disease, and the onset of full-blown AIDS can be delayed for some years through the controlled use of antiretroviral therapies. Such measures will do little to reduce q as strictly defined, but by keeping infected individuals healthier and extending life a bit, they will raise lifetime income and improve the parental care enjoyed by children in affected families.

In the context of the model, therefore, it seems perfectly defensible to interpret these gains as equivalent to a reduction in q . Marseille, Hofmann, and Kahn (2002) put the cost of saving a disability-adjusted life year by such means at \$395, assuming that the drugs take the form of low-cost generics and explicitly neglecting the costs of the technical and human infrastructure needed to support an effective, so-called highly active antiretroviral therapy regimen of this kind. It is assumed here that the highly active antiretroviral therapy regimen is the efficient, marginal form of intervention when a low prevalence rate has resulted from a determined, extensive, and continuing effort at prevention. To complete the specification of this case, the level of aggregate spending at which highly active antiretroviral therapy becomes the best choice at the margin must be determined. Note that in the absence of diminishing returns to preventive measures, it would be possible to attain the status quo ante ($D = 0$) by spending

$$(A2.2) \quad [q(0; D = 1) - q(D = 0)] \cdot (15/0.028) = 278.$$

In fact, diminishing returns will set in as the prevalence rate falls. Where preventing mother-to-child transmission is concerned, for example, a drop in the prevalence rate from 30 percent to 15 percent will almost double the cost of

saving a disability-adjusted life year (Marseille, Hofmann, and Kahn 2002). Since 15 percent hardly counts as a low level of prevalence, it seems fairly safe to assume that highly active antiretroviral therapy will not become cost-efficient until spending on preventive measures, and the treatment of opportunistic infections is at least triple the above estimate. Thus the required fourth condition is:

$$(A2.3) \quad \frac{q'(0; D = 1)}{q'(835; D = 1)} = \frac{395}{15}.$$

The four conditions (8), (9), (A2.1), and (A2.3) may be solved to yield the values of the parameters $a, b, c,$ and d for men, women, and both combined, as set out in table A2-1. Premature adult mortality is defined precisely as death before age 50, conditional on surviving to age 20, the corresponding probability being denoted as ${}_{30}q_{20}$.

By way of sensitivity analysis, to K , the function $q(\cdot)$ is respecified as follows:

$$(A2.4) \quad q(\eta; D = 1) = d - \frac{1}{a + ce^{-b\eta(K/395)}}$$

where K denotes the annual cost of a course of generic drugs. Equations (A2.1) and (A2.3) are then modified to read

$$(A2.5) \quad \frac{dq(0; D = 1)}{d\eta} = -\frac{cb(K/395)}{(a + c)^2} = -\frac{0.028}{15}$$

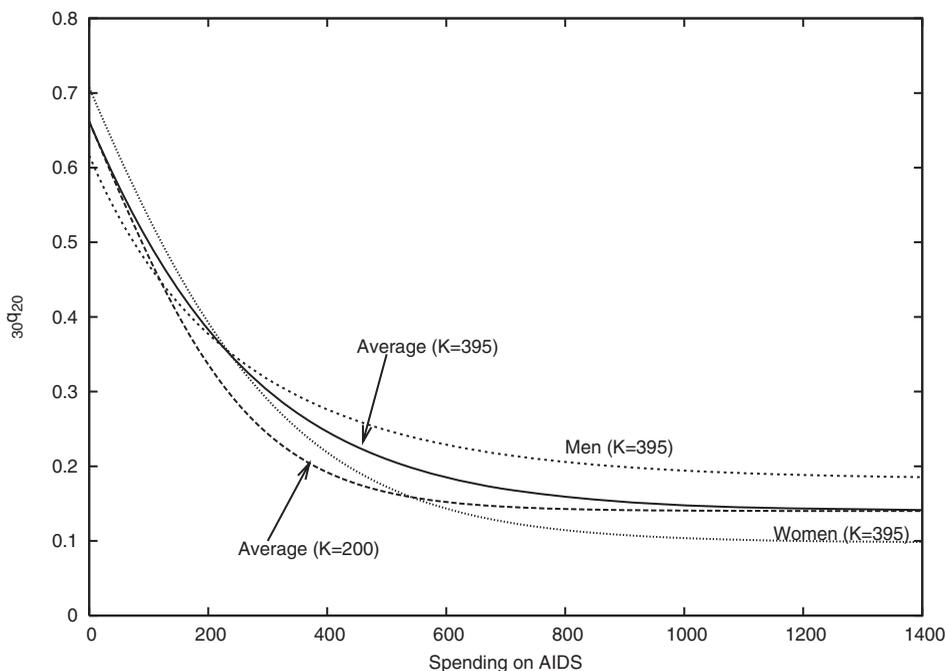
and

$$(A2.6) \quad q(278 + (835 - 278)(K/395), K, D = 1) = q(835, K = 395, D = 1)$$

respectively. Setting $K = 200$ yields the parameters reported in the lower half of Table A2-1 and the associated function in figure A2-1. The shift of $q(\cdot)$ represents the favorable effects of the said reduction in costs on the government's possibilities of reducing premature adult mortality.

TABLE A2-1. Parameters of the $q(\cdot)$ Function

	a	b	c	d
$K = 395$				
Women	0.6613	0.0051	0.4464	1.6101
Men	-0.6555	0.0034	0.1451	-1.3432
Average	0.3562	0.4396	0.8145	2.9450
$K = 200$				
Women	1.0532	0.0169	1.8875	1.0473
Men	0.6469	0.0118	0.2524	1.7280
Average	0.1633	0.0073	0.0152	6.2653

FIGURE A2-1. Premature Adult Mortality ${}_{30}q_{20}$ as a function of η and K 

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