

POLICY RESEARCH WORKING PAPER

WPS 2676

2676

An Alternative Unifying Measure of Welfare Gains from Risk-Sharing

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Unlike the traditional measure of welfare gains from risk-sharing, the new measure presented here does not depend on the horizon, and it is robust to alternative specifications of the consumption stochastic processes and preferences. This measure shows that if international risk-sharing eliminates volatility in aggregate consumption and leads to greater consumption growth, risk-sharing can have a sizable impact on consumer welfare.

The World Bank
Latin America and the Caribbean Region
Economic Policy Sector Unit
September 2001



Summary findings

Following Lucas's (1987) standard approach, welfare gains from international risk-sharing have been measured as the percentage increase in consumption levels that leaves individuals indifferent between autarky and risk-sharing. Auffret proposes to measure welfare gains as the increase in consumption growth instead of consumption levels. When the consumption process is nonstationary, Auffret's proposed measure has several attractive features: it does not depend on the horizon, and it is robust to alternative specifications of the consumption stochastic processes (from geometric Brownian processes to Orstein-Uhlenbeck mean-reverting processes) and preferences (from constant relative risk aversion

preferences to Kreps-Porteus preferences). The author then uses this measure to estimate potential welfare gains from international risk-sharing for a representative U.S. consumer.

Auffret finds that if international risk-sharing leads only to a complete elimination of aggregate consumption volatility (with no impact on consumption growth), it represents gains to a U.S. consumer of only \$12 a year on average. But if international risk-sharing also permits an increase in consumption growth, it may have a sizable impact on welfare. Each 0.5 percentage point increase in consumption growth represents gains to a U.S. consumer of about \$160 a year on average.

This paper—a product of the Economic Policy Sector Unit, Latin America and the Caribbean Region—is part of a larger effort in the region to analyze the impact of catastrophic risks on welfare and determine whether the catastrophic insurance function can serve a key developmental role in disaster prone countries. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Kevin Tomlinson, mail stop I4-403, telephone 202-473-9763, fax 202-676-1494, email address ktomlinson@worldbank.org. Policy Research Working Papers are also posted on the Web at <http://econ.worldbank.org>. The author may be contacted at pauffret@worldbank.org. September 2001. (23 pages)

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**AN ALTERNATIVE UNIFYING MEASURE OF WELFARE GAINS
FROM RISK-SHARING**

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* I want to thank Aart Kraay and Ali Khadr for useful comments on an earlier version of this paper and Jacqueline Mora for her research assistance.

1. Introduction

How big are welfare gains from risksharing? This question has received much interest in the literature starting with Lucas' (1987) seminal paper. Indeed, if unexploited welfare gains from risksharing are small, then small imperfections in the economy would suffice to prevent risksharing mechanisms to develop beyond existing levels. However, the presence of large unexploited welfare gains would lead to the next question: Why do not risksharing mechanisms develop further? Answers to this question may have important implications for public policy. Indeed, if risksharing mechanisms remain incomplete in the presence of large unexploited welfare gains, measures to facilitate their development could enhance economic welfare.

Numerous papers in the international economics literature have sought to derive the welfare gains from international risksharing. To do so, papers have followed Lucas' (1987) approach and measured welfare gains as the percentage increase in consumption, uniform across all dates and states of the world that makes people indifferent between the existing stochastic consumption path and that which would result from international risksharing. However, this measure of welfare gains depends on a number of parameters including the discount rate and the time horizon and is sensitive to alternative specifications of preferences and consumption stochastic processes. Unsurprisingly, van Vincoop mentions that "the reported results varies enormously, ranging from less than 0.1% to over 100%" (van Vincoop, 1999, p.110).

This paper has two objectives. The first is to propose an alternative measure of welfare gains from risksharing. Under this alternative measure, gains from risksharing are defined as the additional percentage points of consumption growth which make people indifferent between the existing stochastic consumption path and that under risksharing. This alternative measure has several attractive features: it does not depend on the discount rate and the horizon, and is robust to alternative specifications of preferences (from constant relative risk aversion (CRRA) preferences to Kreps-Porteus preferences) and stochastic processes (from geometric Brownian processes to Orstein-Uhlenbeck mean-reverting processes) for consumption. The second is to use the alternative measure to estimate potential welfare gains from international risksharing for a representative US consumer. The main finding of the paper is that potential welfare gains from risksharing are small if risksharing leads exclusively to a decrease in volatility but may become substantial if risksharing results in an increase in consumption growth.

The remainder of this paper is organized as follows. Section 2 briefly reviews the literature on welfare gains from international risksharing. Section 3 derives the conventional measure of welfare gains from risksharing as well as the proposed alternative measure under alternative stochastic processes and preferences. Section 4 evaluates the welfare gains from international risksharing for a representative US consumer using the alternative measure. The final section summarizes the main results and suggests directions for future research.

2. Literature on welfare gains

In his 1987 seminal paper, Lucas sought to assess the costs of economic fluctuations. Under the assumption that (log of) per-capita consumption follows a trend-stationary process, Lucas evaluated the welfare gains that would result from a complete elimination of the variability of aggregate consumption with no change in consumption growth for a representative US consumer. Using aggregate consumption data, he found these costs to be less than one tenth of a percentage point of expected consumption which seems "to be an extremely low estimate of the costs of economic instability" (p. 27). Lucas qualified his result. First, he acknowledged that since setting the objective of a complete elimination of *all* consumption variability does not result from an

individual's optimization problem it may not be "a feasible or a desirable objective of policy" (p. 27). Second, he mentioned that eliminating the variability of aggregate consumption "cannot be expected to eliminate more than a small part of the uninsurable risk borne at the individual level" (p. 29).

Papers from the international economics literature have adapted Lucas' method to assess the gains from international risksharing using two approaches.¹ A first approach based on a general equilibrium framework, starts from the observation that the perfect risksharing hypothesis implies that country-specific consumption growth should be perfectly correlated across countries while it should be uncorrelated with country-specific income growth. However, these theoretical findings are generally rejected in regression-based tests.² Whether the imperfect risksharing outcome is surprising or not depends on the size of the potential gains from international risksharing. Consequently, consumption-based models have asked the question: How much welfare gains can be obtained from lowering country-specific consumption variance to the variance which would result from sharing aggregate business cycle risks among countries? To derive the welfare gains from international risksharing, these models have typically assumed that country-specific consumption paths follow exogenously given stochastic processes *with identical means*. A second approach based on a partial equilibrium CAPM framework, starts from the observation that there is a strong home bias in international stock and bond portfolios.³ Papers based on a portfolio approach have sought to assess how much welfare gains would result from maximizing the expected utility of a portfolio made of country-specific securities that follow exogenously given stochastic processes. National stock market indices (Lewis, 1996) and GDP-based securities (Shiller, 1993; Shiller and Athanasoulis, 1997) have been typically taken as proxies for country-specific securities. In these models, the welfare gains from diversification result from changes in *both the means and variance* of the optimal portfolio compared to the currently held portfolio. Gains from diversification have typically been estimated to be about 100 times larger under the portfolio-based approach than under the consumption-based approach.⁴

To understand how changes in growth and variance of (per-capita) consumption translate into measurements of welfare gains, the existing literature has followed Lucas' (1987) standard approach. Welfare gains are measured as the percentage increase consumption (uniform across all dates and states of the world) that leaves individuals indifferent between autarky and the hypothetical economy whose consumption growth and variance result from risksharing. However, while Lucas assumed that consumption followed a trend-stationary process, subsequent papers have assumed that consumption followed non-stationary (unit-root) processes.⁵ While empirically it is hard to say whether consumption is stationary or non-stationary, theory tells us that optimizing behavior leads to a random walk in consumption. However, while the measure of welfare gains proposed by Lucas depends on a few parameters when the consumption stochastic process is stationary, this measure depends on and is sensitive to a large set of parameters once the consumption process is assumed to be non-stationary. Numerous papers have sought to analyze the dependence of welfare gains on the specifications of such parameters including the horizon (van Wincoop, 1999; Athanasoulis and van Wincoop, 2000), alternative stochastic processes (Obstfeld, 1994b; van Wincoop, 1999) and alternative preferences (Obstfeld, 1994b; van Wincoop, 1994; 1999). Consequently, depending on the assumptions they have made, some papers have reported small gains from international risk sharing (Lucas, 1987; Cole and Obstfeld,

¹ For a recent survey, see Lewis (1996).

² Lewis (1999) provides a review of this literature.

³ For a recent paper which documents the home bias puzzle, see Tesar and Werner (1998).

⁴ For a more detailed discussion, see Lewis (1999).

⁵ The only exception we know is Obstfeld (1994b) which considers both stationary and non-stationary processes).

1991; Tesar, 1995) while others (van Wincoop, 1994, 1996; Lewis, 1996; Shiller and Athanasoulis, 1995 and Obstfeld, 1994b, 1995) report much larger welfare gains. This has led to widely different results which have led to considerable confusion over the true magnitude of the welfare gains from international risksharing.

The strategy adopted in this paper is to provide an alternative unifying measure for evaluating the welfare gains from risksharing which only depends on a few parameters when economic uncertainties are generated by non-stationary stochastic processes. The basic intuition is that if welfare gains measured as the percentage increase in consumption *level*, uniform across dates and states, depend on a few parameters when the exogenously assumed consumption process is stationary (Lucas, 1987), measures of welfare gains measured as the percentage points increase in consumption *growth* equally depend on a few parameters when the consumption process is non-stationary. This alternative measure depends on a few parameters, does not depend on the horizon and is robust to changes in the stochastic processes (from geometric Brownian processes to Ornstein-Uhlenbeck mean-reverting processes) and alternative preferences (from constant relative risk aversion (CRRA) preferences to Kreps-Porteus preferences). Additionally, the paper shows that the consumption-based papers that have assumed a non-stationary process (Cole and Obstfeld, 1991; Obstfeld, 1994a; van Wincoop, 1994; Tesar, 1995; Lewis, 1996) would have derived more uniform welfare gains from risksharing had they used this alternative measure. As such the alternative measure of welfare gains unites the literature on welfare gains from international risksharing.

3. Welfare gains from risk sharing: Theory

3.1 Model

The basic model used in the paper is a representative agent multi-country model where the dynamics of (per-capita) consumption is described by an exogenously defined diffusion-type stochastic process. Consider a world economy consisting of N countries with a representative individual in each country. In making plans, individuals take account of the welfare of their descendants with a horizon H . This intergenerational interaction is modeled by assuming that living individuals maximize expected utility over an horizon H that may exceed their lifetime.⁶ Population follows a deterministic process with exogenous constant growth n .⁷ Individuals have time-separable expected utility functions over a single tradable consumption good. It is assumed that a representative individual has a utility function with CRRA preference given by $u(c) = \frac{c^{1-\rho} - 1}{1-\rho}$ where $\rho > 0$ and $\rho \neq 1$ is the coefficient of relative risk aversion or $u(c) = \text{Log} c$ the limiting case when $\rho = 1$.⁸ The multiplication of $u(c)$ by the family size represents the adding up of utils of all family members alive at time t . We define β as the rate of time preference ($\beta > 0$).⁹ Expected utility at time 0 is

⁶ This setting is appropriate if altruistic parents provide transfers to their children, who give in turn to their children. The individual with a finite horizon that we model corresponds to finite-lived individuals who are connected via a pattern of operative intergenerational transfers that are based on altruism.

⁷ The model could be expanded to allow for stochastic population growth. See Merton, 1975.

⁸ The inclusion of the term -1 in the formula is convenient because it implies that $u(c)$ approaches $\text{Log}(c)$ as $\rho \rightarrow 1$.

The term $-1/(1-\rho)$ can, however be omitted without affecting the subsequent results which are invariant to linear transformations of the utility function.

⁹ Kocherlakota (1990) argues that the time discount rate could be negative.

$$V = E_0 \left[\int_0^H e^{(n-\beta)t} \cdot \frac{c_t^{1-\rho} - 1}{1-\rho} \cdot dt \right] \quad (1)$$

Per-capita consumption is assumed to follow an exogenous stochastic process (SP1) defined by

$$c_0; \frac{dc_t}{c_t} = \mu \cdot dt + \sigma \cdot dZ \quad (SP1)$$

where c_0 represents per-capita consumption at time 0 and the second term is the differential representation for the dynamics of c_t where $dZ = N(0, dt)$ is a standard Wiener or Brownian motion process with expectation and variance equal to 0 and dt respectively. We assume that both μ and σ are constant. Under these assumptions, per-capita consumption follows a geometric Brownian motion with drift.

Proposition 1. *When (per-capita) consumption follows the non-stationary stochastic process (SP1), expected utility at time 0 is given by*

$$V(c_0, \mu, \sigma^2, \rho, H) = \frac{c_0^{1-\rho}}{1-\rho} \cdot \frac{1 - e^{[\beta - n + (\rho-1)\bar{\mu}]H}}{\beta - n + (\rho-1)\bar{\mu}} - \frac{1}{1-\rho} \cdot \frac{1 - e^{-(\beta-n)H}}{\beta-n} \quad (2)$$

$$\text{where } \bar{\mu} \equiv \mu - \frac{1}{2} \cdot \rho \cdot \sigma^2$$

The propositions are proved in the appendix. $F(\bar{\mu}, \rho) \equiv \beta - n + (\rho-1)\bar{\mu}$ acts like a discount rate for this economy. We assume that $F(\bar{\mu}, \rho) > 0$. As anticipated, expected utility at time 0 increases with initial consumption c_0 $\left(\frac{\partial V}{\partial c_0} > 0\right)$. Note that since $\left(\frac{\partial V}{\partial \bar{\mu}} > 0\right)$ expected utility increases with $\bar{\mu}$ which implies that higher consumption growth (keeping variance constant) and lower volatility (keeping growth constant) are associated with higher expected utility.

3.2 Welfare gains: definition

We want to assess the potential welfare gains of policies that affect both consumption growth and variance. More specifically we assume that once these policies are implemented, consumption follows the stochastic process (SP2)

$$c_0; \frac{dc_t}{c_t} = \mu' \cdot dt + \sigma' \cdot dZ' \quad \text{where } dZ' = N(0, dt) \quad {}^{10} \quad (SP2)$$

Define $\bar{\mu}' = \mu' - \frac{1}{2} \cdot \rho \cdot \sigma'^2$. To ensure that expected utility improves under (SP2), we assume that $\bar{\mu}' > \bar{\mu}$. We also assume that $F(\bar{\mu}', \rho) > 0$. Note that at this stage we are not asking whether adopting policies which modify the stochastic process is feasible or desirable. Neither are we asking which policies, if adopted, would lead to a change in the stochastic process although, consistent with the literature, we are implicitly assuming that these policies increase international risk-sharing. We are simply focusing on the potential welfare changes that would correspond to a change in the means and variance of the consumption stochastic process. Two different but related definitions of welfare gains are proposed.

Definition 1: *The measure p of welfare gains is implicitly defined as*

¹⁰ Note that we do not impose that the Wiener process dZ' be identical to dZ .

$$V((1+p) \cdot c_0, \mu, \sigma^2, \rho, H) = V(c_0, \mu', \sigma'^2, \rho, H) \quad (3)$$

Under this definition introduced by Lucas (1987) and extensively used in the literature afterwards,¹¹ the measure p of welfare gains is the percentage increase in consumption at time 0 that makes individuals indifferent between the existing consumption dynamics defined by (μ, σ) and the consumption dynamics defined by (μ', σ') . Equivalently, p is such that individuals are indifferent between

$$(1+p) \cdot c_0; \frac{dc_t}{c_t} = \mu \cdot dt + \sigma \cdot dZ \quad (\text{SP3}) \quad \text{and} \quad c_0; \frac{dc_t}{c_t} = \mu' \cdot dt + \sigma' \cdot dZ' \quad (\text{SP2})$$

where first terms represent consumption at time 0 and second terms are the differential representation for the dynamics of c_t . Note that the compensating increase in current consumption results in an increase in expected consumption in all dates and states of the world given by

$$\forall t \leq H; \frac{E_0 \left[c_{(1+p)c_0, \mu, \sigma}(t) \right]}{E_0 \left[c_{c_0, \mu, \sigma}(t) \right]} = 1 + p \quad (4)$$

where $c_{c_0, \mu, \sigma}(t)$ and $c_{(1+p)c_0, \mu, \sigma}(t)$ represent per-capita consumption under (SP1) and (SP3) respectively. We propose to consider the following alternative measure of welfare gains.

Definition 2: The measure τ of welfare gains is implicitly defined as:

$$V(c_0, \mu + \tau, \sigma^2, \rho, H) = V(c_0, \mu', \sigma'^2, \rho, H) \quad (5)$$

This measure of welfare gains does not seem to have been used in the literature. It can be interpreted as the additional percentage points of consumption growth which make individuals indifferent between the consumption dynamics defined by $(\mu + \tau, \sigma)$ and the consumption dynamics defined by (μ', σ') while keeping consumption c_0 unchanged. Equivalently, τ is such that individuals are indifferent between

$$c_0; \frac{dc_t}{c_t} = (\mu + \tau) \cdot dt + \sigma \cdot dZ \quad (\text{SP4}) \quad \text{and} \quad c_0; \frac{dc_t}{c_t} = \mu' \cdot dt + \sigma' \cdot dZ' \quad (\text{SP2})$$

Note that

¹¹ Recent papers that have used this definition include Cole and Obstfeld (1991), Lewis (1996), Obstfeld (1994 a,b), van Wincoop (1994, 1999). Tesar (1995) uses lifetime consumption instead of initial consumption. Consequently, her measure of welfare gain is $p_T = \frac{\mu}{e^{\mu H} - 1} \cdot p \approx \frac{p}{H}$ for $\mu \rightarrow 0$. Since she takes $H = 100$, this helps explain why the welfare gains she derives are about 100 times smaller than that reported in other papers.

$$\forall t \leq H; \frac{E_0 \left[c_{c_0, \mu+\tau, \sigma}(t) \right]}{E_0 \left[c_{c_0, \mu, \sigma}(t) \right]} = e^{\tau \cdot t} \quad (6)$$

Consequently, the compensating increase in the consumption growth rate represents the growth rate of expected consumption in all states of nature.

The two definitions of welfare gains can be interpreted in terms of "willingness to pay". Under this interpretation, p^* represents the percentage decline in (per-capita) consumption at time 0 that individuals are willing to pay to "benefit" from the stochastic process defined by (μ', σ') i.e. p^* is such that individuals are indifferent between

$$c_0; \frac{dc_t}{c_t} = \mu \cdot dt + \sigma \cdot dZ \quad (\text{SP1}) \quad \text{and} \quad (1-p^*) \cdot c_0; \frac{dc_t}{c_t} = \mu' \cdot dt + \sigma' \cdot dZ' \quad (\text{SP5})$$

It can be immediately derived that $1-p^* = \frac{1}{1+p}$ which implies $p \approx p^*$ when p is small ($p \approx 0$).

Similarly, τ^* can be interpreted as the decrease in percentage points of consumption growth that individuals are willing to pay to "benefit" from the stochastic process defined by (μ', σ') i.e. τ^* is such that individuals are indifferent between

$$c_0; \frac{dc_t}{c_t} = \mu \cdot dt + \sigma \cdot dZ \quad (\text{SP1}) \quad \text{and} \quad c_0; \frac{dc_t}{c_t} = (\mu' - \tau^*) \cdot dt + \sigma' \cdot dZ' \quad (\text{SP6})$$

Proposition 2. *The welfare gains τ and p of policies that modify the stochastic process followed by consumption from (SP1) to (SP2) are given by*

$$\tau = \bar{\mu}' - \bar{\mu} \quad (7)$$

$$\text{Log}(1+p) = \frac{1}{\rho-1} \cdot \text{Log} \left[\frac{1 - e^{-[\beta - n + (\rho-1) \cdot \bar{\mu}]H}}{\beta - n + (\rho-1) \cdot \bar{\mu}} \cdot \frac{1 - e^{-[\beta - n + (\rho-1) \cdot \bar{\mu}']H}}{\beta - n + (\rho-1) \cdot \bar{\mu}'} \right] \quad (8)$$

The measure τ of welfare gains only depends on the coefficient of relative risk aversion and the growth and variance of the initial and final stochastic processes for consumption. However, the measure p of welfare gains depends on the degree of risk aversion, the time horizon and the discount rates. This outcome is not surprising. The additional percentage points of consumption growth τ that leave individuals indifferent between the two stochastic processes do not depend on the horizon. Consequently, the conversion of extra percentage points of growth τ into a percentage p of current consumption requires to know the discount rate and the time horizon in addition to the stochastic parameters and the degree of aversion to risk

Using the previous relations, we now evidence the dependency of the measure p of welfare gains on the horizon and the parameters (μ', σ') of the final consumption dynamics.

Proposition 3. *Given initial (SP1) and final (SP2) stochastic processes for consumption, the measure p of welfare gains is an increasing function of the horizon H .*

As the horizon increases, the stream of welfare benefits becomes larger which translates into higher percentage-equivalent p of current consumption. This outcome is consistent with previous papers (van Wincoop, 1999; Athanasoulis and van Wincoop, 2000) which analyze the dependency of p on the horizon. As the horizon becomes longer, the measure p of welfare gains monotonically increases to the asymptotic value $p(H = \infty)$ when horizon is infinite ($H = \infty$) with

$$\text{Log}(1 + p(H = \infty)) = \frac{1}{\rho - 1} \cdot \text{Log} \left[\frac{F(\bar{\mu}', \rho)}{F(\bar{\mu}, \rho)} \right] \quad (9)$$

Obsfeld (1994b) derives a similar relation in a discrete time version of the model. Note that when the initial and final discount rates are close $F(\bar{\mu}, \rho) \approx F(\bar{\mu}', \rho)$ or equivalently $\bar{\mu}' \approx \bar{\mu}$ and the horizon is infinite, the measure p of welfare gains becomes

$$p(H = \infty) \approx \frac{\tau}{F(\bar{\mu}, \rho)} \quad (10)$$

Consequently, at infinite horizon, the measure p of welfare gains is equal to the measure τ divided by the discount rate. In addition

$$\frac{d[\log(1 + p_B)]}{dH} (H = 0) = \frac{1}{2} \cdot (\bar{\mu}' - \bar{\mu}) \cdot \quad (11)$$

Graph 1: Welfare gain and horizon

Proposition 4. *Given an initial stochastic process (SPI) for consumption, the measure p of welfare gains is an increasing function of $\bar{\mu}'$.*

This outcome is not surprising. Higher growth (keeping variance constant) or lower variance (keeping growth constant) of the final stochastic process translates into higher welfare gains in terms of percentage of current consumption. Graph 2 illustrates this point.

Graph 2: Welfare gain and variance

In order to calibrate the model, it is useful to determine the implicit risk-free interest rate that corresponds to this economy. Indeed, as evidenced in the empirical section of the paper, differences between reported welfare gains across papers can largely be explained by the choice of parameters that allow for unrealistically large implicit risk-free interest rates.

Proposition 5: *The implicit risk-free interest rate in this economy is constant and given by*

$$R = \beta - n + \rho \cdot \left(\mu - \frac{1}{2} \cdot \rho \cdot \sigma^2 \right) - \frac{1}{2} \cdot \rho^2 \cdot \sigma^2 \quad (12)$$

The implicit risk-free rate is constant. It decreases as consumption volatility increases. A higher consumption variance σ^2 implies that, other things equal, the expected marginal utility of consumption rises more rapidly over time. Note that this expression is consistent with Obstfeld (1994b).¹²

¹² However, van Wincoop (1994, 1999) and Athanasoulis (2000) refer to $r = \beta - n + \rho \left(\mu - \frac{1}{2} \cdot \rho \sigma^2 \right)$ as the implicit risk-free interest rate.

3.3 Alternative stochastic process

In this section, we derive the two measures τ and p of welfare gains under alternative stochastic processes. We consider the cases where (per-capita) consumption follows mean-reverting and stationary processes.

3.3.1 Mean-reverting process

We consider an alternative stochastic process where (log of) per-capita consumption is assumed to follow a geometric Ornstein-Uhlenbeck process.

$$c_0; \frac{dc_t}{c_t} = (\mu - \gamma \cdot \text{Log}c_t) \cdot dt + \sigma \cdot dZ \quad \text{with } \gamma > 0 \quad (\text{SP7})$$

where $dZ = N(0, dt)$ is the standard Brownian motion. $\text{Log}c$ is a mean-reverting process such that if $\text{Log}c > \left(\mu - \frac{1}{2} \cdot \sigma^2\right) / \gamma$, then $E_t(d\text{Log}c) < 0$ and if $\text{Log}c < \left(\mu - \frac{1}{2} \cdot \sigma^2\right) / \gamma$, then $E_t(d\text{Log}c) > 0$. Note that

$$E_0[\text{Log}c_t] = \frac{\mu - \frac{1}{2} \cdot \sigma^2}{\gamma} + \left(\text{Log}c_0 - \frac{\mu - \frac{1}{2} \cdot \sigma^2}{\gamma} \right) \cdot e^{-\gamma t} \quad (13)$$

In this section, for reasons of tractability, we assume that $\rho = 1$ such that $u(c_t) = \text{Log}c_t$ and $\bar{\mu} = \mu - \frac{1}{2} \cdot \sigma^2$. We also assume that $n = \beta$. A special case is when population remains constant ($n = 0$) and the rate of time preference is zero ($\beta = 0$).¹³ Under these assumptions, expected utility at time 0 is

$$V_{OU} = E_0 \left[\int_0^H \text{Log}c_t \cdot dt \right] \quad (14)$$

Proposition 6. *When (per-capita) consumption follows the geometric Ornstein-Uhlenbeck (OU) stochastic process (SP7), expected utility at time 0 is given by*

$$V_{OU}(c_0, \mu, \sigma^2, \gamma, H) = \frac{1}{\gamma} \cdot \bar{\mu} \cdot H + \left(\text{Log}c_0 - \frac{1}{\gamma} \cdot \bar{\mu} \right) \cdot \frac{1 - e^{-\gamma H}}{\gamma} \quad (15)$$

$$\text{where } \bar{\mu} = \mu - \frac{1}{2} \cdot \sigma^2$$

As when (per-capita) consumption follows a geometric Brownian motion with drift, expected utility at time 0 increases with initial consumption $\left(\frac{\partial V_{OU}}{\partial c_0} > 0 \right)$ and $\bar{\mu}$ $\left(\frac{\partial V_{OU}}{\partial \bar{\mu}} > 0 \right)$ which implies that higher consumption growth and lower volatility are associated with higher expected utility.

¹³ Barro and Sala-i-Martin (1995) mention that Ramsey (1928) assumed $\beta = 0$ and interpreted the optimizing agent as a social planner, rather than a competitive household while the discounting of utility for future generations ($\beta > 0$) was, according to Ramsey, "ethically indefensible". (Barro and Sala-i-Martin, 1995, p.61).

Again, we want to assess the potential welfare gains of policies that affect both consumption growth and variance. We assume that these policies do not affect the mean-reverting coefficient. More specifically we assume that once these policies are implemented, consumption follows the stochastic process (SP8)

$$c_0; \frac{dc_t}{c_t} = (\mu' - \gamma \cdot \text{Log} c_t) \cdot dt + \sigma' \cdot dZ' \quad \text{where } dZ' = N(0, dt) \text{ and } \gamma > 0 \quad (\text{SP8})$$

To ensure that expected utility improves under (SP8), we assume that $\bar{\mu}' > \bar{\mu}$.

Proposition 7. *The welfare gains τ_{OU} and p_{OU} of policies that modify the stochastic process followed by consumption from (SP7) to (SP8) are given by*

$$\tau_{OU} = \bar{\mu}' - \bar{\mu} \quad (16)$$

$$\text{Log}(1 + p_{OU}) = \frac{1}{\gamma} \cdot \left(\frac{\gamma \cdot H}{1 - e^{-\gamma \cdot H}} - 1 \right) \cdot (\bar{\mu}' - \bar{\mu}) \quad (17)$$

Interestingly, the measure τ_{OU} of welfare gains remains identical to that when per-capita consumption follows a geometric Brownian motion with drift ($\tau_{OU} = \tau$) (Equation (6) remains valid when $\rho = 1$) while the measure p_{OU} under OU process differs from the measure p under Brownian motion with drift (set $\beta = n$ in Equation (8)). Interestingly, the measure p_{OU} depends on the mean-reverting coefficient γ . It can also be checked that when $\gamma = 0$ (per-capita consumption follows a Brownian motion with drift), the measure p_{OU} of welfare gains is given by

$$\text{Log}(1 + p_{OU}) = \frac{1}{2} \cdot H \cdot (\bar{\mu}' - \bar{\mu}) \quad (18)$$

which can be obtained from Equation (8) taking the limit as $\rho \rightarrow 1$. Graph 3 shows the dependency of the measure p_{OU} of welfare gains on the mean-reverting coefficient using Equation (16).

Graph 3: Welfare gain and mean-reverting coefficient

Hence, to assume that (log of) consumption follows a Brownian motion with drift ($\gamma = 0$) whereas the true process is O-U ($\gamma > 0$), leads to undervalue the measure p of welfare gains. The undervaluation increase with γ . This finding is consistent with van Wincoop (1999) who discusses the role of alternative stochastic processes on gains from risk sharing using a discrete time approach. When γ is large ($\gamma \gg 0$), the measure of welfare gains under O-U process $p_{OU}(\gamma \gg 0)$ is related to that under Brownian motion with drift $p_{OU}(\gamma = 0)$ by

$$p_{OU}(\gamma \gg 0) = p_{OU}(\gamma = 0) \cdot [2 + p_{OU}(\gamma = 0)] \quad (19)$$

3.3.2 Stationary process

We turn to the case where (log of) per-capita consumption follows a trend stationary process.

$$c_0; Logc_t = Logc_0 + \mu \cdot t + \sigma \cdot G(t) \quad (SP9)$$

where $G(0) \equiv 0$ and $G(t) = N(0,1)$ is distributed Gaussian with mean 0 and variance 1. This was the stochastic process assumed by Lucas in his seminal 1987 paper (Lucas, 1987).

Proposition 8. *When (per-capita) consumption follows the trend stationary stochastic process (SP9), expected utility at time 0 is given by*

$$V_S(c_0, \mu, \sigma^2, \rho, H) = \frac{c_0^{1-\rho}}{1-\rho} \cdot \frac{1-e^{-(\beta-n+(\rho-1)\cdot\mu)H}}{\beta-n+(\rho-1)\cdot\mu} \cdot e^{\frac{1}{2}(\rho-1)^2\sigma^2} - \frac{1}{1-\rho} \cdot \frac{1-e^{-(\beta-n)H}}{\beta-n} \quad (20)$$

$F(\mu, \rho) \equiv \beta - n + (\rho - 1) \cdot \mu$ acts like a discount rate for this economy. We assume that $F(\mu, \rho) > 0$. Consequently, when (per-capita) consumption follows a trend stationary stochastic process, expected utility at time 0 increases with initial consumption c_0 $\left(\frac{\partial V_S}{\partial c_0} > 0\right)$ while it increases with μ $\left(\frac{\partial V_S}{\partial \mu} > 0\right)$. However, expected utility decreases (resp. increases) with consumption variance when $\rho > 1$ (resp. $\rho < 1$).

Again, we want to assess the potential welfare gains of policies that affect both consumption growth and variance. More specifically we assume that once these policies are implemented, consumption follows the stochastic process (SP10)

$$c_0; Logc_t = Logc_0 + \mu' \cdot t + \sigma' \cdot G'(t) \quad (SP10)$$

To ensure that expected utility improves under (SP10), we assume that $V_S(c_0, \mu', \sigma'^2, \rho, H) > V_S(c_0, \mu, \sigma^2, \rho, H)$.

Proposition 9. *The welfare gains τ_S and p_S of policies that modify the stochastic process followed by consumption from (SP9) to (SP10) are given by*

$$\frac{1}{2} \cdot (\rho - 1)^2 \cdot (\sigma^2 - \sigma'^2) = \text{Log} \left[\frac{\frac{1-e^{-(\beta-n+(\rho-1)\cdot\mu')H}}{\beta-n+(\rho-1)\cdot\mu'}}{\frac{1-e^{-(\beta-n+(\rho-1)\cdot(\mu+\tau_S)H}}{\beta-n+(\rho-1)\cdot(\mu+\tau_S)}} \right] \quad (21)$$

$$\text{Log}(1+p_S) = \frac{1}{2} \cdot (\rho - 1) \cdot (\sigma^2 - \sigma'^2) + \frac{1}{\rho - 1} \cdot \text{Log} \left[\frac{\frac{1-e^{-(\beta-n+(\rho-1)\cdot\mu)H}}{\beta-n+(\rho-1)\cdot\mu}}{\frac{1-e^{-(\beta-n+(\rho-1)\cdot\mu')H}}{\beta-n+(\rho-1)\cdot\mu'}} \right] \quad (22)$$

Both measures of welfare gains depend on consumption drifts and variances (μ, σ) and (μ', σ') of the initial and final stochastic processes respectively and on the time horizon H . When the drifts remain unchanged ($\mu' = \mu$), the second term in Equation (21) disappears and the measure p_S of

welfare gains becomes independent of the horizon while the measure τ_S still depends on H . Note that when $\mu' = \mu$, $\sigma' = 0$ and p_S is small ($p_S \approx 0$), $\text{Log}(1 + p_S) \approx p_S$ and

$$p_S \approx \frac{1}{2} \cdot (\rho - 1) \cdot (\sigma^2 - \sigma'^2) \quad (23)$$

which is similar to the relation derived by Lucas (1987, Eq. (8), p.26).¹⁴ However, even when $\mu' = \mu$, the measure τ_S still depends on the horizon. However, when $H = \infty$ and $(\mu', \sigma') \approx (\mu, \sigma)$, there exists a closed form solution for τ_S given by

$$\tau_S \approx (\mu' - \mu) + \frac{1}{2} \cdot (\rho - 1) \cdot (\sigma^2 - \sigma'^2) \cdot [\beta - n + (\rho - 1) \cdot \mu'] \quad (24)$$

Note also that

$$\frac{E_0 \left[c_{(1+p_S)c_0, \mu, \sigma}(t) \right]}{E_0 \left[c_{c_0, \mu, \sigma}(t) \right]} = 1 + p_S \quad (25)$$

where $c_{c_0, \mu, \sigma}(t)$ and $c_{(1+p_S)c_0, \mu, \sigma}(t)$ represent per-capita consumption under (SP9) and (SP10) respectively. Consequently, as Lucas mentioned (1987, p. 25), the compensating increase in current consumption results in an identical increase in expected consumption on all dates and in all states of nature. However, p_S does generally depend on a number of parameters including the horizon except when $\mu = \mu'$ which was the case Lucas considered.

Since this model has been used empirically, it is useful to determine the implicit risk-free rate that corresponds to this economy in order to calibrate the model.

Proposition 10. *In this economy, the implicit risk-free interest rate is stochastic with mean*

$$E(R_{t,t+1}) = \beta - n + \rho \cdot \left(\mu - \frac{1}{2} \cdot \rho \cdot \sigma^2 \right) \quad (26)$$

This outcome is not surprising. When the contemporaneous shock $G(t)$ is positive and consumption follows a trend-stationary process, individuals expect future one-period consumption to decrease to the trend-level and consequently expected marginal utility to increase.

3.4 Alternative preference

We now consider Kreps-Porteus preferences, which allow for a separate rate of relative risk aversion ρ and elasticity of intertemporal substitution $\frac{1}{\psi}$. We assume that (log of) per-capita consumption follows a geometric Brownian motion with drift identical to (SP1). In this section, we assume $H = \infty$. The continuous time version of Kreps-Porteus preferences that we consider implies that U follows the stochastic integral (Svensson, 1989; Duffie and Epstein, 1989, 1992)

$$dU = \left(\frac{1}{1-\psi} \cdot \frac{(\beta-n) \cdot U^{1-\psi} - c^{1-\psi}}{U^{-\psi}} + \frac{1}{2} \cdot \rho \cdot \sigma^2 \cdot U \right) \cdot dt + \sigma \cdot U \cdot dZ \quad (\text{SP11})$$

where $\frac{1}{\psi}$ is the intertemporal elasticity of substitution.

¹⁴ In fact, Lucas found the coefficient $(\rho - 1)$ to be ρ instead. We were not able to reconcile this difference.

Proposition 9. When (per-capita) consumption follows Kreps-Porteus preferences, expected utility at time 0 is given by

$$V_{KP}(c_0, \mu, \sigma^2, n) = [\beta - n + (\psi - 1) \cdot \bar{\mu}]^{\frac{1}{\psi-1}} \cdot c_0 \quad (27)$$

where

$$\bar{\mu} = \mu - \frac{1}{2} \cdot \rho \cdot \sigma^2$$

As mentioned in van Wincoop (1994), note that when $\psi = \rho$, and we apply the transformation

$$\frac{U^{1-\rho}}{1-\rho} \text{ to utility, this is the same equation as (4) with } H = \infty.$$

We want to assess the potential welfare gains of policies that, once implemented, lead to a change in the consumption stochastic process to (SP2).

Proposition 12. The welfare gains τ_{KP} and p_{KP} of policies that modify the stochastic process followed by consumption from (SP1) to (SP2) are given by

$$\tau_{KP} = \bar{\mu}' - \bar{\mu} \quad (28)$$

$$\text{Log}(1 + p_{KP}) = \frac{1}{\psi - 1} \cdot \text{Log} \left[\frac{\beta - n + (\psi - 1) \cdot \bar{\mu}'}{\beta - n + (\psi - 1) \cdot \bar{\mu}} \right] \quad (29)$$

Note the analogy with Proposition 2 when $\psi = \rho$. Under Kreps-Porteus preferences, the measure τ_{KP} of welfare gains remains identical to that under CRRA preferences. It depends only on the rate of relative risk aversion ρ but does not depend on the elasticity of intertemporal substitution $\frac{1}{\psi}$. However, measure p_{KP} of welfare gains does depend on the elasticity of intertemporal substitution as well as the risk-aversion coefficient. This result is identical to Obsfeld (1994b), who derives the measure p_{KP} of welfare gains in a discrete time version of the current model and concludes “unless risk aversion and intertemporal substitutability are carefully separated, attempts to measure the welfare costs of changes in consumption risk can yield misleading conclusions about the role of risk-aversion.” Graph 5 describes measure p_{KP} of welfare gains as a function of ψ holding the rate of relative risk aversion constant at 4. Measure p_{KP} of welfare gains become smaller if we lower the intertemporal substitution elasticity from 1/4 to 1/10 but they become much larger if we raise the intertemporal substitution elasticity from 1/4 to 1. These results are identical those derived in van Wincoop (1994).

Graph 5: Welfare gain and elasticity of inter-temporal substitution

Van Wincoop (1994) mentions that using consumption data for nineteen US states, a point estimate of the elasticity of intertemporal substitution of 1 is found, with a relatively small standard error of 0.3. However, note that measure p_{KP} of welfare gains can be rewritten

$$\text{Log}(1 + p_{KP}) = \frac{1}{\psi - 1} \cdot \text{Log} \left[1 + (\psi - 1) \frac{\bar{\mu}' - \bar{\mu}}{\beta - n + (\psi - 1) \cdot \bar{\mu}} \right] \quad (30)$$

which implies

$$\text{Log}(1 + p_{KP}) \xrightarrow{\psi \rightarrow 1} \frac{\bar{\mu}' - \bar{\mu}}{\beta - n} \quad (31)$$

Consequently, gains from risk sharing as measured by p_{KP} are very sensitive to the difference $(\beta - n)$ when ψ tends to 1 which makes it difficult to use this measure to evaluate welfare gains with precision.

4. Welfare gains from risk sharing: empirical results

Empirical studies of the welfare gains from international risksharing have used two approaches. Under the consumption-based approach, papers have typically assumed that country-specific (per-capita) consumption path follow exogenously given stochastic processes with *identical means*. This literature assumes that risksharing allow countries to reduce consumption variance while maintaining the same consumption growth as before risksharing ($\mu' = \mu$). Consequently, this literature has then focused on the question: What are the potential welfare gains of policies that would reduce the standard deviation of consumption from σ to σ' where σ' is derived from paper-specific consideration. However, the assumption of common consumption growth rate is at odds with the hypothesis of conditional convergence supported by empirical studies.¹⁵ The alternative portfolio-based approach has sought to derive the parameters (μ', σ') of consumption dynamics after risksharing from a partial equilibrium CAPM model. Earlier studies on the gains from international risksharing focused on the benefits from diversifying across international equity markets (Grubel, 1968; Levy and Sarnat, 1970) while Shiller (1993) and Shiller and Athanasoulis (1997) derive the welfare gains that would result from trading hypothetical country-specific GDP-linked securities.

The objective of this paper is not to derive (μ', σ') from an optimization problem. Instead, it is to assess the welfare gains from international risksharing using the proposed alternative measure τ . As we have shown, if (per-capita) consumption dynamics is non-stationary, the measure τ of risksharing is robust to alternative specifications of preferences (from constant relative risk aversion (CRRA) preferences to Kreps-Porteus preferences) and stochastic processes (from geometric Brownian processes to Orstein-Uhlenbeck mean-reverting processes) for consumption. It has also the attractive feature of being independent of the horizon. These features are in sharp contrast with the measure p previously used in the literature that is sensitive to the horizon and alternative specifications of utility and stochastic process.

Table 2 shows a literature comparison of welfare gains from risksharing. Assumptions regarding the stochastic processes and the underlying parameters are taken directly from the papers. Implicit risk-free interest rates are derived from Proposition 5 when the consumption stochastic process is assumed to be stationary while for non-stationary processes it is derived from Proposition 10. Welfare gains are derived using Table 1: the base case scenario corresponds to non-stationary processes while alternative 2 corresponds to stationary processes. Eventually, we report the measure p of welfare gains as reported in the papers themselves. With the exception of Athanasoulis and van Wincoop (2000), the papers that assume non-stationary processes for consumption report a measure p of welfare gains in the range 0.45-2.00 for a ratio of 1 to 4

¹⁵ For example, van Wincoop (1994) who applies this method to derive welfare gains notes that can be achieved through risksharing among twenty OECD countries mentions that average per capita consumption growth ranges from 0.3 percent in Ireland to 4.1 percent in Iceland. To get around this problem van Wincoop sets μ' equal to the unweighted average per capita consumption growth rate among the twenty OECD countries. See Barro and Sala-i-Martin (1995) for more on conditional convergence.

between the lowest and highest value.¹⁶ However, had they used the alternative measure τ , the same papers would have derived welfare gains in a more narrow range of 0.033-0.072 percent with an average of 0.05 percent.

Table 2: Welfare gains from risksharing: Consumption-based approach

	Lucas (1987) ^{2/}	Cole and Obstfeld (1991) ^{3/}	Obstfeld (1994a)	Van Wincoop (1994)	Tesar (1995) ^{4/}	Lewis (1996)	Athanasoulis and van Wincoop (2000) ^{5/}
Stochastic process ^{1/}	S	NS	NS	NS	NS	NS	NS
Parameters							
$\mu = \mu'$	0.0300	0.0270	0.0202	0.0170	0	0.0234	0.0187
σ	0.0130	0.0250	0.0163	0.0219	0.0295	0.0191	0.0503
σ'	0	0.0199	0	0.0110	0.0172	0.0153	0.0277
ρ	5	4	4	4	2	5	3
$e^{-\beta}$	0.95	0.98	0.95	0.99	0.98	0.98	1.03
n	0	0	0	0	0	0	0
H	∞	50	∞	∞	100	∞	35
Risk-free interest rate (percent)							
R	19.9	12.2	12.9	7.3	1.8	13.2	1.3
R'	20.1	12.4	13.2	7.7	1.9	13.4	2.3
Derived welfare gains (percent)							
p	0.034	0.45	0.48	1.22	2.00	0.30	4.62
τ	0.0058	0.046	0.053	0.072	0.057	0.033	0.26
Reported welfare gains (percent)							
p	0.042	0.24	0.36	1.30	1.70	0.080	1.52

1/ S: stationary; NS: non-stationary

2/ Lucas uses a different relation for p (See footnote 11). We set $G(t) = 0$ to derive the risk-free interest rate (See Proposition 9).

3/ The low welfare gains derived by Cole and Obstfeld (1991) in a two-country model is driven by the specific four-state Markov process that they consider. See Athanasoulis and van Wincoop (2000) for more details.

4/ Tesar (1995) reports $\frac{p}{100}$ (See footnote 9).

5/ Data correspond to 21 OECD countries (Table 2, p. 489).

Based on our previous analysis, we now turn to our own estimates of the welfare gains from risksharing for a representative US consumer (Table 3). Empirically, the degree of risk-aversion is often estimated to be quite small in the range of 2 to 4.¹⁷ In this paper, we set $\rho = 3$ and take $e^{-\beta} = 0.98$. From the Penn World Tables (Summers and Heston (1991)), we derive the other parameters including consumption growth ($\mu = 0.0189$) and variance ($\sigma = 0.0157$). We assume that international risksharing results in a complete elimination of consumption volatility (although alternative assumptions could be easily worked out). We consider alternative scenarios where international risksharing does not lead to an increase in consumption growth (Scenario 1) or leads to 0.5 percent or 1 percent increase (Scenario 2 and 3 respectively). We use the relations reported in Table 1 (Base Case) to derive the measure τ of welfare gains both in percentage points of consumption growth and in US dollar equivalent. As Table 3 shows, eliminating all consumption volatility would represent 0.037 percentage points of consumption growth equivalent to US\$12 per annum per person in 1999 to a representative US consumer. This outcome is similar to that derived by Lucas in 1987 (US\$8.50). This is a very small amount. Consequently, if the only benefits brought about by international risksharing is a decrease in volatility, then even minor

¹⁶ The outcome of 4.62 reported by Athanasoulis and van Wincoop (2000) results from an unrealistically high assumption for σ .

¹⁷ However, Mehra and Prescott (1985) find that the degree of risk aversion must be large in order to generate a risk premium on equity returns consistent with US data.

impediments to full risksharing can wipe out these small gains. However, if international risksharing also leads to an increase in consumption growth, then Table 2 shows that any additional ½ percent increase in consumption growth is worth about US\$160 per annum to a representative US consumer. Whether this amount is high or low is a matter of judgment (and personal wealth) but this conclusion clearly shows that what is required to pursue the analysis and reach stronger conclusions regarding the benefits from risksharing is to build a model that endogenizes both initial and final consumption stochastic processes which would allow to derive (μ', σ') from an individual optimization problem.

Table 3: Welfare gains from risk-sharing (United States)

Parameters^{1/}: $\mu = 0.0189$, $\sigma = 0.0157$, $\rho = 3$, $\exp(-\beta) = 0.98$, $n = 0.0116$

	Scenario 1	Scenario 2	Scenario 3
Parameters			
μ'	0.0189	0.0239	0.02891
σ'	0	0	0
Risk-free interest rate (percent)			
R	6.4	6.4	6.4
R'	6.5	8.0	9.5
Welfare gains (percent)			
τ (percent)	0.037	0.54	1.04
US\$ dollars equivalent (1999)	11.8	171.2	330.6
$p(H = 35)$ (percent)	0.48	7.0	13.4
$p(H = \infty)$ (percent)	0.80	11.1	20.6

1/ Parameters are based on consumption data from Summer and Heston 1991 (Penn World Tables), for the period 1960-1997.

The analysis of the paper would be equally applicable to derive the welfare gains from policies that affect the growth and variability of consumption born at the individual level in a model with heterogeneous agents. For example, the standard deviation of annual consumption growth based on PSID data seems to be about 30 percent (Altonji and Siow, 1987). Using the framework proposed in this paper, a complete elimination of this volatility is worth about US\$4,400 per annum to a representative consumer!

5. Summary and conclusions

This paper examines the welfare gains from risksharing in a number of different models and suggests that differences in outcome have resulted from the use of a measure which is very sensitive to a set of parameters and alternative specifications of utility and stochastic processes when uncertainty results from non-stationary processes. However, the difference among papers would not have been so large had the authors used the alternative measure we proposed. Using the latter measure, the welfare gains that would result from a complete elimination of consumption volatility do not appear large enough to outweigh even small transaction or information costs. However, gains could become substantial if international risksharing also lead to an increase in consumption growth.

The paper proposed an alternative measure to assess the welfare gains that would result from adopting policies which modify the (per-capita) consumption stochastic process from (SP1) to (SP2). However, the paper has taken both stochastic processes as exogenous. We believe that future research should seek to apply Merton's (1990) complete-market general equilibrium model

in continuous time (GEMCT) to a multi-country framework to endogenize the stochastic processes. We also believe that the consumption- and portfolio-based approaches can be interpreted within Merton's framework in which the continuous-time version of the Sharpe-Lintner-Mossin CAPM is obtained. But even if this is done, other questions will remain. First, a GEMCT would probably assume that all sources of disturbances are country-specific. Domestic shocks such as monetary or fiscal policy shocks would also need to be considered. Second, a GEMCT would assume that the production dynamics are observable and cannot be affected by domestic policies or coalitions of countries. The assumption that individuals take the "production" side of the economy as given would need to be relaxed if countries are able to "self-insure" by adjusting labor supply, for example. In fact, much research remains to be done to incorporate human capital and incentive in this framework.¹⁸ Third, the model assumes that uncertainties follow a geometric Brownian motion with drift. However, means and variance may evolve with time. For example, a complete model would have to be consistent with the empirical findings of conditional convergence.

¹⁸ Bodie et Al (1992) introduces labor in a continuous time model.

Appendix

Proof of Proposition 1

Per-capita consumption at any time t is given by

$$c_t = c_0 \cdot e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma[Z(t) - Z(0)]}$$

Expected utility is

$$V(c_0, \mu, \sigma^2, \rho, H) = E_0 \left[\int_0^H e^{(n-\beta)t} \cdot \frac{c_t^{1-\rho} - 1}{1-\rho} \cdot dt \right]$$

Substituting c_t and taking expectations

$$V(c_0, \mu, \sigma^2, \rho, H) = \frac{c_0^{1-\rho}}{1-\rho} \cdot \left[\int_0^H e^{(n-\beta)t} \cdot e^{(1-\rho)\left(\mu - \frac{1}{2}\rho\sigma^2\right)t} \cdot dt \right] - \frac{1}{1-\rho} \cdot \left[\int_0^H e^{(n-\beta)t} \cdot dt \right]$$

which can be rewritten

$$V(c_0, \mu, \sigma^2, \rho, H) = \frac{c_0^{1-\rho}}{1-\rho} \cdot \frac{1 - e^{[\beta - n + (\rho-1)\bar{\mu}]H}}{\beta - n + (\rho-1) \cdot \bar{\mu}} - \frac{1}{1-\rho} \cdot \frac{1 - e^{-(\beta-n)H}}{\beta - n}$$

where

$$\bar{\mu} \equiv \mu - \frac{1}{2} \cdot \rho \cdot \sigma^2$$

Proof of Proposition 2

$$\text{Define } \forall F > 0, \forall H \geq 0, G(F, H) = \frac{1 - e^{-F \cdot H}}{F}$$

$$\text{Note that } V(c_0, \mu + \tau, \sigma^2, \rho, H) = V(c_0, \mu', \sigma^2, \rho, H) \Rightarrow G[F(\bar{\mu} + \tau, \rho), H] = G[F(\bar{\mu}', \rho), H]$$

$$\forall H > 0; \frac{\partial G}{\partial F}(F, H) < 0 \text{ implies that } G \text{ is a strictly decreasing function of } F. \text{ Hence,}$$

$$G[F(\bar{\mu} + \tau, \rho), H] = G[F(\bar{\mu}', \rho), H] \Rightarrow F(\bar{\mu} + \tau, \rho) = F(\bar{\mu}', \rho) \text{ which implies } \tau = \bar{\mu}' - \bar{\mu}.$$

Proof of Proposition 3

$$\frac{dp}{1+p} = \frac{1}{\rho-1} \cdot \left[\frac{F(\bar{\mu}, \rho)}{e^{F(\bar{\mu}, \rho) \cdot H} - 1} - \frac{F(\bar{\mu}', \rho)}{e^{F(\bar{\mu}', \rho) \cdot H} - 1} \right] \cdot dH$$

By assumption $\bar{\mu}' > \bar{\mu}$ hence $F(\bar{\mu}', \rho) > F(\bar{\mu}, \rho)$ (resp. $F(\bar{\mu}', \rho) < F(\bar{\mu}, \rho)$) when $\rho > 1$ (resp.

$\rho < 1$). The proposition results from the fact that the function $f(F) = \frac{F}{e^{F \cdot H} - 1}$ is strictly decreasing.

Proof of Proposition 4

Write $\frac{dp}{1+p} = \frac{1-(F' \cdot H + 1) \cdot e^{-F' \cdot H}}{F' \cdot (1 - e^{-F' \cdot H})} \cdot d\bar{\mu}'$ where $F' = F(\bar{\mu}', \rho)$. By assumption, $F' > 0$. The proposition results from $e^{F' \cdot H} > 1 + F' \cdot H$.

Proof of Proposition 5

Define $R_{t,t+h}$ as the risk-free interest rate (assumed known at time t) during the period $[t, t+h]$. Euler equation is

$$\frac{du}{dt}(c_t) = e^{-(R_{t,t+h} + n - \beta) \cdot h} \cdot E_t \left[\frac{du}{dt}(c_{t+h}) \right]$$

which can be rewritten

$$E_t \left(\frac{c_{t+h}^{-\rho}}{c_t^{-\rho}} \right) = e^{-(R_{t,t+h} + n - \beta) \cdot h}$$

Then, the proposition directly results from

$$c_{t+h} = c_t \cdot e^{\left(\mu - \frac{1}{2} \sigma^2 \right) h + \sigma [Z(t+h) - Z(t)]}$$

Proof of Proposition 6

Immediate from Equation (13).

Proof of Proposition 7

Immediate.

Proof of Proposition 8

Per-capita consumption at any time t is given by

$$c_t = c_0 \cdot e^{\mu \cdot t + \sigma \cdot G(t)}$$

Expected utility is

$$V_S(c_0, \mu, \sigma^2, \rho, H) = E_0 \left[\int_0^H e^{(n-\beta)t} \cdot \frac{c_t^{1-\rho}}{1-\rho} \cdot dt \right]$$

Substituting c_t and taking expectations

$$V_S(c_0, \mu, \sigma^2, \rho, H) = \frac{c_0^{1-\rho}}{1-\rho} \cdot \left[\int_0^H e^{(n-\beta)t} \cdot e^{(1-\rho)\mu t + \frac{1}{2}(1-\rho)^2 \sigma^2 t} \cdot dt \right]$$

which can be rewritten

$$V_S(c_0, \mu, \sigma^2, \rho, H) = \frac{c_0^{1-\rho}}{1-\rho} \cdot \frac{1 - e^{[\beta - n + (\rho-1)\mu] \cdot H}}{\beta - n + (\rho-1) \cdot \mu} \cdot e^{\frac{1}{2}(\rho-1)^2 \sigma^2 H}$$

Proof of Proposition 9

Immediate using Equation (19).

Proof of Proposition 10

Follow the proof of Proposition 5 and write

$$\frac{c_{t+h}^{-\rho}}{c_t^{-\rho}} = e^{-\rho \cdot \mu \cdot h - \rho \cdot \sigma \cdot [G(t+h) - G(t)]}$$

Since $G(t)$ is known a time t

$$R_{t,t+h} = \beta - n + \rho \cdot \left(\mu - \frac{1}{2} \cdot \rho \cdot \sigma^2 \right) + \rho \cdot \sigma \cdot \frac{G(t)}{h}$$

Proof of Proposition 11

Immediate.

Proof of Proposition 12

Immediate.

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